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THE RACAH ALGEBRA FOR GROUPS WITH TIME REVERSAL SYMMETRY

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Except where referenced otherwise, the author certifies that the material in this thesis was performed by him.

15th. January 1981

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This thesis is in two parts. Part one is largely theory and is based on two papers* accepted for publication by the Journal of Mathematical Physics. Part two consists of tables of the various coefficients discussed here-in for the grey double point groups.

Attached to this thesis are two papers as supportive material which have already been published**.

* J.D. Newmarch and R.M. Golding, 'The Racah Algebra for Groups with Time Reversal Symmetry', J.Math. Phys. (to appear)

J.D. Newmarch and R.M. Golding, 'The Racah Algebra for Groups with Time Reversal Symmetry II', J. Math. Phys. (to appear)

** R.M. Golding and J.D. Newmarch, 'Symmetry Coupling Coefficients for the double Groups C_n^*, D_n^* and T^*', Mol. Phys. 33 1301 (1977)

J.D. Newmarch and R.M. Golding, 'Ladder Operators for some Spherically Symmetric Potentials in Quantum Mechanics' AM.J. Phys. 46 658 (1978)

ABSTRACT

The aim of this thesis is to construct and determine the properties of the Racah algebra for compact groups of linear/antilinear operators including the Wigner time reversal operator θ (the grey groups). Here θ is an operator which squares to the positive or negative identity, which commutes with all other operators of the group, and is antilinear

$$\theta(\alpha|\psi> + \beta|\phi>) = \alpha^*\theta|\psi> + \beta^*\theta|\phi>$$

This problem has a clear physical basis as for non-magnetic, paramagnetic and diamagnetic crystals in chemistry, θ commutes with the Hamiltonian in the absence of an external magnetic field. Time reversal symmetry is also used in elementary particle physics. The use of a symmetry group for a quantum system can only be fully exploited by extensive use of Racah algebra methods, particularly the Wigner-Eckart theorem.

The mathematical foundation for considering this problem lies in the antilinearity of θ. The very extensive range of group representation theory rests solidly on the assumption that an abstract group is isomorphic to a group of linear operators and hence to include an antilinear operator means that representation theory is not applicable.

One common method of dealing with this difficulty has been to avoid it – that is, to use only the linear subgroup for the major part of any calculation and then consider the effect of time reversal later.

However, not only does this fail to utilize the complete symmetry, it may even give rise to errors if applied injudiciously. A second method, common in chemistry, is to lump pairs of complex conjugate representations together. This is a dangerous path - for as well as causing mathematical problems it also gives incorrect degeneracies for quite simple systems.

The approach by Wigner was to start all over again and to look anew at the multiplication properties of the matrices of linear and antilinear operators. He called the result 'corepresentation theory' and it is the proper framework to deal with groups of linear and antilinear operators. As from the very beginning it differs from representation theory, it is necessary to rework all results from representation theory to determine whether or not they still hold for corepresentations.

The first part of this thesis is concerned with the theory of the Racah algebra for the corepresentations of compact grey groups. From the above, the subject matter is quite straightforward: to take as much as possible of the general Racah algebra for compact linear groups and rework it for the grey groups.

The primary results are:

(a) n-jm symbols may be found with essentially the same properties as their counterparts in representation theory.

(b) If the n-jm symbols of the linear subgroup are known, then they can easily be induced up to give the symbols for the grey group.

(c) The Wigner-Eckart theorem and Racah's lemma both hold for a grey group using the 3jm symbol.

(d) The reduction of two irreducible corepresentations to a third shows a marked difference to the corresponding reduction for representations. This loosens the bond between the coupling coefficient and the 3jm symbol, and also between the recoupling and n-j symbols.

The second part gives various tables and notations for use in the grey double point groups.

It is shown in this thesis that a well-developed Racah algebra exists for all compact grey groups. In many places it parallels the algebra for linear groups but there are some important divergences. By and large the algebra is as tractable as the linear group algebra, but some annoying problems remain in the area of the n-j symbols.

PART ONE:

THEORY

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CHAPTER ONE : INTRODUCTION

1.1 The Racah Algebra for SU(2)

The history of the mathematical techniques generally subsumed under the heading of 'Racah algebra' is long and varied. From the initial introduction by Condon and Shortley [1] of the Clebsch-Gordan coefficients (which in this thesis are called coupling coefficients), Racah [2-5] developed a powerful array of methods for dealing with many electron calculations in a spherically symmetric potential. This symmetry allows the algebra to be treated in terms of the representation theory of the pure rotation group SO(3) (or if spin is included the unimodular group SU(2)) [6]. This interpretation is as follows:

(a) The set of vectors $\{|jm\rangle : m = -j ,\ldots\ldots, j\}$ form a basis for an irreducible representation j of SU(2) (or of SO(3) if j is restricted to be integral).

(b) If two non-interacting electrons with angular momenta j_1 and j_2 respectively are coupled, the total angular momentum j_3 takes the values $|j_1 - j_2| ,\ldots\ldots, j_1 + j_2$. Group theoretically we are reducing the direct (inner) product $j_1 \otimes j_2$ to a sum of IRs through a Clebsch-Gordan series

$$j_1 \otimes j_2 = \sum_{j_3} \otimes C_{12}^{3} j_3$$

The coefficients C_{12}^{3} are termed Clebsch-Gordan coefficients, and the value of a coefficient is often called the multiplicity of j_3 in $j_1 \otimes j_2$.

(c) Whilst the Clebsch-Gordan series can be used to give qualitative results such as simple selection rules, quantitative calculations require a more detailed description of the direct product reduction. The matrix of the direct product $j_1 \otimes j_2$ may be block diagonalized by a unitary transformation, which gives a corresponding transformation of the basis vectors:

$$|j_1m_1\rangle|j_2m_2\rangle = \sum_{j_3m_3} \langle j_1j_2j_3m_3|j_1j_2m_1m_2\rangle|j_3m_3\rangle$$

with inverse

$$|j_3m_3\rangle = \sum_{m_1m_2} \langle j_1j_2m_1m_2|j_1j_2j_3m_3\rangle|j_1m_1\rangle|j_2m_2\rangle$$

The elements of this unitary transformation will be called coupling coefficients.

(d) Algebraic expressions exist for the coupling coefficients [2,6,7] but they are extremely cumbersome. A variety of related coefficients exist which possess simple properties under permutations of the coupling, of which the two most important are probably the 3j symbol of Wigner [6] and the $\bar{V}$ coefficient of Fano and Racah [8]. These permutational properties are closely related to the associativity property of the reduction of three IRs $j_1 \otimes j_2 \otimes j_3$ to $\underset{\sim}{0}$, the identity IR of SU(2) and the behaviour of the IRs under complex conjugation or time reversal [6]

(e) The reduction of the triple product $j_1 \otimes j_2 \otimes j_3$ to a sum of IRs j (and of higher order products) is of interest due to the variety of orders of coupling which may be used. 6j , 9j ,....., symbols may be defined relating these orders, and possess the property of invariance under basis transformations.

(f) Forming eigenvectors of a Hamiltonian is only part of solving a problem in quantum mechanics. To obtain, say, the probability of electromagnetic transitions from one electronic state to another, it is necessary to calculate matrix elements of various operators. This is facilitated by the extremely important Wigner-Eckhart theorem [9,10] which starts with a definition: if a tensor T(jm) transforms under SU(2) as a basis vector $|jm\rangle$ then it forms the m_{th} component of an irreducible tensor T(j). Then for an irreducible tensor, the radial and spherical components 'factor out' to give

$$\langle j_3m_3|T(k_1q_1)|j_2m_2\rangle$$

$$= (-1)^{k_1+j_2+2j_3-m_3} \times \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{pmatrix} \langle j_3||T(k_1)||j_2\rangle$$

where the reduced matrix element $\langle j_3||T(k_1)||j_2\rangle$ represents the radial contribution. By considering various coupled tensors in coupled schemes, recouplings using the 6j and 9j symbols may be used to simplify calculations.

1.2 Groups in Physics and Chemistry

The realization that SO(3) symmetry is responsible for many of the properties of the angular momentum wavefunctions $Y_{lm}(\theta\phi)$ has led to a boom in the use of group theory in all branches of quantum mechanics. In many introductory texts (e.g. Schiff [11]) SO(4) and SU(3) symmetries are used to explain the degeneracies of the hydrogen atom and three dimensional harmonic oscillator respectively. On a wider front, group theory is indispensable for many problems. We cite:

(a) Elementary particles. SU(3) and SU(6) symmetries [12] have succeeded in bringing some measure of order to the classification of elementary particles.

(b) Atomic spectroscopy. Treating the $2 \times (2\ell + 1)$ states of an ℓ electron as basis vectors for $SU(4\ell + 2)$ gives a classification through a chain $SU(4\ell + 2) \supset G \supset G' \supset \ldots \supset SU(2) \otimes SO(3)$ for many electron states ℓ^n, the most notable being the seniority number ν of Racah [6] which corresponds to an IR of $SP(4\ell + 2)$ [13]. Although these additional quantum numbers need not be 'good' [14] they are still useful.

(c) Single crystals. Single crystals are a good approximation to regular structures and thus possess geometric symmetry groups such as K or A_5, the group of the icosahedron. One of the most striking uses of group theory was the demonstration by Jahn and Teller [15,16] that with the exception of a linear molecule, a completely regular crystal is unstable in a first order perturbation of the shape.

(d) Regular solids. Here again geometric symmetry plays an important role through the medium of space groups.

1.3 The Racah algebra for Linear Groups

The group theoretic interpretation of the Racah algebra for SU(2) suggests that the algebra can be extended to all groups and that calculations can be simplified by exploiting this symmetry. However, whilst serving as an 'ideal' model for general discussions of the algebra, SU(2) possesses so many special features which turn out to drastically simplify the algebra that generalizing it has turned out to be a non-trivial problem involving much work.

Firstly, all transformations in SU(2) are linear i.e.

$$T(a|\alpha\rangle + b|\beta\rangle) = aT|\alpha\rangle + bT|\beta\rangle$$

Representation theory is based on this property, and since most operators in quantum mechanics are linear, the vast body of mathematical literature on representations may be used. However, not all operators are linear. Wigner's time reversal operator θ [6] is a prime example of a non-linear operator and others occur in discussing magnetic materials [17]. Although mathematicians have displayed a passing interest in such operators [18] we were once informed by an algebraist that they 'are not an element of the canonical body of mathematical literature' [19].

Secondly, SU(2) is compact. This topological concept is the next best thing to finiteness and loosely implies that any result involving a summation in a finite group can be extended to a compact group via an appropriate integral [20].

In particular, every representation is equivalent to a direct sum of irreducible unitary representations. Non-compact groups do occur in physics (for example the Poincaré group [21]) and have infinite dimensional irreducible unitary representations.

Thirdly, SU(2) is a Lie group. This allows the Lie algebra su(2) generated by $\underline{J}_x$, $\underline{J}_y$ and $\underline{J}_z$ to be used as for example was originally done by Racah [2] .

These aside, two further properties of SU(2) have greatly influenced the development of Racah algebra methods. A group is called simply reducible (SR) if (a) Every IR is equivalent to its complex conjugate and (b) If every direct product is multiplicity free (MF) ie. if the Clebsch-Gordan coefficients C^{3}_{12} equal zero or one only. Wigner [22] showed that many of the properties of the Racah algebra of SU(2) hold for any simply reducible compact group.

The removal of the two SR restrictions from compact or finite groups has little consequence for the coupling coefficients : it is necessary to include a 'multiplicity label' and to recall that an IR is not always equivalent to its conjugate. It is in the construction of the 3jm and n-j symbols that difficulties arise. The most fundamental work in this area was performed in the mid-sixties by Derome and Sharp [23,24] who elegantly derived many of their properties, but for a long time chemists were primarily inspired by the work of Griffith [25] . At present there is a bewildering variety of papers, conventions and tables dealing just with the point groups [26 - 53] .

1.4 Time Reversal Symmetry

We became interested in these techniques in extending the work of Golding [26,27] on the octahedral and icosahedral double groups to the remaining double point groups T^*, D_n^* and C_n^*. Although we were successful in this [28] a number of points remained incomplete. The method we used, in common with several other authors cited was by descent in symmetry from SU(2), a method validated by Racah's lemma [4]. The Wigner time reversal operator [6] with its clearly defined action in, say, Fano-Racah standardization [8] proved useful in a purely mathematical sense.

A very common ploy in chemistry is to group a pair of complex conjugate IRs into a real representation. Mathematically this is fine, as such a representation is irreducible over the real numbers but using this as a reason for the move is physically indefensible. This increase in degeneracy is not caused by restricting the field but by time reversal being a symmetry operator of a simple electro-static Hamiltonian. If the two methods caused the same consequences it would possibly be excusable to confuse the methods, but a simple example shows that this is not true. Consider C_3^* with the Z-axis as symmetry axis. Then the two vectors $|{}^3/_2\ {}^3/_2>$ and $|{}^3/_2 - {}^3/_2>$ both carry the IR generated by

$$\Gamma(C_3^z) = -1$$

However, $\theta\,|\,{}^3/_2 \pm {}^3/_2> = \mp|\,{}^3/_2 \mp {}^3/_2>$

and it is simple to show that no linear combination of these is left invariant by θ . Hence for this real representation, the degeneracy is increased by time reversal.

Given, then, that θ is a physically important operator with non-trivial consequences and that Racah algebra techiques are very useful, it thus becomes important to determine the properties of a Racah algebra for groups containing this operator. As mentioned before, θ is anti-linear and hence representation theory in its usual form is not applicable. Wigner [6] when faced with this problem defined 'corepresentations' in analogy to representations and the general theory of corepresentations has proved to be useful particularly in the study of magnetic materials [17] .

Despite this, the development of Racah algebra methods for corepresentations is lagging far behind that for representations. A large number of papers have been published concerning the corepresentations of linear/anti-linear groups [54-61] but moving beyond that only a few papers have to our knowledge been published on the coupling coefficients [62-64] , and the Wigner-Eckart theorem has been proved [65]. We have as yet seen no discussion of the 3jm and 6j symbols, nor of Racah's lemma.

In this thesis we develop the Racah algebra for compact groups with time reversal symmetry. We take time reversal θ as a commuting operator of the group [66] and $\theta^2 = I$ for bosons or an even number of fermions, $\theta^2 = -I$ for an odd number of fermions. These two properties allow us to simplify the development from that which would be needed for a general linear/anti-linear group.

In practice, we usually start from a compact group of linear operators H and extend it to G by adding in the antilinear operators $a = \theta u$ where u is linear. Since linear times antilinear is antilinear, and antilinear times antilinear is linear, the coset group G/H is isomorphic to C_2. As H is compact, from the homeomorphism $f_\theta(u) = f(\theta u)$, G − H is compact and hence G is compact. We shall see later that all irreducible corepresentations of G are obtained from irreducible representations of H, and so the compactness of H guarantees a complete set of corepresentations of G. Finally, we shall often need to integrate over the group. G and H both possess invariant integrals, so we may write

$$\int_G f(x)dx = \int_H f(u)du + \int_{G-H} f(a)\, da.$$

By simple substitution we have in particular

$$|H| = \int_H 1\, du = \int_{G-H} 1\, da = \frac{1}{2}\int_G 1\, dx = \frac{1}{2}|G|$$

For finite groups these integrals reduce to sums.

CHAPTER TWO : A COVARIANT NOTATION

2.1 Basic Concepts

A wide variety of notations exist in representation and corepresentation theory regarding matrices, coupling coefficients, 3jm symbols, etc. A simple notation leads to ease of reading but can hide many subtle and important points as is shown by the papers by Derome and Sharp [23,24] . They, and others following them [48,49] have used a covariant notation but unfortunately not one that is easily adapted to corepresentation theory. For this thesis we use a notation borrowed from spinor and rotor calculus [67,68] which handles in an elegant fashion all the 'book-keeping' aspects of linear and anti-linear operators.

Let S be a complex vector space with basis $\{e_{(m)} : m = 1,2, ...\}$ and S* be the complex conjugate space with basis $\{e_{(\dot{m})} : \dot{m} = 1,2,.....\}$. We suppose S to be extended by elements of S* so that S may be taken as equal to S*. Examples of such spaces are Hilbert spaces of square integrable functions. If P is a change of basis matrix in S to a new basis $\{e_{(m')} : m' = 1,2,.....\}$ then using the summation convention,

$$e_{(m')} = e_{(m)} P^{m}{}_{m'} \tag{2.1}$$

This induces a corresponding transformation in S* by P* :

$$e_{(\dot{m}')} = e_{(\dot{m})} P^{\dot{m}}{}_{\dot{m}'} \tag{2.2}$$

Thus in this notation

$$(P^{m}{}_{m'})^* = P^{\dot{m}}{}_{\dot{m}'} \quad \text{and} \quad (P^{\dot{m}}{}_{\dot{m}'})^* = P^{m}{}_{m'} \tag{2.3}$$

so that dotting of indices represents the operation of complex conjugation.

The inverse of $P^{m}{}_{m'}$ is given by $P^{m'}{}_{m}$:

$$(P^{m}{}_{m'})^{-1} = P^{m'}{}_{m} \tag{2.4}$$

so that equations (2.1) and (2.2) may be inverted to

$$e_{(m)} = e_{(m')} P^{m'}{}_{m} \tag{2.5}$$

and

$$e_{(\dot{m})} = e_{(\dot{m}')} P^{\dot{m}'}{}_{\dot{m}} \tag{2.6}$$

This notation, defined strictly for basis transformations will be used freely whenever there is no risk of confusion. Thus the inverse of, say, the coupling coefficient to be defined later will be

$$(< j_1 j_2 | j_3 >^{m_1 m_2}{}_{r m_3})^{-1} = < j_1 j_2 | j_3 >^{r m_3}{}_{m_1 m_2}$$

Whenever there is such a risk either the minus one will be included or a more detailed notation to be described shortly will be used. For example, in Racah's lemma we shall have occassion to use the inverse of a permutation matrix M(12,3), and shall write

$$(M(12,3)^{r_1}{}_{r_2})^{-1} = M^{-1}(12,3)^{r_2}{}_{r_1}$$

The transpose of a tensor will be formed by changing each inner (outer) index to an outer (inner) index. Thus

$$(P^{m}{}_{m'})^{T} = P_{m'}{}^{m} \quad \text{and} \quad (P^{\dot{m}}{}_{\dot{m}'})^{T} = P_{\dot{m}'}{}^{\dot{m}} \tag{2.7}$$

This operation may also be performed by use of the tensors $\delta^{m}{}_{m'}$, $\delta_{m'}{}^{m}$ and their complex conjugates, which are numerically one if m equals m' and zero otherwise. They have the effect of changing inner, or in matrix terms row, indices to outer or column indices and vice versa. This gives an alternative formulation of equations (2.7)

$$P_{m'}{}^{m} = \delta_{n}{}^{m} \delta^{n'}{}_{m'} P^{n}{}_{n'} \tag{2.8}$$

and

$$P_{\dot{m}'}{}^{\dot{m}} = \delta_{\dot{n}}{}^{\dot{m}} \delta^{\dot{n}'}{}_{\dot{m}'} P^{\dot{n}}{}_{\dot{n}'} \tag{2.9}$$

2.2 Unitary Transformations

We are now in a position to define unitary tensors which are of prime importance in group theory. If we have an orthonormal basis $\{e_{(m)} : m = 1,2,.....\}$ then the metric in this space

$$\delta_{\dot{n}\, m} = e_{(\dot{n})} e_{(m)} = (e_{(n)})^{T^{*}} e_{(m)} \tag{2.10}$$

equals one if n = m and zero otherwise. Under a change of basis

$$e_{(\dot{n}')} e_{(m')} = P^{m}{}_{m'} P_{\dot{n}'}{}^{\dot{n}} e_{(\dot{n})} e_{(m)} \tag{2.11}$$

For P to be unitary, that is to preserve the metric,

$$\delta_{\dot{n}'\, m'} = P^{m}{}_{m'} P_{\dot{n}'}{}^{\dot{n}} \delta_{\dot{n}\, m} \tag{2.12}$$

which by simple manipulations may also be cast into

$$\delta^{m\,\dot{n}} = P^{m}{}_{m'}\, P_{\dot{n}'}{}^{\dot{n}}\, \delta^{m'\,\dot{n}'} \qquad (2.13)$$

giving a 'two-sided' form of the unitary condition.

These equations display one of the points which separate the tensor and matrix notations. In matrix notation the Hermitian adjoint P^{+} of P is defined by

$$P^{+} = P^{T*}$$

This becomes

$$(P^{m}{}_{m'})^{+} = P_{\dot{m}'}{}^{\dot{m}} \qquad (2.14)$$

showing that P^{+} acts on the basis vectors of S*. Thus P^{+} cannot be identified with P^{-1} as it acts on the basis vectors of S. It is not hard however to find expressions for P^{-1} which also act as a check on the convention of equation (2.4). From equation (2.12)

$$\delta^{p'\,\dot{n}'}\, \delta_{\dot{n}'\,m'} = P^{m}{}_{m'}\, P_{\dot{n}'}{}^{\dot{n}}\, \delta_{\dot{n}\,m}\, \delta^{p'\,\dot{n}'}$$

Contracting the tensor on the left and expanding the right-hand side by equation (2.9)

$$\delta^{p'}{}_{m'} = P^{m}{}_{m'}\, P^{\dot{q}}{}_{\dot{q}'}\, \delta^{\dot{q}'}_{\dot{n}'}\, \delta^{\dot{n}}_{\dot{q}}\, \delta_{\dot{n}\,m}\, \delta^{p'\,\dot{n}'}$$

$$= P^{m}{}_{m'}\, P^{\dot{q}}{}_{\dot{q}'}\, \delta^{\dot{q}'p'}\, \delta_{\dot{q}m}$$

with $\delta^{\dot{q}'p'} = \delta^{\dot{q}'}_{\dot{n}'}\, \delta^{p'\,\dot{n}'}$. From this,

$$(P^{m}{}_{m'})^{-1} = P^{m'}{}_{m} = P^{\dot{n}}{}_{\dot{n}'}\, \delta^{m'\dot{n}'}\, \delta_{m\dot{n}} \qquad (2.15)$$

This equation, which equally well expresses the unitarity of P, will be used for the inverse whenever the simpler devices of equation (2.4) or the statements following equation (2.6) prove inadequate.

2.3 Unitary and Anti-Unitary Operators

The matrix of a linear operator $u : S \rightarrow S$ is defined in the usual way by

$$u\, e_{(n)} = e_{(m)}\, j(u)^{m}{}_{n} \tag{2.16}$$

Under a change of basis the usual transformation law holds

$$j(u)^{m'}{}_{n'} = P^{m'}{}_{m}\, j(u)^{m}{}_{n}\, P^{n}{}_{n'} \tag{2.17}$$

A unitary linear operator in addition possesses the properties of equations (2.12), (2.13) and (2.15)

$$\delta_{\dot{n}_2 n_1} = j(u)^{m_1}{}_{n_1}\, j(u)_{\dot{n}_2}{}^{\dot{m}_2}\, \delta_{\dot{m}_2 m_1} \tag{2.18}$$

$$\delta^{m_1 \dot{m}_2} = j(u)^{m_1}{}_{n_1}\, j(u)_{\dot{n}_2}{}^{\dot{m}_2}\, \delta^{\dot{n}_2 n_1} \tag{2.19}$$

and

$$(j(u)^{m_1}{}_{n_1})^{-1} = j(u)^{\dot{m}_2}{}_{\dot{n}_2}\, \delta^{n_1 \dot{n}_2}\, \delta_{m_1 \dot{m}_2} \tag{2.20}$$

The matrix of an anti-linear operator $a : S \rightarrow S^{*}$ is given by

$$a\, e_{(n)} = e_{(m)}\, j(a)^{m}{}_{\dot{n}} \tag{2.21}$$

with now one dotted and one undotted variable.

Under a change of basis it transforms according to

$$j(a)^{m'}{}_{\dot{n}'} = P^{m'}{}_{m}\, j(a)^{m}{}_{\dot{n}}\, P^{\dot{n}}{}_{\dot{n}'} \tag{2.22}$$

In ordinary matrix notation this is

$$j\,(a)' = P\,j(a)\,P^{-1*}$$

a rule which would normally need care in remembering. The dotted and undotted tensor notation takes care of this automatically.

Time reversal, in addition to being anti-linear, is also anti-unitary [6]. That is,

$$\langle a\,\psi \mid a\,\phi \rangle = \langle \psi \mid \phi \rangle^{*}$$

In our notation this is

$$(a\,e_{(n_2)})^{*T}\,(a\,e_{(n_1)}) = ((e_{(n_2)})^{*T}\,e_{(n_1)})^{*}$$

which gives the anti-unitary conditions

$$\delta_{n_2\,\dot{n}_1} = j(a)^{m_1}{}_{\dot{n}_1}\,j(a)_{n_2}{}^{\dot{m}_2}\,\delta_{\dot{m}_2\,m_1} \tag{2.23}$$

$$\delta^{m_1\,\dot{m}_2} = j(a)^{m_1}{}_{\dot{n}_1}\,j(a)_{n_2}{}^{\dot{m}_2}\,\delta^{\dot{n}_1\,n_2} \tag{2.24}$$

and

$$(j(a)^{m_1}{}_{\dot{n}_1})^{-1} = j(a)^{\dot{m}_2}{}_{n_2}\,\delta^{\dot{n}_1 n_2}\,\delta_{m_1\dot{m}_2} \tag{2.25}$$

The multiplication rules for linear and anti-linear operators are easily found from the definitions (2.16) and (2.21) to be

$$j(u_1u_2)^{m}{}_{n} = j(u_1)^{m}{}_{m_1}\,j(u_2)^{m_1}{}_{n} \tag{2.26}$$

$$j(ua)^{m}{}_{\dot{n}} = j(u)^{m}{}_{m_1}\,j(a)^{m_1}{}_{\dot{n}} \tag{2.27}$$

$$j(au)^{m}{}_{\dot{n}} = j(a)^{m}{}_{\dot{m}_1}\, j(u)^{\dot{m}_1}{}_{\dot{n}} \tag{2.28}$$

and

$$j(a_1a_2)^{m}{}_{n} = j(a_1)^{m}{}_{\dot{m}_1}\, j(a)^{\dot{m}_1}{}_{n} \tag{2.29}$$

Again the tensor notation easily handles the complex conjugates which prove a nuisance when using a matrix notation.

It is easily seen from these that the product of two linear or two anti-linear operators is also linear, while the product of one linear and one anti-linear operator is anti-linear. It is readily checked that the same scheme holds for unitary and anti-unitary operators: that the product of two unitary or two anti-unitary operators is unitary, while the product of one unitary and one anti-unitary operator is anti-unitary.

CHAPTER THREE: IRREDUCIBLE COREPRESENTATIONS

3.1 Reducibility and Orthogonality Relations

Let G be a compact group of operators over a complex vector space S, with a subgroup H of linear unitary operators $\{u : u \in H\}$ of index two in G, and a left/right coset G–H of anti-linear anti-unitary operators $\{a : a \in G-H\}$. Then the set of matrices $j = \{j(u), j(a) : u \in H, a \in G-H\}$ generated by equations (2.16) and (2.21) satisfy equations (2.26) to (2.29) and are called a corepresentation of G.
Jansen and Boon [66] have shown that it is not necessary for most results to use the underlying space S, but as our main use of the theory will be for G as the symmetry group of a quantum mechanical system we shall retain it.

As Maschke's theorem still holds [54], each corepresentation of G may be reduced by a unitary transformation to a direct sum of irreducible corepresentations (ICRs) of G. Schur's lemma only holds in a weak form for ICRs though, giving the following rather unpleasant orthogonality relations [55]

$$\int_H j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{\dot{m}_2}{}_{\dot{n}_2}\, du = 0 \qquad (3.1)$$

$$\int_{G-H} j_1(a)^{m_1}{}_{\dot{n}_2}\, j_2(a)^{\dot{m}_2}{}_{n_1}\, da = 0 \qquad (3.2)$$

if j_1 and j_2 are non-equivalent ICRs, and

$$\int_H j(u)^{m_1}{}_{n_1}\, j(u)^{\dot{m}_2}{}_{\dot{n}_2}\, du + \int_{G-H} j(a)^{m_1}{}_{\dot{n}_2}\, j(a)^{\dot{m}_2}{}_{n_1}\, da$$

$$= \frac{|G|}{[j]}\, \delta^{m_1 \dot{m}_2}\, \delta_{n_1 \dot{n}_2} \qquad (3.3)$$

if j is irreducible, and $[j]$ equals the dimension of j. Note how n_1 and $\dot{n}_2$ are interchanged in the last integral.

Setting $m_1 = n_1$, $\dot{m}_2 = \dot{n}_2$ gives a character test for irreducibility

$$\int_H |\chi_j(u)|^2\, du + \int_{G-H} \chi_j(a^2)\, da = |G| \qquad (3.4)$$

An important ICR which we shall use very frequently is the identity ICR $\underset{\sim}{0}$, with

$$\underset{\sim}{0}\,(u) = \underset{\sim}{0}\,(a) = 1 \qquad (3.5)$$

Letting $j_2 = \underset{\sim}{0}$ in equations (3.1) and (3.2) gives

$$\int_H j_1(u)^{m_1}{}_{n_1}\, du = \int_{G-H} j_1(a)^{m_1}{}_{\dot{n}_2}\, da = 0 \qquad (3.6)$$

if $j_1 \not\equiv \underset{\sim}{0}$

Further, on restriction to H, $\underset{\sim}{0}$ is an irreducible representation (IR) of H which possesses an orthogonality property as an IR. Hence both integrals in equation (3.3) are equal to $|G|/2$ for $j = \underset{\sim}{0}$.

3.2 Classification of ICRs

On restriction to H, each ICR j of G subduces to a possibly reducible representation k of H.

Only a limited number of possibilities exist and give this classification for ICRs [6] .

type (a) k is irreducible and equivalent to $\bar{k}$ where $\bar{k} = k(a^{-1}ua)^*$ for arbitrary fixed a. The character test is

$$\int_H |\chi_j(u)|^2 \, du = \int_{G-H} \chi_j(a^2) \, du = \frac{|G|}{2} \qquad (3.7)$$

type (b) k reduces to $k_1 \oplus k_1$ with $k_1 \equiv \bar{k}_1$. The character test is

$$\int_H |\chi_j(u)|^2 \, du = 2|G| \qquad (3.8)$$

type (c) k reduces to $k_1 \oplus \bar{k}_1$ with $k_1 \not\equiv \bar{k}_1$. Here

$$\int_H |\chi_j(u)|^2 \, du = |G| \qquad (3.9)$$

For the grey groups time reversal θ commutes with all elements of G [66] so that $\bar{k} = k^*$ and equivalence (non-equivalence) is the equivalence (non-equivalence) of complex conjugate representations. A further classification may be made according to the Frobenius-Schur invariant.

$$C_k = \frac{1}{|H|} \int_H \chi_k(u^2) \, du \qquad (3.10)$$

Thus k is of the first kind if it is equivalent to k* and to a real IR ($C_k = 1$). k is of the second kind if it is equivalent to k* but not to a real IR ($C_k = -1$), and k is of the third kind if it is not equivalent to k* ($C_k = 0$).

Further, θ^2 is either the positive or negative identity and an examination of the method of construction of types (a) to (c) gives exactly six types of grey ICR [66]

$\underline{\theta^2 = I}$

(A) k is irreducible and of the first kind.

Then $$j(u) = k(u) \tag{3.11}$$

$$j(\theta) = \beta \tag{3.12}$$

and

$$j(a) = k(a\theta^{-1})\beta \tag{3.13}$$

with

$$\beta\beta^* = I \tag{3.14}$$

The character test is

$$\int_H |\chi_j(u)|^2 \, du = \int_H \chi_j(u^2) \, du = \frac{|G|}{2} \tag{3.15}$$

(B) k reduces to $k_1 \oplus k_1$ with k_1 of the second kind

$$j(u) = \begin{pmatrix} k_1(u) & 0 \\ 0 & k_1(u) \end{pmatrix} \tag{3.16}$$

$$j(\theta) = \begin{pmatrix} 0 & \beta \\ -\beta & 0 \end{pmatrix} \tag{3.17}$$

and $$j(a) = \begin{pmatrix} 0 & k_1(a\theta^{-1})\beta \\ -k_1(a\theta^{-1})\beta & 0 \end{pmatrix} \tag{3.18}$$

with $$\beta\beta^* = -I \tag{3.19}$$

$$\int_H |\chi_j(u)|^2 \, du = 2|G| \text{ and } \int_H \chi_j(u^2) \, du = -|G| \tag{3.20}$$

(C) k reduces to $k_1 \oplus \overset{*}{k}_1$ with k_1 of the third kind

$$j(u) = \begin{pmatrix} k_1(u) & 0 \\ 0 & \overset{*}{k}_1(u) \end{pmatrix} \quad (3.21)$$

$$j(\theta) = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad (3.22)$$

and
$$j(a) \begin{pmatrix} 0 & \overset{*}{k}_1(\theta^{-1}\alpha) \\ k_1(a\theta) & 0 \end{pmatrix} \quad (3.23)$$

with

$$\int_H |\chi_j(u)|^2 \, du = |G| \quad \text{and} \quad \int_H \chi_j(u^2) \, du = 0 \quad (3.24)$$

<u>$\theta^2 = -I$</u>

(D) k is irreducible and of the second kind

$$j(u) = k(u) \quad (3.25)$$

$$j(\theta) = \beta \quad (3.26)$$

and $$j(a) = k(a\theta^{-1})\beta \quad (3.27)$$

where $$\beta\beta^* = -I \quad (3.28)$$

Also

$$\int_H |\chi_j(u)|^2 \, du = \left|\frac{G}{2}\right| \quad \text{and} \quad \int_H \chi_j(u^2) \, du = -\left|\frac{G}{2}\right| \quad (3.29)$$

(E) k reduces to $k_1 \oplus k_1$ with k of the first kind

$$j(u) = \begin{pmatrix} k_1(u) & 0 \\ 0 & k_1(u) \end{pmatrix} \quad (3.30)$$

$$j(\theta) = \begin{pmatrix} 0 & \beta \\ -\beta & 0 \end{pmatrix} \tag{3.31}$$

$$j(a) = \begin{pmatrix} 0 & k_1(a\theta^{-1})\beta \\ k_1(a\theta^{-1})\beta & 0 \end{pmatrix} \tag{3.32}$$

where $\beta\beta^* = I$ (3.33)

The character test is

$$\int_H |\chi_j(u)|^2 \, du = 2|G| \quad \text{and} \quad \int_H \chi_j(u^2) \, du = |G| \tag{3.34}$$

(F) k reduces to $k_1 \oplus k_1^*$ with k_1 of the third kind

$$j(u) = \begin{pmatrix} k_1(u) & 0 \\ 0 & k_1^*(u) \end{pmatrix} \tag{3.35}$$

$$j(\theta) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{3.36}$$

$$j(a) = \begin{pmatrix} 0 & k_1^*(\theta^{-1}a) \\ k_1(a\theta) & 0 \end{pmatrix} \tag{3.37}$$

and

$$\int_H |\chi_j(u)|^2 \, du = |G| \quad \text{and} \quad \int_H \chi_j(u^2) \, du = 0 \tag{3.38}$$

Conversely, if we start from the IRs of H, the ICRs of G are constructed exactly according to the above scheme. Thus time reversal only fails to increase the degeneracy for types (A) and (D). We observe that these ICRs are equivalent to their complex conjugates by the character tests of Rudra [56], a fact we shall verify explicitly later.

3.3 Reduction of Direct Products

The direct product of two ICRs may be formed in the usual way

$$j_1 \otimes j_2(u)^{m_1 m_2}{}_{n_1 n_2} = j_1(u)^{m_1}{}_{n_1} j_2(u)^{m_2}{}_{n_2} \qquad (3.39)$$

and

$$j_1 \otimes j_2(a)^{m_1 m_2}{}_{\dot{n}_1 \dot{n}_2} = j_1(a)^{m_1}{}_{\dot{n}_1} j_2(a)^{m_2}{}_{\dot{n}_2} \qquad (3.40)$$

It is clear that the direct product is a corepresentation of G and hence can be written as a direct sum of ICRs. An important first step to establishing a Racah algebra is to determine the Clebsch-Gordan coefficients of this reduction, i.e. the values d^3_{12} in

$$j_1 \otimes j_2 = \sum_{j_3} d^3_{12} \; j_3 \qquad (3.41)$$

This is not quite as straightforward as in representation theory as the orthogonality relations are not so simple. However, for the unitary operators we do have

$$\chi_{j_1}(u) \; \chi_{j_2}(u) = \sum_{j_3} d^3_{12} \; \chi_{j_3}(u) \qquad (3.42)$$

with of course higher order analogues

$$\chi_{j_1}(u) \; \chi_{j_2}(u) \; \chi_{j_3}(u) = \sum_{j_4} d^4_{123} \; \chi_{j_4}(u) \qquad (3.43)$$

etcetera. We do not have such relations for the anti-linear operators as their trace is not invariant under unitary transformations.

From equation (3.6) and the remarks following, these last two equations give

$$d^{\underset{\sim}{0}}_{12} = \frac{2}{|G|} \int_H \chi_{j_1}(u)\, \chi_{j_2}(u)\, du \tag{3.44}$$

and

$$d^{\underset{\sim}{0}}_{123} = \frac{2}{|G|} \int_H \chi_{j_1}(u)\, \chi_{j_2}(u)\, \chi_{j_3}(u)\, du \tag{3.45}$$

We note from equation (3.1) that $d^{\underset{\sim}{0}}_{12} = 0$ unless $j_1 \equiv j_2^*$. For grey ICRs this takes the simpler form that $d^{\underset{\sim}{0}}_{12} = 0$ unless $j_1 \equiv j_2$. Now consider the triple product with an intermediate coupling:

$$j_1 \otimes j_2 \otimes j_3 = \sum_{j_4} d^4_{12}\, j_4 \otimes j_3$$

$$= \sum_{j_4, j_5} d^4_{12}\, d^5_{43}\, j_5 \tag{3.46}$$

It follows immediately that

$$d^{\underset{\sim}{0}}_{123} = d^{3^*}_{12}\, d^{\underset{\sim}{0}}_{3^*3}$$

or that

$$d^3_{12} = d^{\underset{\sim}{0}}_{123^*} \Big/ d^{\underset{\sim}{0}}_{33^*} \tag{3.47}$$

For grey ICRs this is

$$d^3_{12} = d^{\underset{\sim}{0}}_{123} \Big/ d^{\underset{\sim}{0}}_{33} \tag{3.48}$$

These equations display in form no difference to the corresponding equations in representation theory, for there the multiplicity of $\underset{\sim}{0}$ in $k \otimes k^*$ is always one. However, the difference in <u>content</u> is remarkable for $d^{\underset{\sim}{0}}_{33}$ is not necessarily one. To see this, examine each type of ICR in turn. For a type (a) ICR, $\chi_{j_3}(u) = \chi_{k_3}(u)$ from which

$$d^{\underset{\sim}{0}}_{33} = 1 \tag{3.49}$$

For a type (b) ICR, $\chi_{j_3}(u) = 2\,\chi_{k_3}(u)$ and thus

$$d_{33}^{\tilde{0}} = 4 \qquad (3.50)$$

For a type (c) ICR, $\chi_{j_3}(u) = \chi_{k_1}(u) + \chi_{k_1}(u)^*$ so that

$$d_{33}^{\tilde{0}} = \frac{1}{|H|}\int \chi_{k_3}(u)\,\chi_{k_3}(u) + 2\,\chi_{k_3}(u)\chi_{k_3}(u)^* + \chi_{k_3}(u)^*\,\chi_{k_3}(u)^*\,du \qquad (3.51)$$

$$= 2$$

This has the immediate consequence that the multiplicity of, say, j_3^* in $j_1 \otimes j_2$ need not be the same as the multiplicity of j_2^* in $j_1 \otimes j_3$. Thus the coupling coefficient matrix which reduces $j_1 \otimes j_2$ to j_3^* will not in general be related by a unitary transformation to the coupling coefficient matrix which reduces $j_1 \otimes j_3$ to j_2^*. This result is peculiar to corepresentation theory and marks the major departure from representation theory. It will be studied in detail later.

The Clebsch-Gordan coefficient $d_{123}^{\tilde{0}}$ on the other hand, does have the symmetry properties.

$$d_{123}^{\tilde{0}} = d_{132}^{\tilde{0}} = d_{213}^{\tilde{0}} \quad \text{etc.} \qquad (3.52)$$

Thus we may expect that the 3jm symbol which reduces the triple product to $\tilde{0}$ will have similar permutation properties to the 3jm symbol in representation theory [23,24] and this will indeed turn out to be the case.

In many cases, the anti-linear operators G-H will be 'tacked on' to an already known linear group H. Bradley and Davies [54] have given the Clebsch-Gordan coefficients d^{k}_{ij} in terms of those for H, and we complement this by giving in table 1 the coefficients $d^{\underset{\sim}{0}}_{33}$ and $d^{\underset{\sim}{0}}_{123}$ for the grey groups in terms of those for H. In table 2 we also give the non-zero multiplicities when H is quasi-SR i.e. the minimum non-zero Clebsch-Gordan coefficients for ICR couplings.

Table 1 : Clebsch-Gordan Coefficients d^{0}_{123} for ICRs in terms of the Clebsch-Gordan coefficients C^{0}_{123} of the linear subgroup

j_1	j_2	j_3	d^{0}_{123}
(a)	(a)	-	C^{0}_{12}
(b)	(b)	-	$4\ C^{0}_{12}$
(c)	(c)	-	$2\ C^{0}_{12^*}$
(a)	(a)	(a)	C^{0}_{123}
(a)	(a)	(b)	$2\ C^{0}_{123}$
(a)	(a)	(c)	$2\ C^{0}_{123}$
(a)	(b)	(b)	$4\ C^{0}_{123}$
(a)	(b)	(c)	$4\ C^{0}_{123}$
(a)	(c)	(c)	$2\ C^{0}_{123^*} + 2\ C_{123}$
(b)	(b)	(b)	$8\ C^{0}_{123}$
(b)	(b)	(c)	$8\ C^{0}_{123}$
(b)	(c)	(c)	$4\ C^{0}_{123} + 4\ C^{0}_{123^*}$
(c)	(c)	(c)	$2\ C^{0}_{123} + 2\ C^{0}_{123^*} + 2\ C^{0}_{12^*3} + 2\ C^{0}_{12^*3^*}$

Table 2: The minimum non-zero multiplicity of the identity ICR in double and triple products.

Multiplicity-free couplings of IRs give the following multiplicities for the ICRs. If the IR coupling is not multiplicity free, table 1 should be used.

j_1	j_2	j_3	multiplicity
(a)	(a)	-	1
(b)	(b)	-	4
(c)	(c)	-	2
(a)	(a)	(a)	1
(a)	(a)	(b)	2
(a)	(a)	(c)	2
(a)	(b)	(b)	4
(a)	(b)	(c)	4
(b)	(b)	(b)	8
(b)	(b)	(c)	8
(b)	(c)	(c)	4
(c)	(c)	(c)	2

3.4 Schur's Lemma

We are now in a position to state Schur's Lemma for grey groups in its most general form although part (b) has already been used to find the orthogonality relations of section 3.1 [55]

Lemma Let j be an ICR and P a matrix commuting with j in the following sense

$$j(u)^{m_1}{}_{n_1} \; P^{n_1}{}_{m_2} = P^{m_1}{}_{n_1} \; j(u)^{n_1}{}_{m_2} \tag{3.53}$$

and

$$j(a)^{m_1}{}_{\dot{n}_1} \; P^{\dot{n}_1}{}_{\dot{m}_2} = P^{m_1}{}_{n_1} \; j(a)^{n_1}{}_{\dot{m}_2} \tag{3.54}$$

Then

(a) The intertwining number $[j,j]$ equals the multiplicity of $\underset{\sim}{0}$ in $j \otimes j$

(b) If P has at least one real eigenvalue, it is a real constant matrix.

Part (a) is shown by taking each type of ICR in turn and applying Schur's lemma for unitary groups. Thus for type (a), $j(u) = k(u)$ and therefore $P = x\,I$. Substitution into equation (3.53) gives x real and hence the intertwining algebra is isomorphic to $\mathbb{R}$ with intertwining number one. Similarly, for an ICR of type (b), commutation with j(u) gives

$$P = \begin{pmatrix} \lambda_1 \, I & \lambda_2 \, I \\ \lambda_3 \, I & \lambda_4 \, I \end{pmatrix}$$

and then with $j(\theta)$

$$P = x_1 \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} + x_2 \begin{pmatrix} iI & 0 \\ 0 & -iI \end{pmatrix} + x_3 \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} + x_4 \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix} \quad (3.55)$$

with x_i real. This algebra is isomorphic to the quarternions $\mathbb{Q}$ with intertwining number four. Lastly, for type (c)

$$P = x_1 \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} + x_2 \begin{pmatrix} iI & 0 \\ 0 & -iI \end{pmatrix} \quad (3.56)$$

Here the intertwining algebra is isomorphic to $\mathbb{C}$ with intertwining number two. Comparing these with the Clebsch-Gordan coefficients of the last section gives the result.

Part (b) follows immediately as for example, the existence of a real eigenvalue for P of equation (3.55) implies $x_2 = x_3 = x_4 = 0$.

These properties are invariant under any change of basis, completing the proof.

One consequence of the lemma has already been seen in the orthogonality properties. Another is that, unlike representation theory, an irreducible space can have two or more bases all carrying the same matrix corepresentation. For example consider grey C_3^* with the z-axis as symmetry axis acting on SU(2) ket vectors with Fano-Racah standardization. Then the four sets of vectors

$$\{|{}^3/2 \;\; {}^3/2\rangle, -|{}^3/2 \; -{}^3/2\rangle\}, \; \{i|{}^3/2 \;\; {}^3/2\rangle, \; i|{}^3/2 \; -{}^3/2\rangle\}$$

$$\{i|{}^3/2 \; -{}^3/2\rangle, -i|{}^3/2 \;\; {}^3/2\rangle\} \text{ and } \{|{}^3/2 \; -{}^3/2\rangle, \; |{}^3/2 \;\; {}^3/2\rangle\}$$

each form a basis set for the ICR

$$E'(C_3) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{and} \quad E'(\theta) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

This will produce further anomalies in the development of the algebra.

CHAPTER FOUR : ICRs OF THE DOUBLE GREY POINT GROUPS

In this section we give the ICRs of each of the double grey point groups and give a notation similar to Mulliken's [69] which will be used throughout. For the single group the grey group is the direct product of the point group and C_2 (Dimmock and Wheeler [70]) but this is not true for the double group, and we shall abuse notation by referring to a grey group by the usual double point group notation of Griffith [71].

(a) SU(2), O^* and K^*

Each IR of these groups induces an ICR of types (A) or (D) and hence the Mulliken notation may be used to label the ICRs. The basis vectors of the IRs for O^* and K^* given by Griffith [71], Golding [26, 27] and McLellan [72] are also basis vectors of the ICRs.

Table 3: Reduction of the Direct Products of grey T^*

	A	E	T	E'	U'
A	A				
E	E	2A+E			
T	T	2T	A+E+2T		
E'	E'	U'	E'+U'	A+T	
U'	U'	2E'+U'	2E'+2U'	E+2T	2A+E+4T

Table 4: Multiplicity of A in triple products in grey T^*

Product	Multiplicity
AAA	1
EEA	2
EEE	1
TTA	1
TTE	2
TTT	2
E'E'A	1
E'E'T	1
U'E'E	2
U'E'T	2
U'U'A	2
U'U'E	1
U'U'T	4

Table 5: Multiplicity of A_1 in triple products in D^*_n (n=2m+1)

Product	Multiplicity	Product	Multiplicity
$A_1A_1A_1$	1	$E'_jE'_jA_1$	1
$A_2A_2A_1$	1	$E'_jE'_jA_2$	1
$E_jE_jA_1$	1	$E'_jE'_jE_{j-k}$	1
$E_jE_jA_2$	1	$E'_jE'_kE_{j+k}$	1
$E_jE_kE_{j-k}$	1	$E'E'_jE_{n/2-j}$	2
$E_jE_kE_{j+k}$	1	$E'E'_jE_{n/2+j}$	2
		$E'E'A_1$	2
		$E'E'A_2$	2

(b) The tetrahedral double grey group T*

The pair of representations Γ_2 and Γ_3 [71] are of the third kind and hence induce the ICR E of type (C) and similarly E" and E"' induce the ICR U' of type (F). The reduction of the direct products is given in Table 3 and the multiplicity of A in the triple product in table 4. It may be seen that the multiplicity problem for these groups is much worse than in groups without time reversal.

(c) The dihedral double groups D^*_n with n odd (n = 2m + 1)

The single group D_n has two one-dimensional IRs A_1 and A_2 and m two-dimensional IRs $E_1 \ldots\ldots E_m$ all of the first kind. Thus they all induce ICRs of type (A) and the generating matrices are:

$$A_1(C_n) = A_1(C_2) = A_1(\theta) = 1$$

$$A_2(C_n) = -A_2(C_2) = A_2(\theta) = 1$$

$$E_j(C_n) = \begin{bmatrix} \exp(ij\phi) & 0 \\ 0 & \exp(-ij\phi) \end{bmatrix},$$

$$E_j(C_2) = E_j(\theta) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

where $\phi = 2\pi/n$.

The double group D^*_n has in addition m two-dimensional IRs $E'_{\frac{1}{2}} \ldots\ldots E'_{m-\frac{1}{2}}$ which induce ICRs of type (D) and a pair of IRs A' and B' which induce an ICR E' of type (F). Typical generators are

$$E'_j(C_n) = \begin{bmatrix} \exp(ij\phi) & 0 \\ 0 & \exp(-ij\phi) \end{bmatrix}, \quad E'_j(\theta) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$E'(C_n) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad E'(C_2) = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$$

and

$$E'(\theta) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The multiplicity of A_1 in each triple product is given in Table 5.

(d) D^*_n with n even (n = 2m)

The single group D_n has four one-dimensional IRs A_1, A_2, B_1 and B_2 and (m-1) two-dimensional IRs $E_1 \ldots\ldots E_{(m-1)}$. The double group has the additional m two-dimensional representations $E'_{\frac{1}{2}} \ldots\ldots E'_{(m-\frac{1}{2})}$. Each of these induces an ICR. The generators for B_1 and B_2 are.

$$-B_1(C_n) = B_1(C_2) = B_1(\theta) = 1 \quad \text{and} \quad -B_2(C_n) = -B_2(C_2) = B_2(\theta) = 1$$

The generators for the other ICRs follow $D^*_{(2m+1)}$. There is no point giving a table of triple products as the multiplicity of A_1 is always one or zero.

(e) The cyclic double grey group C^*_n

The cyclic group C^*_n is isomorphic to C_{2n} [66] and hence has 2n one-dimensional representations $\Gamma_{m/2}$ with $-n < m \leq n$ where

$$\Gamma_{m/2}(C_n) = \exp(im\phi/2)$$

$$\text{with } \phi = 2\pi/n$$

$A = \Gamma_o$ induces an ICR of type (A). For $1 \leq m \leq n - 1$ the character is complex and hence $\Gamma_{m/2}$ and $\Gamma_{-m/2}$ induce a two dimensional ICR $E_{m/2}$ or $E'_{m/2}$. If n is even, $\Gamma_{n/2}$ induces an ICR A_2, but if n is odd $\Gamma_{n/2}$ is a spin representation of the first kind and $\Gamma_{n/2} \oplus \Gamma_{n/2}$ induces the ICR E'. The generators are

$$E_{m/2}(C_n) = \begin{bmatrix} \exp(im\phi/2) & 0 \\ 0 & \exp(-im\phi/2) \end{bmatrix}, \quad E_{m/2}(\theta) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$E'_{m/2}(C_n) = \begin{bmatrix} \exp(im\phi/2) & 0 \\ 0 & \exp(-im\phi/2) \end{bmatrix}, \quad E'_{m/2}(\theta) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

and for n even

$$A_2(C_n) = -1, \quad A_2(\theta) = 1$$

whereas for n odd

$$E'(C_n) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad E'(\theta) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The triple direct product is given in Table 6.

Multiplicity of A_1 in triple products in C_n^*

Table 6: Triple product in C_n^*

Product	Multiplicity	Product	Multiplicity
$A_1A_1A_1$	1	$E_k' E_\ell' E_{k+1}$	2
$A_2A_2A_1$	1	$E_k' E_\ell' E_{k-1}$	2
$E_k E_\ell E_{k+1}$	2	$E_k' E_k' A_1$	2
$K_k E_\ell E_{k-\ell}$	2	$E_k' E'_{n/2-k} A_2$	2
$E_k E_k A_1$	2		
$E_k E_{n/2-k} A_2$	2	$E'E_k' E_{n/2+k}$	2
		$E'E'_k E_{n/2-k}$	2
		$E'E'A_1$	4

CHAPTER FIVE : THE WIGNER TENSOR

5.1 The Wigner Tensor and Time-Reversal

The Wigner tensor [22] otherwise known as the 1-j symbol [23] or the 1-jm symbol [48] plays an important rôle in representation theory by relating an IR to its complex conjugate IR in ambivalent groups, and hence in relating a coupling coefficient to a 3jm symbol. Since all grey ICRs are ambivalent (i.e. equivalent to their complex conjugates) it may be expected that the Wigner tensor be equally important here. It is straightforward to show that time reversal may be used to give this equivalence by using the commutativity of θ:

$$j(u)^{m_1}{}_{m_2} = j(\theta u \theta^{-1})^{m_1}{}_{m_2}$$

$$= j(\theta)^{m_1}{}_{\dot{n}_1}\, j(u)^{\dot{n}_1}{}_{\dot{n}_2}\, j(\theta^{-1})^{\dot{n}_2}{}_{m_2} \qquad (5.1)$$

Now $\qquad \delta^{m_1}{}_{m_2} = j(\theta\theta^{-1})^{m_1}{}_{m_2} = j(\theta)^{m_1}{}_{\dot{n}_2}\, j(\theta^{-1})^{\dot{n}_2}{}_{m_2}$

So $\qquad j(\theta^{-1})^{\dot{n}_2}{}_{m_2} = j(\theta)^{-1\,\dot{n}_2}{}_{m_2} \qquad (5.2)$

This gives

$$j(u)^{m_1}{}_{m_2} = j(\theta)^{m_1}{}_{\dot{n}_1}\, j(u)^{\dot{n}_1}{}_{\dot{n}_2}\, j(\theta)^{-1\dot{n}_2}{}_{m_2} \qquad (5.3)$$

and similarly

$$j(a)^{m_1}{}_{\dot{m}_2} = j(\theta)^{m_1}{}_{\dot{n}_1}\, j(a)^{\dot{n}_1}{}_{n_2}\, j(\theta)^{-1n_2}{}_{\dot{m}_2} \qquad (5.4)$$

giving an explicit form for the equivalence relation. It may be worth-while commenting on the matrix form of these equations for clarity on this tensor notation. Equation (5.1) is $j(u) = j(\theta)j(u)^* \, j(\theta^{-1})^*$ and equation (5.2) is $j(\theta^{-1})^* = j(\theta)^{-1}$-

an entirely reasonable result since $j(\theta^{-1})$ is an operator from S to S^*, whereas $j(\theta)^{-1}$ is an operator from S^* to S. Equations (5.3) and (5.4) are then $j(u) = j(\theta)\ j(u)^*\ j(\theta)^{-1}$ and $j(a) = j(\theta)\ j(a)^*\ {j(\theta)^{-1}}^*$ respectively.

The tensor giving equivalence shares the same defect as a matrix commuting with all j : it is not unique. For it is clear that if P is one of the commuting matrices of chapter 3 section 4 then P $j(\theta)$ also gives equivalence of j and j^*. It is easy to show that any matrix must be of this form. This holds little (or probably no) consequence if we restrict our attention to the ICRs alone, but frequently in quantum mechanics the basis vectors themselves are of as much importance. Now an early assumption was that our vector space S equalled its conjugate, that is, there is an anti-unitary operator K such that

$$e_{(\dot{m})} = K^{n}{}_{\dot{m}}\, e_{(n)} \tag{5.5}$$

Thus

$$K^{n}{}_{\dot{m}} = P^{n}{}_{m_1}\, j(\theta)^{m_1}{}_{\dot{m}} \tag{5.6}$$

Now K^2 necessarily equals I, whereas θ^2 may equal either plus or minus I. From the commutativity of P and the explicit forms given earlier, it follows easily that if $\theta^2 = I$ then $K = \pm\ \theta$, whereas if $\theta^2 = -I$, $K \neq \lambda\theta$ for any value of λ.

Despite this need to distinguish between K and θ, we regard θ as the more important operator since in physically important problems such as SU(2) and its subgroups, the action of θ is clearly defined whereas the action of K depends an a particular realization of the basis vectors. We emphasize this by taking as the Wigner tensor

$$j(\theta)^{m}{}_{\dot{n}} = \begin{pmatrix} m & \\ & \dot{n} \end{pmatrix} \quad \text{and} \quad j(\theta)^{-1\,\dot{n}}{}_{m} = \begin{bmatrix} \dot{n} & \\ & m \end{bmatrix} \qquad (5.7)$$

so that

$$j(u)^{m_1}{}_{m_2} = \begin{pmatrix} m_1 & \\ & \dot{n}_1 \end{pmatrix} j(u)^{\dot{n}_1}{}_{\dot{n}_2} \begin{bmatrix} \dot{n}_2 & \\ & m_2 \end{bmatrix} \qquad (5.8)$$

and

$$j(a)^{m_1}{}_{\dot{m}_2} = \begin{pmatrix} m_1 & \\ & \dot{n}_1 \end{pmatrix} j(a)^{\dot{n}_1}{}_{n_2} \begin{bmatrix} n_2 & \\ & \dot{m}_2 \end{bmatrix} \qquad (5.9)$$

5.2 Normal Forms of the Wigner Tensor

We shall now investigate to what extent $j(\theta)$ may be cast into a simple form. Ideally this would be diagonal but we shall see that this is not always possible. It is not necessary to use the covariant notation for this, as we are not considering components in any detail and we simplify notation by considering an antilinear operator T satisfying $TT^* = \pm I$ and $T^\dagger = T^{-1}$. The invariant eigenvector equation for an antilinear operator is

$$Tv = \lambda\bar{v}$$

with $T^*\bar{v} = \lambda^* v$.

We first deal with the case $TT^* = I$. Choose any v in S and let

$$\bar{w} = T v$$

Then $Tw = \bar{v}$. If $v = w$ we have an eigenvector. If $v \neq w$, set $u = v + w$. Trivially

$$Tu = \bar{u}$$

giving an eigenvector. If V is the subspace generated by the eigenvector, we turn to the orthogonal subspace $V^\perp$ and repeat the process. Thus we can find an orthogonal eigenvector basis and diagonalize T. The eigenvalues are ±1 from $TT^* = I$.

When $TT^* = -I$ we have a different situation, for we can no longer produce eigenvectors as above. But as before, we let

$$T v = \bar{w}$$

from which

$$T\,w = -\bar{v}$$

Again, if $w = \lambda v$ we have an eigenvector. If not, we search for vectors which preserve the above form and are also orthogonal. By setting

$$v_1 = v + aw \quad \text{and} \quad v_2 = w - av$$

we have $Tv_1 = \bar{v}_2$ and $Tv_2 = -\bar{v}_1$. We may normalize v so that it has modulus one, and from the unitarity of T, w also has modulus one. If v_1 and v_2 are to be orthogonal,

$$0 = \langle v_1 | v_2 \rangle = \bar{a}\langle w | w \rangle - a\langle v | v \rangle + \langle v | w \rangle - a\bar{a}\langle w | v \rangle^*$$

or

$$a - \bar{a} = \langle v | w \rangle - a\bar{a}\langle w | v \rangle.$$

If we let $a\bar{a} = 1$, this is

$$\mathrm{Im}(a) = \mathrm{Im}(\langle v | w \rangle)$$

since v , w are unit vectors. This equation can certainly be solved for a since $a\bar{a} = 1$, and we have orthogonal vectors. We continue as before by considering the orthogonal subspaces. Thus we may transform T to

$$\begin{pmatrix} \lambda_1 & & & & & & \\ & \lambda_2 & & & & & \\ & & \ddots & & & & \\ & & & 0 & 1 & & \\ & & & -1 & 0 & & \\ & & & & & 0 & 1 \\ & & & & & -1 & 0 \\ & & & & & & & \ddots \end{pmatrix} \tag{5.10}$$

where from $T^* T v = -v$ each eigenvalue λ is $\pm i$.

The forms given here are not always the same as those used in practice. For example, in SU(2) with j half-odd-integral, the Fano-Racah standardization [8] gives $j(\theta)$ as

$$\begin{pmatrix} 0 & & & & & 1 \\ & & & & -1 & \\ & & & 1 & & \\ & & -1 & & & \\ & \cdot & & & & 0 \\ \cdot & & & & & \end{pmatrix}$$

which may be obtained from the above by a real orthogonal transformation. It is convenient to have $j(\theta)$ as a matrix with only one entry in each row and column, and we allow for such variations by taking

$$j(\theta)^m{}_{\dot{n}} = \begin{pmatrix} m \\ \dot{n} \end{pmatrix}$$

to be non-zero for only one $\dot{n}$ for each m, and similarly for

$$j(\theta)^{-1\,\dot{n}}{}_m = \begin{bmatrix} \dot{n} \\ m \end{bmatrix}$$

CHAPTER SIX : THE 3jm SYMBOL

6.1 Definition

In representation theory it matters little whether we reduce $j_1 \otimes j_2$ to j_3^* or $j_1 \otimes j_2 \otimes j_3$ to the identity IR $\underset{\sim}{0}$. However we noted in chapter 3 section 3 that the Clebsch-Gordan coefficients $d^{3^*}_{12}$ and $d^{\underset{\sim}{0}}_{123}$ need not be the same, so that there is a fundamental difference between the two reductions. Further, $d^{3^*}_{12}$ need not equal $d^{2^*}_{13}$ etc., so any attempt to base the 3jm symbols on the double product will impose very restrictive properties on permutations of the ICRs. But the multiplicity of $\underset{\sim}{0}$ in $j_1 \otimes j_2 \otimes j_3$ is independent of the order of coupling and so we may expect a 'well-behaved' 3jm symbol from this reduction.

We consider the unitary transformation which reduces the triple product $j_1 \otimes j_2 \otimes j_3$ and define the 3jm symbol to be the part of this matrix which reduces the product to the identity $\underset{\sim}{0}$:

$$j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, j_3(u)^{m_3}{}_{n_3}$$

$$= (j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\, \delta^{r_1}{}_{r_2}\, (j_1j_2j_3)^{r_2}{}_{n_1n_2n_3} \oplus \ldots (6.1)$$

and

$$j_1(a)^{m_1}{}_{\dot{n}_1}\, j_2(a)^{m_2}{}_{\dot{n}_2}\, j_3(a)^{m_3}{}_{\dot{n}_3}$$

$$= (j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\, \delta^{r_1}{}_{\dot{r}_2} (j_1j_2j_3)^{\dot{r}_2}{}_{\dot{n}_1\dot{n}_2\dot{n}_3} \oplus \ldots (6.2)$$

Here $\delta^{r_1}{}_{r_2}$ and $\delta^{r_1}{}_{\dot{r}_2}$ are the linear and anti-linear matrices respectively of the identity ICR, and are unit matrices of dimension equal to the Clebsch-Gordan coefficient $d_{123}^{\underset{\sim}{0}}$.

The inverse notation of equation (2.4) has been used in this definition, which is not strictly correct as the 3jm tensor is not square. The 'invertibility' property is

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\,(j_1j_2j_3)^{r_2}{}_{m_1m_2m_3} = \delta^{r_2}{}_{r_1} \tag{6.3}$$

or in the longer form of equation (2.12)

$$\delta_{\dot{r}_2\,r_1} = (j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}(j_1j_2j_3)_{\dot{r}_2}{}^{\dot{n}_1\dot{n}_2\dot{n}_3}$$

$$\times\ \delta_{\dot{n}_1\dot{n}_2\dot{n}_3\,m_1m_2m_3} \tag{6.4}$$

It is important to note that the orthogonality property only holds in a sum over all three m values. We shall see later that orthogonality in a sum over only two of the m values only holds under very special circumstances, and then only over subspaces of the 3jm multiplicity space. Thus in general,

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}(j_1j_2j_3)^{r_2}{}_{m_1m_2m_3'} \neq \delta^{r_2}{}_{r_1}$$

$$\times\ \delta^{m_3}{}_{m_3'}\,[j_3]^{-1} \tag{6.5}$$

From the unitarity of the reducing matrix, equations (6.1) and (6.2) give

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1} = j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, j_3(u)^{m_3}{}_{n_3}$$

$$\times\ (j_1j_2j_3)^{n_1n_2n_3}{}_{r_1} \tag{6.6}$$

and

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\, \delta^{r_1}{}_{\dot{r}_2}$$

$$= j_1(a)^{m_1}{}_{\dot{n}_1}\, j_2(a)^{m_2}{}_{\dot{n}_2}\, j_3(a)^{m_3}{}_{\dot{n}_3} (j_1j_2j_3)^{\dot{n}_1\dot{n}_2\dot{n}_3}{}_{\dot{r}_2} \tag{6.7}$$

Integration of them and use of the orthogonality equations (3.6) gives the important equations

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\, (j_1j_2j_3)^{r_1}{}_{n_1n_2n_3}$$

$$= \frac{2}{|G|} \int_H j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, j_3(u)^{m_3}{}_{n_3}\, du \tag{6.8}$$

and

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\, \delta^{r_1}{}_{\dot{r}_2}\, (j_1j_2j_3)^{\dot{r}_2}{}_{n_1n_2n_3}$$

$$= \frac{2}{|G|} \int_{G-H} j_1(a)^{m_1}{}_{\dot{n}_1}\, j_2(a)^{m_2}{}_{\dot{n}_2}\, j_3(a)^{m_3}{}_{\dot{n}_3}\, da \tag{6.9}$$

6.2. Completeness of the 3jm tensor

Suppose that $(j_1 j_2 j_3)$ and $[j_1 j_2 j_3]$ are two 3jm tensors of G. That is they both reduce $j_1 \otimes j_2 \otimes j_3$ by equations (6.1) and (6.2). Then as they also satisfy equations (6.8) and (6.9),

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1} (j_1 j_2 j_3)^{r_1}{}_{n_1 n_2 n_3}$$

$$= [j_1 j_2 j_3]^{m_1 m_2 m_3}{}_{r_2} [j_1 j_2 j_3]^{r_2}{}_{n_1 n_2 n_3} \tag{6.10}$$

and

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1}\ \delta^{r_1}{}_{\dot{r}_2} (j_1 j_2 j_3)^{\dot{r}_2}{}_{\dot{n}_1 \dot{n}_2 \dot{n}_3}$$

$$= [j_1 j_2 j_3]^{m_1 m_2 m_3}{}_{r_3}\ \delta^{r_3}{}_{\dot{r}_4} [j_1 j_2 j_3]^{\dot{r}_4}{}_{\dot{n}_1 \dot{n}_2 \dot{n}_3} \tag{6.11}$$

By equation (6.3) these may be written

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1}$$

$$= (j_1 j_2 j_3)^{n_1 n_2 n_3}{}_{r_1} [j_1 j_2 j_3]^{r_2}{}_{n_1 n_2 n_3}$$

$$\times \quad [j_1 j_2 j_3]^{m_1 m_2 m_3}{}_{r_2} \tag{6.12}$$

and

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1}$$

$$= \delta^{\dot{r}_2}{}_{r_1} (j_1 j_2 j_3)^{\dot{n}_1 \dot{n}_2 \dot{n}_3}{}_{\dot{r}_2} [j_1 j_2 j_3]^{\dot{r}_4}{}_{n_1 n_2 n_3}$$

$$\times \quad \delta^{r_3}{}_{\dot{r}_4} [j_1 j_2 j_3]^{m_1 m_2 m_3}{}_{r_3} \tag{6.13}$$

setting

$$U^{r_2}{}_{r_1} = (j_1 j_2 j_3)^{n_1 n_2 n_3}{}_{r_1} \begin{bmatrix} j_1 j_2 j_3 \end{bmatrix}^{r_2}{}_{n_1 n_2 n_3} \tag{6.14}$$

gives

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1} = U^{r_2}{}_{r_1} \begin{bmatrix} j_1 j_2 j_3 \end{bmatrix}^{m_1 m_2 m_3}{}_{r_2} \tag{6.15}$$

$$= \delta^{\dot{r}_2}{}_{r_1} U^{\dot{r}_4}{}_{\dot{r}_2} \delta^{r_3}{}_{\dot{r}_4}$$

$$\times \begin{bmatrix} j_1 j_2 j_3 \end{bmatrix}^{m_1 m_2 m_3}{}_{r_3} \tag{6.16}$$

This shows that any two reductions of the triple product are related by a transformation which is easily shown to be unitary, and since $\delta^{\dot{r}_2}{}_{r_1}$ and $\delta^{r_3}{}_{\dot{r}_4}$ are numerically the identity, it is also real. Thus any two reductions of the triple product are related by an orthogonal transformation. This reality condition, which is stronger than in representation theory, will be shown to be shared by many of the tensors dealt with later.

6.3 Conjugation of the 3jm Tensor

A partly conjugated 3jm tensor may be found by replacing j_3 in equations (6.8) and (6.9) by its conjugate j_3^* through equations (5.8) and (5.9) to give

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\;(j_1j_2j_3)^{r_1}{}_{n_1n_2n_3}$$

$$= \begin{pmatrix} m_3 & \\ & \dot{m}_4 \end{pmatrix} (j_1j_2j_3)^{m_1m_2\dot{m}_4}{}_{r_3}\;(j_1j_2j_3)^{r_3}{}_{n_1n_2\dot{n}_4} \begin{bmatrix} \dot{n}_4 & \\ & n_3 \end{bmatrix}$$

and

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1}\;\delta^{r_1}{}_{\dot{r}_2}\;(j_1j_2j_3)^{\dot{r}_2}{}_{\dot{n}_1\dot{n}_2\dot{n}_3}$$

$$= \begin{pmatrix} m_3 & \\ & \dot{m}_4 \end{pmatrix} (j_1j_2j_3)^{m_1m_2\dot{m}_4}{}_{r_3}\;\delta^{r_3}{}_{\dot{r}_4}\;(j_1j_2j_3)^{\dot{r}_4}{}_{\dot{n}_1\dot{n}_2n_4}$$

$$\times \begin{bmatrix} n_4 & \\ & \dot{n}_3 \end{bmatrix}$$

From the orthogonality of the 3jm tensor these may be written as

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1} = A^{r_3}{}_{r_1}\;(j_1j_2j_3)^{m_1m_2\dot{m}_4}{}_{r_3} \begin{pmatrix} m_3 & \\ & \dot{m}_4 \end{pmatrix} \qquad (6.17)$$

$$= \delta^{\dot{r}_2}{}_{r_1}\;A^{\dot{r}_4}{}_{\dot{r}_2}\;\delta^{r_3}{}_{\dot{r}_4}\;(j_1j_2j_3)^{m_1m_2\dot{m}_4}{}_{r_3}$$

$$\times \begin{pmatrix} m_3 & \\ & \dot{m}_4 \end{pmatrix} \qquad (6.18)$$

with $A^{r_3}{}_{r_1} = (j_1j_2j_3)^{n_1n_2n_3}{}_{r_1} \begin{bmatrix} \dot{n}_4 & \\ & n_3 \end{bmatrix} (j_1j_2j_3)^{r_2}{}_{n_1n_2\dot{n}_4}$ (6.19) as a real unitary (orthogonal) tensor.

In representation theory, orthogonality of the 3jm symbol in a sum over n_1 and n_2 gives A as a diagonal tensor so that equation (6.17) assumes a very simple form. As this does not holds in general in corepresentation theory, A is not diagonal.

A tight relation between the fully conjugated 3jm tensor and the non-conjugated tensor already exists-namely equation (6.7). This may be specialized by setting a = θ to give

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1} = \delta^{\dot{r}_2}{}_{r_1} \begin{pmatrix} m_1 & \\ & \dot{n}_1 \end{pmatrix} \begin{pmatrix} m_2 & \\ & \dot{n}_2 \end{pmatrix} \begin{pmatrix} m_3 & \\ & \dot{n}_3 \end{pmatrix} \times (j_1j_2j_3)^{\dot{n}_1\dot{n}_2\dot{n}_3}{}_{\dot{r}_2} \quad (6.20)$$

Suppose now that one column of the 3jm tensor is real, that is

$$(j_1j_2j_3)^{\dot{n}_1\dot{n}_2\dot{n}_3}{}_{\dot{r}_2} = \delta^{\dot{n}_1}{}_{n_4}\delta^{\dot{n}_2}{}_{n_5}\delta^{\dot{n}_3}{}_{n_6}\delta^{r_3}{}_{\dot{r}_2}(j_1j_2j_3)^{n_4n_5n_6}{}_{r_3}$$

Equation (6.7) then gives

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1} = \delta^{\dot{n}_1}{}_{n_4}\delta^{\dot{n}_2}{}_{n_5}\delta^{\dot{n}_3}{}_{n_6}(j_1j_2j_3)^{n_4n_5n_6}{}_{r_1}$$

$$\times\ j_1(a)^{m_1}{}_{\dot{n}_1}\, j_2(a)^{m_2}{}_{\dot{n}_2}\, j_3(a)^{m_3}{}_{\dot{n}_3} \quad (6.21)$$

This is not analytically soluble, but some simple known solutions correspond to physically important groups.

For example two solutions are

(a) If there exists some anti-linear operator $\bar{\theta}$ for which $j_1(\bar{\theta})$, $j_2(\bar{\theta})$ and $j_3(\bar{\theta})$ are all numerically the identity, then equation (6.2) is satisfied and the complete 3 jm tensor is real

(b) If there exists $\bar{\theta}$ for which one of $j_1(\bar{\theta})$, $j_2(\bar{\theta})$ and $j_3(\bar{\theta})$ is I, and the other two -I, the 3jm tensor is again real

A case where (a) occurs is in grey SU(2) with Fano-Racah standardization [8] where

$$\bar{\theta} = C_y^2 \bar{\theta}$$

is equal to I for all ICRs. Thus all 3jm symbols of grey SU(2) are real.

CHAPTER SEVEN : SYMMETRIES AND MULTIPLICITIES OF THE n-jm SYMBOLS

7.1 The 1jm Symbol or Wigner Tensor

The 1-jm symbol or Wigner tensor, which transforms j to its complex conjugate j^* has already been dealt with in chapter three section two and chapter five, and from these its symmetry properties under transposition may be readily found. For if $k = \beta k^* \beta^{-1}$ then $\beta^T = \beta$ if k is of the first kind and $\beta^T = -\beta$ if k is of the second. Then from chapter three section two, the Wigner tensor $j(\theta)$ is symmetric for ICRs of types (A), (B) and (C) and anti-symmetric for types (D), (E) and (F). These properties are preserved under any transformation $P\, j(\theta) P^{-1*}$. However, it should be noted that these symmetry properties are not necessarily possessed by every matrix which transforms j to j^*. For as mentioned before, if P commutes with j, then $Pj(\theta)$ also gives the transition to j^*. This is intimately connected with the fact that $d^{j}_{j\underset{\sim}{0}}$ need not equal $d^{\underset{\sim}{0}}_{jj}$. A further clarification will be seen in the next section.

7.2 The 2jm Symbol

Due to the equivalence of j and j^*, we define the 2jm symbol to be the tensor which reduces $j \otimes j$ to $\underset{\sim}{0}$:

$$j(u)^{m_1}{}_{n_1}\ j(u)^{m_2}{}_{n_2} = (jj)^{m_1m_2}{}_{r_1}\ (jj)^{r_1}{}_{n_1n_2} \oplus \ldots\ldots (7.1)$$

and

$$j(u)^{m_1}{}_{\dot{n}_1}\ j(u)^{m_2}{}_{\dot{n}_2} = (jj)^{m_1m_2}{}_{r_1}\ \delta^{r_1}{}_{\dot{r}_2} (jj)^{\dot{r}_2}{}_{\dot{n}_1\dot{n}_2} \oplus \ldots (7.2)$$

or

$$(jj)^{m_1m_2}{}_{r_1}\ (jj)^{r_1}{}_{n_1n_2} = \frac{2}{|G|} \int_H j(u)^{m_1}{}_{n_1}\ j(u)^{m_2}{}_{n_2} du \quad (7.3)$$

and

$$(jj)^{m_1m_2}{}_{r_1} (jj)^{\dot{r}_2}{}_{\dot{n}_1\dot{n}_2}\ \delta^{r_1}{}_{\dot{r}_2} = \frac{2}{|G|} \int_{G-H} j(a)^{m_1}{}_{\dot{n}_1}$$

$$\times \quad j(a)^{m_2}{}_{\dot{n}_2}\ da \quad (7.4)$$

This is a special case of the 3jm symbol with one ICR equal to the identity $\underset{\sim}{0}$. The multiplicities of the reduction were given in chapter three section three and are one, four and two for ICRs of types (a), (b) and (c) respectively.

Under a permutation of the m values, the permuted symbol must also satisfy equation (7.3), and so

$$(jj)^{m_1m_2}{}_{r_1}\ (jj)^{r_1}{}_{n_1n_2} = (jj)^{m_2m_1}{}_{r_2}\ (jj)^{r_2}{}_{n_2n_1} \quad (7.5)$$

or

$$(jj)^{m_1m_2}{}_{r_1} = M(12)^{r_2}{}_{r_1}\ (jj)^{m_2m_1}{}_{r_2} \quad (7.6)$$

with $$M(12)^{r_2}{}_{r_1} = (jj)^{r_2}{}_{n_2 n_1} (jj)^{n_1 n_2}{}_{r_1} \tag{7.7}$$

Thus the 2jm symbol generates a representation M(I), M(12) of the permutation group S_2, which has the symmetric IR {2} and the anti-symmetric IR $\{1^2\}$. The multiplicities of these are respectively

$$m_{\{2\}} = \frac{1}{2}\left[\chi_M(I) + \chi_M(12)\right]$$

$$= \frac{1}{2}\left[M(I)^{r_1}{}_{r_1} + M(12)^{r_1}{}_{r_1}\right]$$

and $$m_{\{1^2\}} = \frac{1}{2}\left[M(I)^{r_1}{}_{r_1} - M(12)^{r_1}{}_{r_1}\right]$$

But from equations (7.7) and (7.3),

$$M(I)^{r_1}{}_{r_1} = (jj)^{r_1}{}_{n_1 n_2} (jj)^{n_1 n_2}{}_{r_1}$$

$$= \frac{2}{|G|} \int_H j(u)^{n_1}{}_{n_1}\, j(u)^{n_2}{}_{n_2} du$$

$$= \frac{2}{|G|} \int_H (\chi_j(u))^2\, du$$

and $$M(12)^{r_1}{}_{r_1} = \frac{2}{|G|} \int_H \chi_j(u^2)\, du$$

Therefore

$$m_{\{2\}} = \frac{1}{|G|} \int_H (\chi_j(u))^2 + \chi_j(u^2)\, du \tag{7.8}$$

and

$$m_{\{1^2\}} = \frac{1}{|G|} \int_H (\chi_j(u))^2 - \chi_j(u^2)\, du \tag{7.9}$$

As each of these integrals is only over the linear subgroup M, each ICR j may be written as a sum of IRs of H. Thus for example, if j is of type (B), $\chi_j(u) = 2\chi_k(u)$, with k of the second kind. Since the multiplicity of {2} in k ⊗ k is then zero and of $\{1^2\}$ is one, this gives

$$m_{\{2\}} = 1 \quad \text{and} \quad m_{\{1^2\}} = 3$$

A Frobenius-Schur invariant may be defined by

$$C_j = \frac{2}{|G|} \int_H \chi_j(u^2)\, du \qquad (7.10)$$

and by working through each type of ICR in turn it is found that C_j completely characterizes the symmetry of the 2jm symbol. The rather curious results are given in table 7.

Before concluding this section, we make a few remarks concerning the relation between the 2jm and 1jm symbols. In representation theory, since the multiplicity of $\underset{\sim}{0}$ in k ⊗ k is one if k is equivalent to k^* the 1jm and 2jm symbols are frequently treated as almost identical. Here the multiplicity may be higher, but comparison of the symmetries of the two symbols shows that the 1jm symbol is closely related to one column of the symmetrized 2jm symbol. This relation, involving as it does the coupling coefficient,will be dealt with in the next chapter.

Table 7: Symmetry structure of the 2jm symbol

Type of ICR	Multiplicity of $\underset{\sim}{0}$ in $j \otimes \{2\}$	Multiplicity of $\underset{\sim}{0}$ in $j \otimes \{1^2\}$	Frobenius-Schur invariant
A	1	0	1
B	1	3	-2
C	1	1	0
D	0	1	-1
E	3	1	2
F	1	1	0

7.3 The 3jm Symbol

The triple product $j_1 \otimes j_2 \otimes j_3$ may be reduced to the identity $\underset{\sim}{0}$ by a variety of differently ordered 3jm tensors. However since each tensor must satisfy equation (6.8) we have

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1} = M(12,3)^{r_2}{}_{r_1} (j_2 j_1 j_3)^{m_2 m_1 m_3}{}_{r_2}$$

$$= M(1,23)^{r_3}{}_{r_1} (j_1 j_3 j_2)^{m_1 m_3 m_2}{}_{r_3} \tag{7.11}$$

etcetera with

$$M(12,3)^{r_2}{}_{r_1} = (j_1 j_2 j_3)^{n_1 n_2 n_3}{}_{r_1} (j_2 j_1 j_3)^{r_2}{}_{n_2 n_1 n_3} \tag{7.12}$$

etcetera, while the anti-linear equation (6.9) shows that each permutation matrix M is a real orthogonal matrix. This reality forms the only difference between the analyses of Derome and Sharp [23,24] and the corresponding analysis required here, and since their results are not dependent on M being non-real, we may quote them without modification. They, and we, distinguish between three major cases

(a) If none of j_1, j_2 and j_3 are equivalent, every M may be simultaneously diagonalized, with the diagonal elements arbitrary. Common choices for these elements are one, so that each transposition causes no sign change or $(-1)^{j_1 + j_2 + j_3}$ for transpositions, as in SU(2).

(b) If exactly two are equivalent, then every transposition may be diagonalized to

$$\begin{pmatrix} I_s & 0 \\ 0 & -I_A \end{pmatrix}$$

and every cyclic permutation equals the identity

(c) If $j_1 \equiv j_2 \equiv j_3$, every M may be diagonalized iff

$$\int_H (\chi_j(u))^3 \, du = \int_H \chi_j(u^3) \, du \qquad (7.13)$$

As with the 2jm symbol, we may also establish the 3jm transformation properties in terms of those of the linear subgroup H. Consider for example case (c) with all three j's equivalent. The matrices M now generate a representation of S_3 with IRs {3}, {21}, $\{1^3\}$. The characters of typical elements are

$$\chi_M(I) = M(I)^{r_1}{}_{r_1} = (jjj)^{n_1 n_2 n_3}{}_{r_1} (jjj)^{r_1}{}_{n_1 n_2 n_3}$$

$$= \frac{2}{|G|} \int_H (\chi_j(u))^3 \, du$$

$$\chi_M(12,3) = \frac{2}{|G|} \int_H (\chi_j(u))^2 \chi_j(u) \, du$$

$$\chi_M(123) = \frac{2}{|G|} \int_H \chi_j(u^3) \, du$$

where M(123) is a cyclic permutation of the three j's.

The multiplicities of the IRs of S_3 are then given by

$$m^j_{\{3\}} = \frac{1}{3|G|} \int_H (\chi_j(u))^3 + 3\chi_j(u^2)\chi_j(u) + \chi_j(u^3))\,du \qquad (7.14)$$

$$m^j_{\{21\}} = \frac{2}{3|G|} \int_H (\chi_j(u))^3 - \chi_j(u^3))\,du \qquad (7.15)$$

and

$$m^j_{\{1^3\}} = \frac{1}{3|G|} \int_H (\chi_j(u))^3 - 3\chi_j(u^2)\chi_j(u) + 2\chi_j(u^3))du \qquad (7.16)$$

If j is of type (a) then these multiplicities equal those of the linear subgroup H, and writing Γ_j and Γ_k for the representations generated by j and k respectively,

$$\Gamma_j = \Gamma_k \qquad (7.17)$$

If j is of type (b), then $\chi_j(u) = 2\chi_k(u)$ from which

$$m^j_{\{3\}} = 4\, m^k_{\{3\}} + 2\, m^k_{\{21\}} \qquad (7.18)$$

$$m^j_{\{21\}} = 2\, m^k_{\{3\}} + 6\, m^k_{\{21\}} + 2\, m^k_{\{1^3\}} \qquad (7.19)$$

$$m^j_{\{1^3\}} = 2\, m^k_{\{21\}} + 4\, m^k_{\{13\}} \qquad (7.20)$$

or

$$\Gamma_j = (4\,\{3\} \oplus 2\,\{21\}) \otimes \Gamma_k \qquad (7.21)$$

From equation (7.19) the multiplicity of the mixed symmetry term must be non-zero and hence it is impossible to transform the 3jm tensor to a form in which the rows merely change sign under permutations. The best that can be achieved is that M takes the form

$$M = \begin{pmatrix} M^{\{3\}} & & & & & & & & 0 \\ & \ddots & & & & & & & \\ & & M^{\{3\}} & & & & & & \\ & & & M^{\{21\}} & & & & & \\ & & & & \ddots & & & & \\ & & & & & M^{\{21\}} & & & \\ & & & & & & M^{\{1^3\}} & & \\ & & & & & & & \ddots & \\ 0 & & & & & & & & M^{\{1^3\}} \end{pmatrix}$$

where

$$M^{\{3\}}(12,3) = M^{\{3\}}(1,23) = -M^{\{1^3\}}(12,3) = -M^{\{1^3\}}(1,23) = 1$$

and

$$M^{\{21\}}(12,3) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad M^{\{21\}}(1,32) = \begin{pmatrix} -1/2 & \sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$$

Finally, if j is of type (c), restriction to H will give the IR couplings $k \otimes k \otimes k$ (with $k^* \otimes k^* \otimes k^*$ yielding an equivalent representation of S_3) and $k \otimes k \otimes k^*$ (with $k^* \otimes k^* \otimes k$ yielding an equivalent representation). Setting $m^k{}_{\{3\}}$ etcetera as before, and $m^k{}_{\{2\}}$ to be multiplicity of k in $k \otimes \{2\}$, $m^k{}_{\{1^2\}}$ to be the multiplicity of k in $k \otimes \{1^2\}$

$$m^j{}_{\{3\}} = 2\, m^k{}_{\{3\}} + 2\, m^k{}_{\{2\}}$$

$$m^j{}_{\{21\}} = 2\, m^k{}_{\{21\}} + 2\, m^k{}_{\{1^2\}} + 2m^k\, \{2\}$$

$$m^j{}_{\{1^3\}} = 2\, m^k{}_{\{1^3\}} + 2\, m^k{}_{\{1^2\}}$$

Writing Γ_{kkk} to be the representation of S_3 generated by $k \otimes k \otimes k$, and Γ_{kkk^*} to be the representation of S_2 generated by $k \otimes k \otimes k^*$, this is

$$\Gamma_j = 2\{3\} \otimes \Gamma_{kkk} \oplus 2\{2\} \odot \Gamma_{kkk^*}$$

where $\odot$ represents the outer product $S_n \odot S_m \rightarrow S_{n+m}$ [14].

In a similar manner the permutation properties for $j_1 \equiv j_2 \not\equiv j_3$ may be found in terms of those of the linear subgroup. They are summarized in table 8.

Table 8: Symmetry structure Γ_j of the 3jm tensor $(j_1 j_1 j_2)$ in terms of the symmetry structure of the tensor $(k_1 k_1 k_2)$ of the linear subgroup

j_1	j_1	j_2	Γ_j
(a)	(a)	(a)	$\Gamma_{k_1k_1k_2}$
(a)	(a)	(b)	$2\Gamma_{k_1k_1k_2}$
(a)	(a)	(c)	$2\Gamma_{k_1k_1k_2}$
(b)	(b)	(a)	$(3\{2\} \oplus \{1^2\}) \otimes \Gamma_{k_1k_1k_2}$
(b)	(b)	(b)	$(6\{2\} \oplus 2\{1^2\}) \otimes \Gamma_{k_1k_1k_2}$
(b)	(b)	(c)	$(6\{2\} \oplus 2\{1^2\}) \otimes \Gamma_{k_1k_1k_2}$
(c)	(c)	(a)	$2\{2\} \otimes \Gamma_{k_1k_1k_2} \oplus 2\{1\} \odot \Gamma_{k_1k_1^*k_2}$
(c)	(c)	(b)	$4\{2\} \otimes \Gamma_{k_1k_1k_2} \oplus 4\{1\} \odot \Gamma_{k_1k_1^*k_2}$
(c)	(c)	(c)	$2\{2\} \otimes \Gamma_{k_1k_1k_2} \oplus 2\{2\} \otimes \Gamma_{k_1k_1k_2^*} \oplus 4\{1\} \odot \Gamma_{k_1k_1^*k_2}$

CHAPTER EIGHT : THE COUPLING COEFFICIENT

8.1 Definition and a Difficulty

At this stage in the development of the Racah algebra it is appropriate to discuss the coupling coefficient which is the unitary transformation reducing $j_1 \otimes j_2$ to a sum of ICRs j_3:

$$j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2} = \sum_{j_3} \langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}\, j_3(u)^{m_3}{}_{n_3} \times \langle j_1 j_2 | j_3 \rangle^{r_1 n_3}{}_{n_1 n_2} \tag{8.1}$$

and

$$j_1(a)^{m_1}{}_{\dot{n}_1}\, j_2(a)^{m_2}{}_{\dot{n}_2} = \sum_{j_3} \langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}\, \delta^{r_1}{}_{\dot{r}_2} \times j_3(a)^{m_3}{}_{\dot{n}_3}\, \langle j_1 j_2 | j_3 \rangle^{\dot{r}_2 \dot{n}_3}{}_{\dot{n}_1 \dot{n}_2} \tag{8.2}$$

The unitarity of the reduction gives the orthogonality relations

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r m_3}\, \langle j_1 j_2 | j_3 \rangle^{r' m_3'}{}_{m_1 m_2} = \delta^{r'}{}_{r}\, \delta^{m_3'}{}_{m_3} \tag{8.3}$$

and

$$\sum_{r j_3} \langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r m_3}\, \langle j_1 j_2 | j_3 \rangle^{r m_3}{}_{m_1' m_2'} = \delta^{m_1}{}_{m_1'}\, \delta^{m_2}{}_{m_2'} \tag{8.4}$$

These allow the defining equations to be rewritten as

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}\, j_3(u)^{m_3}{}_{n_3} = j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, \langle j_1 j_2 | j_3 \rangle^{n_1 n_2}{}_{r_1 n_3} \tag{8.5}$$

and

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}\, j_3(a)^{m_3}{}_{\dot{n}_3}$$

$$= \delta^{\dot{r}_2}{}_{r_1}\, j_1(a)^{m_1}{}_{\dot{n}_1}\, j_2(a)^{m}{}_{2}{}_{\dot{n}_2} \langle j_1 j_2 | j_3 \rangle^{\dot{n}_1 \dot{n}_2}{}_{\dot{r}_2 \dot{n}_3} \quad (8.6)$$

One problem has already been mentioned in connection with the coupling coefficients, namely that the Clebsch-Gordan coefficient d_{12}^{3} need not equal the coefficient d_{13}^{2}. There is another however, which is far worse: if there are two reductions of the product $j_1 \otimes j_2$ to j_3, the coupling coefficient tensors <u>need not be related by a unitary transformation in the multiplicity label</u>. This is simply seen by setting $j_3 = j_1$ and $j_2 = \underset{\sim}{0}$ in these last two equations to give

$$\langle j\ \underset{\sim}{0} | j \rangle^{m_1 0}{}_{r_1 m_3}\, j(u)^{m_3}{}_{n_3}$$

$$= j(u)^{m_1}{}_{n_1} \langle j\ \underset{\sim}{0} | j \rangle^{n_1 0}{}_{r_1 n_3} \quad (8.7)$$

and

$$\langle j\ \underset{\sim}{0} | j \rangle^{m_1 0}{}_{r_1 m_3}\, j(a)^{m_3}{}_{\dot{n}_3}$$

$$= j(a)^{m_1}{}_{\dot{n}_1}\, \delta^{\dot{r}_2}{}_{r_1} \langle j\ \underset{\sim}{0} | j \rangle^{\dot{n}_1 0}{}_{\dot{r}_2 \dot{n}_3} \quad (8.8)$$

$\langle j\ \underset{\sim}{0} | j \rangle$ thus commutes with j and for a type (b) ICR is a real linear combination of

$$\begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \begin{pmatrix} iI & 0 \\ 0 & -iI \end{pmatrix}, \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & iI \\ iI & 0 \end{pmatrix}$$

Indeed any of these four matrices form perfectly adequate coupling coefficients $\langle j\, \underset{\sim}{0}\, | j\rangle$. Since trivially $d^{1}_{1\underset{\sim}{0}}$ is one and these matrices are not simple multiples of one another, the result follows.

A notational device will be useful to distinguish between these different coupling coefficients, and we write

$$\langle j\, \underset{\sim}{0}\; |\; j\rangle^{m_1 0}{}_{(r)m_3} \tag{8.9}$$

The label (r) is not a multiplicity label as the multiplicity is only one but (and here is the reason for stating Schur's lemma in the particular form of chapter 3 section 4) (r) = 1, ..., $d^{\underset{\sim}{0}}_{11}$. As $\langle j\, \underset{\sim}{0}\, |\, j\rangle_{(r)}$ commutes with j, the general result follows as

$$\langle j_1 j_2 | j_3\rangle^{m_1 m_2}{}_{r_1 m_4}\, \langle j_3\, \underset{\sim}{0}\, | j_3\rangle^{m_4 0}{}_{(r)m_3} \tag{8.10}$$

and

$$\langle j_1 j_2 | j_3\rangle^{m_1 m_2}{}_{r_1 m_4}\, \langle j_3\, \underset{\sim}{0}\, | j_3\rangle^{m_4 0}{}_{(r')m_3}$$

are both coupling coefficient tensors not related by a transformation in the label r_1 unless $r = r'$.

8.2 Relation Between the 3jm and Coupling Coefficient Tensors

Despite the fact that d_{12}^{3} and $d_{12\tilde{3}}^{0}$ are not always equal, a variety of relations exist between the 3jm and coupling coefficient tensors. The first is found by by using the unitary and anti-unitary equations (2.20) and (2.25) in equations (8.5) and (8.6) to give

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}\ \delta^{m_3 \dot{m}_4}$$

$$= \quad j_1(u)^{m_1}{}_{n_1}\ j_2(u)^{m_2}{}_{n_2}\ j_3(u)^{\dot{m}_4}{}_{\dot{n}_4}$$

$$\times\ \langle j_1 j_2 | j_3 \rangle^{n_1 n_2}{}_{r_1 n_3}\ \delta^{n_3 \dot{n}_4} \qquad (8.11)$$

and

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}\ \delta^{m_3 \dot{m}_4}$$

$$= \quad j_1(a)^{m_1}{}_{\dot{n}_1}\ j_2(a)^{m_2}{}_{\dot{n}_2}\ j_3(a)^{\dot{m}_4}{}_{n_4}\ \delta^{\dot{r}_2}{}_{r_1}$$

$$\times\ \langle j_1 j_2 | j_3 \rangle^{\dot{n}_1 \dot{n}_2}{}_{\dot{r}_2 \dot{n}_3}\ \delta^{\dot{n}_3 n_4} \qquad (8.12)$$

Using the Wigner tensor to replace j_3^* by j_3, integrating over H and using equation (6.8) gives for equation (8.11)

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}\ \delta^{m_3 \dot{m}_4}$$

$$= \quad (j_1 j_2 j_3)^{m_1 m_2 m_5}{}_{r_2} (j_1 j_2 j_3)^{r_2}{}_{n_1 n_2 n_5} \begin{bmatrix} \dot{m}_4 & \\ & m_5 \end{bmatrix} \begin{pmatrix} n_5 & \\ & \dot{n}_4 \end{pmatrix}$$

$$\times\ \langle j_1 j_2 | j_3 \rangle^{n_1 n_2}{}_{r_1 n_3}\ \delta^{n_3 \dot{n}_4} \qquad (8.13)$$

Setting

$$U^{r_2}{}_{r_1} = [j_3]^{-\frac{1}{2}} (j_1 j_2 j_3)^{r_2}{}_{n_1 n_2 n_5} \begin{bmatrix} n_5 & \\ & \dot{n}_4 \end{bmatrix}$$

$$\times \ \langle j_1 j_2 | j_3 \rangle^{n_1 n_2}{}_{r_1 n_3} \ \delta^{n_3 \dot{n}_4} \tag{8.14}$$

simplifies this to

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}$$

$$= [j_3]^{\frac{1}{2}} \ U^{r_2}{}_{r_1} \ \delta_{m_3 \dot{m}_4} \begin{bmatrix} \dot{m}_4 & \\ & m_5 \end{bmatrix} (j_1 j_2 j_3)^{m_1 m_2 m_5}{}_{r_2} \tag{8.15}$$

The anti-linear equation shows that the rectangular tensor $U^{r_2}{}_{r_1}$ is real. If required an orthogonal transformation may be applied to the 3jm multiplicity space to bring this to the form

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3} = \pm [j_3]^{\frac{1}{2}} \delta_{m_3 \dot{m}_4} \begin{bmatrix} \dot{m}_4 & \\ & m_5 \end{bmatrix}$$

$$(j_1 j_2 j_3)^{m_1 m_2 m_5}{}_{r_1} \tag{8.16}$$

where the normalization of the 3jm and coupling coefficient tensors has been used to give $|U| = 1$.

It is clear from this last equation that the coupling coefficient tensor spans a subspace of the 3jm multiplicity space and that different tensors of the form (8.10) span different subspaces, which may easily be shown to be orthogonal in a sum over all three m-values. Since the 3jm tensor is complete in the sense that any other 3jm tensor is related by an orthogonal transformation, it follows that any coupling coefficient tensor is an orthogonal combination of tensors of the form (8.10)

$$<j_1j_2|j_3>^{m_1m_2}{}_{r_1m_3} = U^{r_2(r)}{}_{r_1} <j_1j_2|j_3>^{m_1m_2}{}_{r_2m_4} <j_3\underset{\sim}{0}|j_3>^{m_40}{}_{(r)m_3} \qquad (8.17)$$

A relation between the coupling coefficient and partly conjugated 3jm tensors may be found by not using the Wigner tensor in proceeding from equations (8.11) and (8.12) to give

$$<j_1j_2|j_3>^{m_1m_2}{}_{r_1m_3} = [j_3]^{\frac{1}{2}} V^{r_2}{}_{r_1} \delta_{m_3\dot{m}_4} (j_1j_2j_3)^{m_1m_2\dot{m}_4}{}_{r_2} \qquad (8.18)$$

with $V^{r_2}{}_{r_1} = [j_3]^{-\frac{1}{2}} (j_1j_2j_3)^{r_2}{}_{n_1n_2\dot{n}_4} \delta^{n_3\dot{n}_4}$

$$\times <j_1j_2|j_3>^{n_1n_2}{}_{r_1n_3} \qquad (8.19)$$

The two tensors U and V are related by the tensor A of equation (6.19) by

$$V^{r_2}{}_{r_1} = A^{r_3}{}_{r_1} U^{r_2}{}_{r_3} \qquad (8.20)$$

The second main relation between these two tensors acts in a complementary sense to equation (8.15) and the remarks preceeding equation (8.17) by using rather than $\langle j\underset{\sim}{0}|j\rangle$, the tensor $\langle jj|\underset{\sim}{0}\rangle$. In equations (6.8) and (6.9), if the triple product $j_1 \otimes j_2 \otimes j_3$ is treated by the intermediate coupling of j_1 and j_2 to j_{12} then coupled to j_3, on integration all ICRs apart from $\underset{\sim}{0}$ vanish to give

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1} = W^{r_2r_3}{}_{r_1} \langle j_1j_2|j_3\rangle^{m_1m_2}{}_{r_2m_4}$$

$$\times \quad \langle j_3j_3|\underset{\sim}{0}\rangle^{m_4m_3}{}_{r_30} \qquad (8.21)$$

with W as the orthogonal tensor

$$W^{r_2r_3}{}_{r_1} = (j_1j_2j_3)^{n_1n_2n_3}{}_{r_1} \langle j_1j_2|j_3\rangle^{r_2n_4}{}_{n_1n_2}$$

$$\times \quad \langle j_3j_3|\underset{\sim}{0}\rangle^{r_30}{}_{n_4n_3} \qquad (8.22)$$

8.3 The 1jm and 2jm Symbols Revisited

Let us specialize equation (8.16) by setting $j_1 = j_3$ and $j_2 = \underset{\sim}{0}$. For simplicity we also choose the positive sign although this is not important. Then

$$< j\underset{\sim}{0}\,|j>^{m_1 0}{}_{1 m_3} = [j]^{\frac{1}{2}}\,\delta_{m_3 \dot{m}_4} \begin{bmatrix} \dot{m}_4 & \\ & m_5 \end{bmatrix} (j\underset{\sim}{0}j)^{m_1 0 m_5}{}_1$$

There is of course an essential arbitrariness in the coupling coefficient, but choosing the simplest case when it equals the identity,

$$\delta^{m_1}{}_{m_3} = [j]^{\frac{1}{2}}\,\delta_{m_3 \dot{m}_4} \begin{bmatrix} \dot{m}_4 & \\ & m_5 \end{bmatrix} (j\,\underset{\sim}{0}\,j)^{m_1\,0\,m_5}{}_1$$

But $(j\,\underset{\sim}{0}\,j)$ is nothing more than the 2jm symbol, so

$$\delta^{m_1 \dot{m}_4} \begin{pmatrix} m_5 & \\ & \dot{m}_4 \end{pmatrix} = [j]^{\frac{1}{2}}\,(jj)^{m_1 m_5}{}_1$$

In representation theory this immediately gives the 2jm symbol if the 1jm symbol is known or vice-versa. In corepresentation theory, as the multiplicity of the 2jm symbol may be greater than one, the complete correspondence is lost. However, it still exists in a limited sense as an examination of the symmetries of both symbols shows. For if $\theta^2 = I$, the 1jm symbol is symmetric under interchange of the m-values and from table 7 the symmetric IR $\{2\}$ also only occurs once in the 2jm symbol, whereas the anti-symmetric IR $\{1^2\}$ may occur up to three times.

Similarly, if $\theta^2 = -I$, the 1jm symbol is anti-symmetric and $\{1^2\}$ only occurs once in the symmetry representation of the 2jm symbol. Thus there is a one-to-one correspondence between the 1jm symbol and the unique column of the 2jm symbol having the same symmetry.

CHAPTER NINE : RECOUPLING AND n-j SYMBOLS

9.1 The Recoupling of Three ICRs

The difference between the coupling coefficient tensor and the 3jm symbol forebodes that a variety of distinct recoupling and n-j symbols exist, and indeed this turns out to be the case. The first one we shall deal with is the recoupling of three ICRs j_1, j_2 and j_3 to j_4. Performing this through the intermediate couplings of $j_1 \otimes j_2$ to j_{12} and $j_2 \otimes j_3$ to j_{23} by equation (8.1) gives respectively

$$j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, j_3(u)^{m_3}{}_{n_3}$$

$$= \sum_{j_{12}} \langle j_1 j_2 | j_{12}\rangle^{m_1 m_2}{}_{r_1 m_{12}}\, j_{12}(u)^{m_{12}}{}_{n_{12}}\, \langle j_1 j_2 | j_{12}\rangle^{r_1 n_{12}}{}_{n_1 n_2}$$

$$= \sum_{j_{12} j_4} \langle j_1 j_2 | j_{12}\rangle^{m_1 m_2}{}_{r_1 m_{12}}\, \langle j_{12} j_3 | j_4\rangle^{m_{12} m_3}{}_{r_2 m_4}$$

$$\times\ j(u)^{m_4}{}_{n_4}\, \langle j_1 j_2 | j_{12}\rangle^{r_1 n_{12}}{}_{n_1 n_2}\, \langle j_{12} j_3 | j_4\rangle^{r_2 n_4}{}_{n_{12} n_3} \tag{9.1}$$

and

$$j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, j_3(u)^{m_3}{}_{n_3}$$

$$= \sum_{j_{23} j_5} \langle j_2 j_3 | j_{23}\rangle^{m_2 m_3}{}_{r_3 m_{23}}\, \langle j_1 j_{23} | j_5\rangle^{r_4 m_1 m_{23}}{}_{r_4 m_5}$$

$$\times\ j_5(u)^{m_5}{}_{n_5}\, \langle j_2 j_3 | j_{23}\rangle^{r_3 n_{23}}{}_{n_2 n_3}\, \langle j_1 j_{23} | j_5\rangle^{r_4 n_5}{}_{n_1 n_{23}} \tag{9.2}$$

Use of the orthogonality equation (8.4) gives

$$j_4(u)^{m_4}{}_{n_4} \; \langle j_1 j_2 | j_{12} \rangle^{r_1 n_{12}}{}_{n_1 n_2} \; \langle j_{12} j_3 | j_4 \rangle^{r_2 n_4}{}_{n_{12} n_3}$$

$$= \sum_{j_{23} j_5} \langle j_1 j_2 | j_{12} \rangle^{r_1 m_{12}}{}_{m_1 m_2} \; \langle j_{12} j_3 | j_4 \rangle^{r_2 m_4}{}_{m_{12} m_3} \; \langle j_2 j_3 | j_{23} \rangle^{m_2 m_3}{}_{r_3 m_{23}}$$

$$\times \; \langle j_1 j_{23} | j_5 \rangle^{r_4 m_1 m_{23}}{}_{r_4 m_5} \; j_5(u)^{m_5}{}_{n_5} \; \langle j_2 j_3 | j_{23} \rangle^{r_3 n_{23}}{}_{n_2 n_3}$$

$$\times \; \langle j_1 j_{23} | j_5 \rangle^{r_4 n_5}{}_{n_1 n_3} \qquad (9.3)$$

or

$$j_4(u)^{m_4}{}_{n_4} \; \langle j_1 j_2 | j_{12} \rangle^{r_1 n_{12}}{}_{n_1 n_2} \; \langle j_{12} j_3 | j_4 \rangle^{r_2 n_4}{}_{n_{12} n_3}$$

$$\times \; \langle j_2 j_3 | j_{23} \rangle^{n_2 n_3}{}_{r_3 n_{23}} \; \langle j_1 j_{23} | j_5 \rangle^{n_1 n_{23}}{}_{r_4 n_5}$$

$$= \langle j_1 j_2 | j_{12} \rangle^{r_1 m_{12}}{}_{m_1 m_2} \; \langle j_{12} j_3 | j_4 \rangle^{r_2 m_4}{}_{m_{12} m_3} \; \langle j_2 j_3 | j_{23} \rangle^{m_2 m_3}{}_{r_3 m_{23}}$$

$$\times \; \langle j_1 j_{23} | j_5 \rangle^{m_1 m_{23}}{}_{r_4 m_5} \; j_5(u)^{m_5}{}_{n_5} \qquad (9.4)$$

The tensor composed of the four coupling coefficients on either side of this equation satisfies

$$j_4(u) \; R = R \; j_5(u) \qquad (9.5)$$

and also $\qquad j_4(a) \; R^* = R \; j_5(a)$

and hence is zero unless $j_4 = j_5$ and so we may define the recoupling coefficient as

$$\langle (j_1j_2)j_{12}\ j_3\ j_4 | j_1(j_2j_3)j_{23}\ j_4\rangle^{r_1r_2m_4}{}_{r_3r_4m_5}$$

$$= \langle j_1j_2|j_{12}\rangle^{r_1m_{12}}{}_{m_1m_2} \langle j_{12}j_3|j_4\rangle^{r_2m_4}{}_{m_{12}m_3}$$

$$\times \langle j_2j_3|j_{23}\rangle^{m_2m_3}{}_{r_3m_{23}} \langle j_1j_{23}|j_4\rangle^{m_1m_{23}}{}_{r_4m_5} \qquad (9.6)$$

This may be inserted into equation (9.3) and u set equal to the identity of G to give an alternative definition

$$\langle j_1j_2|j_{12}\rangle^{r_1n_{12}}{}_{n_1n_2} \langle j_{12}j_3|j_4\rangle^{r_2n_4}{}_{n_{12}n_3}$$

$$= \sum_{j_{23}} \langle (j_1j_2)j_{12}j_3j_4|j_1(j_2j_3)j_{23}j_4\rangle^{r_1r_2n_4}{}_{r_3r_4n_5}$$

$$\times \langle j_2j_3|j_{23}\rangle^{r_3n_{23}}{}_{n_2n_3} \langle j_1j_{23}|j_4\rangle^{r_4n_5}{}_{n_1n_{23}} \qquad (9.7)$$

From this immediately follows the orthogonality property

$$\delta^{r_1}{}_{r_1'}\ \delta^{r_2}{}_{r_2'}\ \delta^{n_4}{}_{n_4'}\ \delta(j_{12}, j_{12}')$$

$$= \sum_{j_{23}} \langle (j_1j_2)j_{12}j_3j_4|j_1(j_2j_3)j_{23}j_4\rangle^{r_1r_2n_4}{}_{r_3r_4n_5}$$

$$\times \langle (j_1j_2)j_{12}'j_3j_4|j_1(j_2j_3)j_{23}j_4\rangle^{-1\,r_3r_4n_5}{}_{r_1'r_2'n_4'} \qquad (9.8)$$

If we were using representation theory, the commutativity of the recoupling coefficient with j_4 would give directly from Schur's lemma that it was diagonally constant in the m-values.

But we have seen many times that, unless j_4 is an ICR of type (a), non-constant matrices commute with j_4. It is only necessary to find one suitable example to show that in general the recoupling coefficient is not diagonally constant in the m-values. Such an example is almost trivially found in grey C_4 where we have

$$\begin{aligned} \langle A_1E|E\rangle^{a_1x}{}_{(1)x} &= \langle A_1E|E\rangle^{a_1y}{}_{(1)y} = 1 \\ \langle A_1E|E\rangle^{a_1x}{}_{(2)x} &= -\langle A_1E|E\rangle^{a_1y}{}_{(2)y} = i \end{aligned} \qquad (9.9)$$

and

$$\begin{aligned} \langle EA_2|E\rangle^{xa_2}{}_{(1)y} &= \langle EA_2|E\rangle^{ya_2}{}_{(1)x} = 1 \\ \langle EA_2|E\rangle^{xa_2}{}_{(2)y} &= -\langle EA_2|E\rangle^{ya_2}{}_{(2)x} = i \end{aligned} \qquad (9.10)$$

(Bracketed numbers are used here in conformity with (8.9) as each coupling coefficient only has multiplicity one)

Then

$$\langle (A_1E)EA_2E|A_1(EA_2)EE\rangle^{(1)(1)}{}_{(1)(2)} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \qquad (9.11)$$

as required.

In the definition of the recoupling coefficient, the order in the coupling coefficients is extremely important. For since the coupling coefficient does not generally span the 3jm multiplicity space, permuted orders of the j's will usually give coupling coefficient tensors not related by a unitary transformation and hence inequivalent recoupling coefficients.

As a consequence of this, neither the Biedenharn identity [73] nor the Racah back-coupling rule [3] hold. Even if a special choice of coupling coefficient were made which did possess permutational symmetry, the non-diagonal form of the recoupling coefficient would still block the Biedenharn identity.

Using similar methods one can go on to develop higher order recoupling coefficients which also need not be diagonal in the m-values. This again causes many of the standard results to fail. For example, unless the coefficient of this section is diagonal, it is not possible to express the recoupling coefficient of four ICRs as a product of three recoupling coefficients of three ICRs.

9.2 The Reduction of Four ICRs to $\underset{\sim}{0}$

Since the reduction of three ICRs to $\underset{\sim}{0}$ led to the 3jm symbol possessing many of the properties holding in representation theory, it is natural to consider the reduction of four ICRs to $\underset{\sim}{0}$ in searching for a satisfactory 6j symbol. In fact we do not find one, as we shall see. Consider the coupling schemes $j_1, j_2 \to j_{12}; j_{12}, j_3, j_4 \to \underset{\sim}{0}$ and $j_2, j_3 \to j_{23}; j_1, j_{23}, j_4 \to \underset{\sim}{0}$. Then from equations (3.6), (6.1) and (8.1)

$$\frac{2}{|G|} \int_H j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, j_3(u)^{m_3}{}_{n_3}\, j_4(u)^{m_4}{}_{n_4}\, du$$

$$= \sum_{j_{12}} \langle j_1 j_2 | j_{12}\rangle^{m_1 m_2}{}_{r_1 m_{12}} (j_{12} j_3 j_4)^{m_{12} m_3 m_4}{}_{r_2}$$

$$\times\ (j_{12} j_3 j_4)^{r_2}{}_{n_{12} n_3 n_4} \langle j_1 j_2 | j_{12}\rangle^{r_1 n_{12}}{}_{n_1 n_2}$$

$$= \sum_{j_{23}} \langle j_2 j_3 | j_{23}\rangle^{m_2 m_3}{}_{r_3 m_{23}} (j_1 j_{23} j_4)^{m_1 m_{23} m_4}{}_{r_4}$$

$$\times\ (j_1 j_{23} j_4)^{r_4}{}_{n_1 n_{23} n_4} \langle j_2 j_3 | j_{23}\rangle^{r_3 n_{23}}{}_{n_2 n_3} \qquad (9.12)$$

Orthogonality of the coupling coefficient and 3jm tensors gives

$$\langle j_1 j_2 | j_{12}\rangle^{r_1 n_{12}}{}_{n_1 n_2} (j_{12} j_3 j_4)^{r_2}{}_{n_{12} n_3 n_4}$$

$$= \sum_{j_{23}} [j_{12}]^{\frac{1}{2}} [j_{23}]^{\frac{1}{2}} \begin{Bmatrix} j_1 & j_2 & \vdots & j_{12} \\ j_3 & j_4 & \vdots & j_{23} \end{Bmatrix}^{r_1 r_2}{}_{r_3 r_4}$$

$$\times\ \langle j_2 j_3 | j_{23}\rangle^{r_3 n_{23}}{}_{n_2 n_3} (j_1 j_{23} j_4)^{r_4}{}_{n_1 n_{23} n_4} \qquad (9.13)$$

where

$$\begin{Bmatrix} j_1 & j_2 & \vdots & j_{12} \\ j_3 & j_4 & \vdots & j_{23} \end{Bmatrix}^{r_1 r_2}_{\quad r_3 r_4}$$

$$= [j_{12}]^{-\frac{1}{2}} [j_{23}]^{-\frac{1}{2}} \langle j_1 j_2 | j_{12} \rangle^{r_1 m_{12}}{}_{m_1 m_2}$$

$$\times \; (j_{12} j_3 j_4)^{r_2}{}_{m_{12} m_3 m_4} \langle j_2 j_3 | j_{23} \rangle^{m_2 m_3}{}_{r_3 m_{23}}$$

$$\times \; (j_1 j_{23} j_4)^{m_1 m_{23} m_4}{}_{r_4} \qquad (9.14)$$

The anti-linear equations give the tensor as real.

From equation (9.13) we find the orthogonality property

$$\delta^{r_1 \dot{r}_1} \delta^{r_2 \dot{r}_2} \delta(j_{12}, j_{12}')$$

$$= \sum_{j_{23}} [j_{12}] [j_{23}] \begin{Bmatrix} j_1 & j_2 & \vdots & j_{12} \\ j_3 & j_4 & \vdots & j_{23} \end{Bmatrix}^{r_1 r_2}_{\quad r_3 r_4}$$

$$\times \; \delta^{r_3 \dot{r}_3} \delta^{r_4 \dot{r}_4} \begin{Bmatrix} j_1 & j_2 & \vdots & j_{12}' \\ j_3 & j_4 & \vdots & j_{23} \end{Bmatrix}^{\dot{r}_1 \dot{r}_2}_{\quad \dot{r}_3 \dot{r}_4} \qquad (9.15)$$

The Racah back-coupling rule <u>almost</u> holds for this tensor. If the definition were changed by transposing the order of j_2 and j_3 in the coupling coefficient $\langle j_2 j_3 | j_{23} \rangle$, then j_2 would occupy a more symmetric position and no permutations of the coupling coefficient would be required to give the result. However the standard ordering has become so firmly embedded in the literature that it would be difficult to change it now.

9.3 The n-j Symbols

Two alternative definitions may be given for the 6j symbol. The first is probably of more relevance to general corepresentation theory and is

definition one

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}_{r_1 r_2 r_3 r_4}$$

$$= \delta_{m_1 \dot{m}_1} \delta_{m_2 \dot{m}_2} \delta_{m_3 \dot{m}_3} \delta_{m_4 \dot{m}_4} \delta_{m_5 \dot{m}_5} \delta_{m_6 \dot{m}_6}$$

$$\times \; (j_1 j_2 j_3)^{\dot{m}_1 \dot{m}_2 m_3}{}_{r_1} \; (j_3 j_4 j_5)^{\dot{m}_3 \dot{m}_4 \dot{m}_5}{}_{r_2}$$

$$\times \; (j_2 j_4 j_6)^{m_2 m_4 \dot{m}_6}{}_{r_3} \; (j_1 j_6 j_5)^{m_1 m_6 m_5}{}_{r_4} \qquad (9.16)$$

The second which explicitly uses the Wigner tensor is

definition two

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}_{r_1 r_2 r_3 r_4}$$

$$= \delta_{m_1 \dot{m}_1} \delta_{m_2 \dot{m}_2} \delta_{m_3 \dot{m}_3} \delta_{m_4 \dot{m}_4} \delta_{m_5 \dot{m}_5} \delta_{m_6 \dot{m}_6}$$

$$\times \begin{bmatrix} \dot{m}_1 & \\ & m_1' \end{bmatrix} \begin{bmatrix} \dot{m}_2 & \\ & m_2' \end{bmatrix} \begin{bmatrix} \dot{m}_3 & \\ & m_3' \end{bmatrix} \begin{bmatrix} \dot{m}_4 & \\ & m_4' \end{bmatrix} \begin{bmatrix} \dot{m}_5 & \\ & m_5' \end{bmatrix} \begin{bmatrix} \dot{m}_6 & \\ & m_6' \end{bmatrix}$$

$$\times \; (j_1 j_2 j_3)^{m_1' m_2' m_3}{}_{r_1} \; (j_3 j_4 j_5)^{m_3' m_4' m_5'}{}_{r_2}$$

$$\times \; (j_2 j_4 j_6)^{m_2 m_4 m_6'}{}_{r_3} \; (j_1 j_6 j_5)^{m_1 m_6 m_5}{}_{r_4} \qquad (9.17)$$

These two definitions, equivalent in representation theory, are related by the non-diagonal A tensor of chapter six section three:

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}^{def.1}_{r_1 r_2 r_3 r_4}$$

$$= A^{*\,r_1'}_{j_3\;\;\;r_1}\, A^{\;\;r_3'}_{j_6\;\;\;r_3} \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}^{def.2}_{r_1' r_2 r_3' r_4} \qquad (9.18)$$

Due to the completeness of the 3jm tensor, any permuted 6j symbol is related to either of the above tensors by a unitary transformation. Thus for example from equation (9.17)

$$\begin{Bmatrix} j_1^* & j_5^* & j_6 \\ j_4^* & j_2^* & j_3 \end{Bmatrix}_{r_1 r_2 r_3 r_4} = \delta_{m_1 \dot{m}_1}\, \delta_{m_2 \dot{m}_2}\, \delta_{m_3 \dot{m}_3}\, \delta_{m_4 \dot{m}_4}\, \delta_{m_5 \dot{m}_5}\, \delta_{m_6 \dot{m}_6}$$

$$\times \begin{pmatrix} m_1 & \\ & \dot{m}_1' \end{pmatrix} \begin{pmatrix} m_5 & \\ & \dot{m}_5' \end{pmatrix} \begin{bmatrix} \dot{m}_6 & \\ & m_6' \end{bmatrix} \begin{pmatrix} m_4 & \\ & \dot{m}_4' \end{pmatrix} \begin{pmatrix} m_2 & \\ & \dot{m}_2' \end{pmatrix} \begin{bmatrix} \dot{m}_3 & \\ & m_3' \end{bmatrix}$$

$$\times\ (j_1 j_5 j_6)^{\dot{m}_1' \dot{m}_5' m_6}{}_{r_1}\ (j_6 j_4 j_2)^{m_6' \dot{m}_4' \dot{m}_2'}{}_{r_2}$$

$$\times\ (j_5 j_4 j_3)^{\dot{m}_5' \dot{m}_4' m_3'}{}_{r_3}\ (j_1 j_3 j_2)^{\dot{m}_2' \dot{m}_4' m_6'}{}_{r_4} \qquad (9.19)$$

The complex conjugates may be removed by equation (6.18), and since the A tensors square to the identity, they cancel. Eight of the Wigner tensors cancel in pairs and permutational symmetry of the 3jm tensor gives

$$\begin{Bmatrix} j_1^* & j_5^* & j_6 \\ j_4^* & j_2^* & j_3 \end{Bmatrix}_{r_1 r_2 r_3 r_4}$$

$$= M(1\,,32)^{r_4'}{}_{r_4}\, M(4\,,53)^{r_3'}{}_{r_3}\, M(4\,,62)^{r_2'}{}_{r_2}\, M(1\,,56)^{r_1'}{}_{r_1}$$

$$\times \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}_{r_4' r_3' r_2' r_1'} \qquad (9.20)$$

Similarly

$$\begin{Bmatrix} j_2 & j_1 & j_3 \\ j_5 & j_4 & j_6^* \end{Bmatrix}_{r_1 r_2 r_3 r_4}$$

$$= M(12\,,3)^{r_1'}{}_{r_1}\, M(3\,,54)^{r_2'}{}_{r_2}\, M(2\,,64)^{r_4'}{}_{r_4}\, M(1\,,56)^{r_3'}{}_{r_3}$$

$$\times \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}_{r_1' r_2' r_4' r_3'} \qquad (9.21)$$

The complex conjugates may be dropped in these as each ICR is equivalent to its conjugate, and thus all permuted 6j symbols may be found.

The motivation for the definitions of equations (9.16) and (9.17) came from trying to find a satisfactory 3jm form of the tensor (9.14) of the last section. By making the special choice of 3jm symbols from the coupling coefficient of equation (8.16) and rewriting the inverse 3jm symbols as

$$(j_1 j_2 j_{12})^{r_1}{}_{m_1 m_2 m_{12}} = \delta^{r_1 \dot{r}_1}\, \delta_{m_1 \dot{m}_1}\, \delta_{m_2 \dot{m}_2}\, \delta_{m_{12} \dot{m}_{12}}$$

$$\times \quad (j_1 j_2 j_{12})^{\dot{m}_1 \dot{m}_2 \dot{m}_{12}}{}_{\dot{r}_1}$$

$$= \delta^{r_1 \dot{r}_1'} \delta_{m_1 \dot{m}_1} \delta_{m_2 \dot{m}_2} \delta_{m_{12} \dot{m}_{12}}$$

$$\times \begin{bmatrix} \dot{m}_1 & \\ & m_1' \end{bmatrix} \begin{bmatrix} \dot{m}_2 & \\ & m_2' \end{bmatrix} \begin{bmatrix} \dot{m}_{12} & \\ & m_{12}' \end{bmatrix} (j_1 j_2 j_{12})^{m_1' m_2' m_{12}'}{}_{r_1'}$$

by equations (2.20) and (6.20), we find

$$\left\{ \begin{matrix} j_1 & j_2 & \vdots & j_{12} \\ j_3 & j_4 & \vdots & j_{34} \end{matrix} \right\}^{r_1 r_2}{}_{r_3 r_4} = \delta^{r_1 r_1'} \delta^{r_2 r_2'} \left\{ \begin{matrix} j_1 & j_2 & j_{12} \\ j_3 & j_4 & j_{34} \end{matrix} \right\}_{r_1' r_2' r_3 r_4}$$

Of course, this only holds for a particular set of 3jm tensors.

A similar relation may be found for the trace of the recoupling coefficient.

9.4 Comments

Three coefficients have been introduced in this chapter to replace the essentially unique recoupling or 6j symbol of representation theory, and none of them possess anything like the full range of properties found in representation theory. This is obviously very unsatisfactory and the reasons for these breakdowns lie in three places: in general (a) the recoupling coefficient is not diagonally constant; (b) it is not possible to permute the order of coupling in the coupling coefficient (c) the 3jm tensor need not possess orthogonality properties in a sum over only two m-values. It thus becomes an important problem to see if coefficients can be found which do possess the three required properties. This is a non-trivial problem of choice-basically in the coupling coefficient,and is at present unsolved for the general case.

CHAPTER TEN : THE COUPLING OF BASIS VECTORS

10.1 The Coupling of Three Basis Vectors to $\underset{\sim}{0}$

In the previous chapters attention has been mainly directed to the ICR matrices and various products of these matrices. In applications of group theory to quantum mechanics it is not so much these matrices which are of interest, but the basis vectors carrying the ICRs. We may expect that the differences between the coupling of two ICRs to a third, and of three ICRs to $\underset{\sim}{0}$ will also be reflected in the coupling of basis vectors. Because of this we commence with the coupling of three to $\underset{\sim}{0}$ rather than of two to a third.

It is trivial to verify that if

$$\{e^1{}_{(m)} : m = 1,\ldots\ldots, [j_1]\},\ \{e^2{}_{(m)} : m = 1,\ldots\ldots,[j_2]\}$$

and

$$\{e^3{}_{(m)} : m = 1,\ldots\ldots, [j_3]\}$$

form bases for j_1, j_2 and j_3 respectively, then

$$\{e^1{}_{(m_1)}\ e^2{}_{(m_2)}\ e^3{}_{(m_3)} :\ m_1, m_2, m_3 = 1, 2,\ldots\} \quad (10.1)$$

form a basis for the direct product $j_1 \otimes j_2 \otimes j_3$. Suppose there exists a linear combination of these which forms a basis for $\underset{\sim}{0}$ so that

$$u\ (a^{n_1 n_2 n_3}{}_{\underset{\sim}{0}}\ e^1{}_{(n_1)}\ e^2{}_{(n_2)}\ e^3{}_{(n_3)})$$

$$= a^{m_1 m_2 m_3}{}_{\underset{\sim}{0}}\ e^1{}_{(m_1)}\ e^2{}_{(m_2)}\ e^3{}_{(m_3)} \quad (10.2)$$

and

$$a\,(a^{n_1 n_2 n_3}{}_{\underset{\sim}{Q}}\; e^1{}_{(n_1)}\; e^2{}_{(n_2)}\; e^3{}_{(n_3)})$$

$$= \delta^{\underset{\sim}{Q}}{}_{\underset{\sim}{\dot{Q}}}\; a^{m_1 m_2 m_3}{}_{\underset{\sim}{Q}}\; e^1{}_{(m_1)}\; e^2{}_{(m_2)}\; e^3{}_{(m_3)} \qquad (10.3)$$

By using the linearity of u in the left-hand side of equation (10.2)

$$a^{m_1 m_2 m_3}{}_{\underset{\sim}{Q}}\; e^1{}_{(m_1)}\; e^2{}_{(m_2)}\; e^3{}_{(m_3)}$$

$$= a^{n_1 n_2 n_3}{}_{\underset{\sim}{Q}}\; u\,(e^1{}_{(n_1)}\; e^2{}_{(n_2)}\; e^3{}_{(n_3)})$$

$$= a^{n_1 n_2 n_3}{}_{\underset{\sim}{Q}}\; j_1(u)^{m_1'}{}_{n_1}\; j_2(u)^{m_2'}{}_{n_2}\; j_3(u)^{m_3'}{}_{n_3}$$

$$\times\; e^1{}_{(m_1')}\; e^2{}_{(m_2')}\; e^3{}_{(m_3')} \qquad (10.4)$$

As this is independent of u, it may be integrated over H and equation (6.8) used to give

$$a^{m_1 m_2 m_3}{}_{\underset{\sim}{Q}}\; e^1{}_{(m_1)}\; e^2{}_{(m_2)}\; e^3{}_{(m_3)}$$

$$= a^{n_1 n_2 n_3}{}_{\underset{\sim}{Q}}\; (j_1 j_2 j_3)^{m_1' m_2' m_3'}{}_r\; (j_1 j_2 j_3)^r{}_{n_1 n_2 n_3}$$

$$\times\; e^1{}_{(m_1')}\; e^2{}_{(m_2')}\; e^3{}_{(m_3')} \qquad (10.5)$$

But as the basis vectors are orthogonal,

$$a^{m_1 m_2 m_3}{}_{\underset{\sim}{Q}} = B^r{}_{\underset{\sim}{Q}}\; (j_1 j_2 j_3)^{m_1 m_2 m_3}{}_r \qquad (10.6)$$

with $B^r{}_{\underset{\sim}{Q}} = a^{n_1 n_2 n_3}{}_{\underset{\sim}{Q}}\; (j_1 j_2 j_3)^r{}_{n_1 n_2 n_3}$ (10.7)

B is real from the anti-linear equation.

It is trivial that $a^{m_1 m_2 m_3}{}_{\underset{\sim}{0}}$ may be taken as any column of a 3jm tensor giving these results:

(a) $(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r}\; e^1{}_{(m_1)}\; e^2{}_{(m_2)}\; e^3{}_{(m_3)}$ (10.8)

forms a basis vector for $\underset{\sim}{0}$

(b) If $a^{m_1 m_2 m_3}{}_{\underset{\sim}{0}}\; e^1{}_{(m_1)}\; e^2{}_{(m_2)}\; e^3{}_{(m_3)}$ is a basis vector for $\underset{\sim}{0}$, a is a real linear combination of columns of the 3jm tensor $(j_1 j_2 j_3)$

10.2 The Coupling of Two Basis Vectors to a Third

It is elementary to verify that if we have bases for j_1 and j_2 as in the last section, then

$$e^3{}_{r(m_3)} = \langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r m_3}\, e^1{}_{(m_1)}\, e^2{}_{(m_2)} \qquad (10.9)$$

is a basis vector for j_3. This can be inverted to give

$$e^1{}_{(m_1)}\, e^2{}_{(m_2)} = \sum_{j_3} \langle j_1 j_2 | j_3 \rangle^{r m_3}\, e^3{}_{r(m_3)} \qquad (10.10)$$

However, not every linear combination forming a basis for j_3 can be put into the form of equation (10.9). To establish a completeness condition, consider

$$e^3{}_{(m_3)} = a^{m_1 m_2}{}_{m_3}\, e^1{}_{(m_1)}\, e^2{}_{(m_2)} \qquad (10.11)$$

Then

$$\delta_{\dot{m}_3\, m_3} = e^3{}_{(\dot{m}_3)}\, e^3{}_{(m_3)} = a^{m_1 m_2}{}_{m_3}\, e^1{}_{(m_1)}\, e^2{}_{(m_2)}\, e^3{}_{(\dot{m}_3)}$$

or $$1 = \delta^{m_3\, \dot{m}_3}\, a^{m_1 m_2}{}_{m_3}\, e^1{}_{(m_1)}\, e^2{}_{(m_2)}\, e^3{}_{(\dot{m}_4)}\, \delta^{\dot{m}_4}{}_{\dot{m}_3}$$

$$= a^{m_1 m_2}{}_{m_3}\, \delta^{m_3 \dot{m}_4}\, e^1{}_{(m_1)}\, e^2{}_{(m_2)}\, e^3{}_{(\dot{m}_4)} \qquad (10.12)$$

But 1 is a basis for $\underset{\sim}{0}$ and hence from the last section,

$$a^{m_1 m_2}{}_{m_3}\, \delta^{m_3 \dot{m}_4} = B^r{}_{\underset{\sim}{0}}\, (j_1 j_2 j_3)^{m_1 m_2 \dot{m}_4}{}_r \qquad (10.13)$$

Using the relation (6.17) between the partly conjugated and unconjugated 3jm symbols, we find

$$a^{m_1 m_2}{}_{m_3} = C^r{}_{\underset{\sim}{0}}\, \delta_{m_3 \dot{m}_4} \begin{bmatrix} \dot{m}_4 & \\ & m_5 \end{bmatrix} (j_1 j_2 j_3)^{m_1 m_2 m_5}{}_r \qquad (10.14)$$

Not only does this give the desired completeness, but as B can be chosen to pick out any column of a 3jm symbol, and the tensor A of equation (6.17) is unitary over the whole of the 3jm multiplicity space, we have that

$$e^{3}{}_{r(m_3)} = \delta_{m_3\dot{m}_4} \begin{bmatrix} \dot{m}_4 & \\ & m_5 \end{bmatrix} (j_1 j_2 j_3)^{m_1 m_2 m_5}{}_{r}\, e^{1}{}_{(m_1)}\, e^{2}{}_{(m_3)} \qquad (10.15)$$

is a basis vector of j_3 for any column of a 3jm tensor. It is not a very good basis vector as it need not be orthogonal to $e^{3}{}_{r(m_3')}$ when $m_3 \neq m_3'$ from the non-orthogonality of the 3jm tensor in a sum over only two m values. For this reason we shall not use it in later chapters.

CHAPTER ELEVEN : THE WIGNER-ECKART THEOREM

11.1 The Wigner-Eckart theorem

In any calculations involving operators acting on a system the Wigner-Eckart theorem [9,10], which separates out the symmetry properties of the matrix elements from the non-symmetry properties, gives great simplifications. In representation theory it can be formulated in terms of the coupling coefficient tensor or the equivalent 3jm tensor. Aviran and Zak [62] have demonstrated the theorem for corepresentations, but in doing so used the coupling coefficient with its essential incompleteness. We give a proof similar to theirs but which instead uses the more satisfactory 3jm symbol. It is based on the method of Koster [74] for linear groups.

Firstly we define an irreducible tensor operator T(kq) to be a tensor which satisfies

$$u\, T(kq_1)\, u^{-1} = k(u)^{q_2}{}_{q_1}\, T(kq_2) \tag{11.1}$$

and

$$a\, T(kq_1)\, a^{-1} = k(a)^{q_2}{}_{\dot{q}_1}\, T(kq_2) \tag{11.2}$$

where k is an ICR of G. A general matrix element in a symmetry adapted system may be taken as

$$\langle j_1 m_1 | T(kq_1) | j_2 m_2 \rangle$$

where $|j_1 m_1\rangle$ and $|j_2 m_2\rangle$ are basis vectors for the ICRs j_1 and j_2 respectively. In the notation of chapter two this may be written as

$$<j_1m_1|\,T(kq_1)\,|\,j_2m_2> \;=\; e^{j_1}{}_{(\dot{m}_1)}\;T(kq_1)\;e^{j_2}{}_{(m_2)} \tag{11.3}$$

The matrix element being a scalar, is invariant under any unitary operator of the group and hence

$$<j_1m_1|\,T(kq_1)\,|\,j_2m_2> \;=\; <u\,j_1m_1|\,u\,T(kq_1)u^{-1}\,|uj_2m_2>$$

$$= \; j_1(u)_{\dot{m}_1}{}^{\dot{n}_1}\; e^{j_1}{}_{(\dot{n}_1)}\quad k(u)^{q_2}{}_{q_1}\; T(kq_2)\; j_2(u)^{n_2}{}_{m_2}\; e^{j_2}{}_{(n_2)}$$

$$= \; j_1(u)_{\dot{m}_1}{}^{\dot{n}_1}\; k(u)^{q_2}{}_{q_1}\; j_2(u)^{n_2}{}_{m_2} <j_1n_1|\,T(kq_2)\,|\,j_2n_2> \tag{11.4}$$

Similarly as an anti-unitary operator transforms the matrix element to its complex conjugate,

$$<j_1m_1|\,T(kq_1)\,|\,j_2m_2>$$

$$= \; j_1(a)_{\dot{m}_1}{}^{n_1}\; k(a)^{\dot{q}_2}{}_{q_1}\; j_2(a)^{\dot{n}_2}{}_{m_2}\; <j_1n_1|\,T(kq_2)\,|\,j_2n_2>^{*} \tag{11.5}$$

As both of these equations are independent of the particular choice of group elements, we can integrate over H and G-H respectively and divide by the order H. Using the transposition tensors as in equation (2.9) gives

$$<j_1m_1|\,T(kq_1)\,|\,j_2m_2>$$

$$= \frac{2}{|G|}\int_H \delta^{\dot{n}_1}_{\dot{n}_3}\;\delta^{\dot{m}_3}_{\dot{m}_1}\; j_1(u)^{\dot{n}_3}{}_{\dot{m}_3}\; k(u)^{q_2}{}_{q_1}\; j_2(u)^{n_2}{}_{m_2}$$

$$\times <j_1n_1|\,T(kq_2)\,|\,j_2n_2>\, du \tag{11.6}$$

$$= \frac{2}{|G|} \int_{G-H} \delta^{n_1}_{n_3} \delta^{\dot{m}_3}_{\dot{m}_1} \; j_1(a)^{n_3}{}_{\dot{m}_3} \; k(a)^{\dot{q}_2}{}_{q_1} \; j_2(a)^{\dot{n}_2}{}_{m_2}$$

$$\times \; \langle j_1 n_1 | T(kq_2) | j_2 n_2 \rangle^* \; da \qquad (11.7)$$

At this point we part company with Aviran and Zak. For they reduce the triple product by using a coupling coefficient - a path strewn with problems. It should be reduced by the 3jm symbol which is the path we take. To avoid a partly conjugated 3jm symbol we also replace j_1^* by j_1 using the Wigner tensor, to give for equation (11.6)

$$\langle j_1 m_1 | T(kq_1) | j_2 m_2 \rangle$$

$$= \delta^{\dot{n}_1}_{\dot{n}_3} \delta^{\dot{m}_3}_{\dot{m}_1} \begin{bmatrix} \dot{n}_3 & \\ & n_4 \end{bmatrix} \begin{pmatrix} m_4 & \\ & \dot{m}_3 \end{pmatrix} (j_1 k \; j_2)^{n_4 q_2 n_2}{}_r$$

$$\times \; (j_1 \; k \; j_2)^r{}_{m_4 q_1 m_2} \langle j_1 n_1 | T(kq_1) | j_2 n_2 \rangle \qquad (11.8)$$

Defining the reduced matrix element to be

$$\langle j_1 || T(k) || j_2 \rangle_r = \delta^{\dot{n}_1}_{\dot{n}_3} \begin{bmatrix} \dot{n}_3 & \\ & n_4 \end{bmatrix} (j_1 \; k \; j_2)^{n_4 q_2 n_2}{}_r$$

$$\times \; \langle j_1 n_1 | T(kq_1) | j_2 n_2 \rangle \qquad (11.9)$$

gives

$$\langle j_1 m_1 | T(kq_1) | j_2 m_2 \rangle$$

$$= \delta^{\dot{m}_3}_{\dot{m}_1} \begin{pmatrix} m_4 & \\ & \dot{m}_3 \end{pmatrix} (j_1 \; k \; j_2)^r{}_{m_4 q_1 m_2} \langle j_1 || T(k) || j_2 \rangle_r \qquad (11.10)$$

A similar analysis applied to equation (11.7) shows that the reduced matrix element is real.

11.2 Coupled Tensor Operators

The tensor calculus does not stop at this point: products of tensor operators acting on coupled basis vectors are frequently used (as for example the spin-orbit in L-S coupled functions). The case we consider here is for matrix elements of

$$<(j_3j_4)j_1m_1 \mid (TU)\ X\ (kq)\,|(j_5j_6)j_2m_2> \qquad (11.11)$$

where T acts only between j_3 and j_5, and U between j_4 and j_6. In representation theory the reduced matrix element $<j_1||X(k)||j_2>$ may be

given in terms of $<j_3||T(k_1)||j_5>$ and $<j_4||U(k_2)||j_6>$ by a 9j symbol but from the last chapter there will be a variety of such symbols here, and we establish exactly which one it is.

It is extremely unlikely that a purposeful choice would be made to give non-orthogonal basis vectors of j_1, j_2 and X. Hence we perform the couplings by equation (10.9) rather than (10.15) to give

$$<(j_3j_4)r_1\ j_1m_1|(TU)r_3\ X\ (kq)|(j_5j_6)r_2\ j_2m_2>$$

$$= <j_3j_4|j_1>_{\dot{r}_1}^{\dot{m}_3\dot{m}_4}<k_1k_2|k>^{q_1q_2}_{r_3q}\quad <j_5j_6|j_2>^{m_5m_6}_{r_2m_2}$$

$$x\quad <j_3m_3|T(k_1q_1)|j_2m_2>\ <j_4m_4|U(k_2q_2)|j_6m_6> \qquad (11.12)$$

All matrix elements may be evaluated by the Wigner-Eckart theorem to give

$$\langle (j_3 j_4) r_1 j_1 || (TU) r_3 X(k) || (j_5 j_6) r_2 j_2 \rangle_{r_4}$$

$$= [j_1]^{\frac{1}{2}} [j_2]^{\frac{1}{2}} [k]^{\frac{1}{2}} \begin{Bmatrix} j_3 & j_4 & \vdots & j_1 \\ k_1 & k_2 & \vdots & k \\ j_5 & j_6 & \vdots & j_2 \end{Bmatrix}^{r_5 r_6}_{\dot{r}_1 \quad r_2 r_3 r_4}$$

$$\times \langle j_3 || T(k_1) || j_5 \rangle_{r_5} \langle j_4 || U(k_2) || j_6 \rangle_{r_6} \qquad (11.13)$$

with

$$\begin{Bmatrix} j_3 & j_4 & \vdots & j_1 \\ k_1 & k_2 & \vdots & k \\ j_5 & j_6 & \vdots & j_2 \end{Bmatrix}^{r_5 r_6}_{\dot{r}_1 \quad r_2 r_3 r_4}$$

$$= [j_1]^{-\frac{1}{2}} [j_2]^{-\frac{1}{2}} [k]^{-\frac{1}{2}} \langle j_3 j_4 | j_1 \rangle_{r_1}^{\dot{m}_3 \dot{m}_4}$$

$$\times \langle k_1 k_2 | k \rangle^{q_1 q_2}_{r_3 q} \langle j_5 j_6 | j_2 \rangle^{m_5 m_6}_{r_2 m_2}$$

$$\times \delta^{\dot{m}_1}_{\dot{m}_7} \begin{bmatrix} \dot{m}_7 & \\ & m_8 \end{bmatrix} (j_1 k \, j_2)^{m_8 q m_2}_{r_4}$$

$$\times \delta^{\dot{m}_9}_{\dot{m}_3} \begin{pmatrix} m_{10} & \\ & \dot{m}_9 \end{pmatrix} (j_3 k_1 j_5)^{r_5}_{m_{10} q_1 m_5}$$

$$\times \delta^{\dot{m}_{11}}_{\dot{m}_4} \begin{pmatrix} m_{12} & \\ & \dot{m}_{11} \end{pmatrix} (j_4 k_2 j_6)^{r_6}_{m_{12} q_2 m_6} \qquad (11.14)$$

Rather laboriously this may be shown to be a special case of a 9j symbol.

Important special cases of equation (11.13) are when one of k_1, k_2 or k is the identity ICR $\underset{\sim}{0}$. In representation theory any of these reduce the 9j symbol to a 6j symbol. However there are sums over the 3jm multiplicity labels r_5 and r_6 in the equation, and over r_4 in expanding equation (11.11) directly and so the symbol here will not collapse down to a 6j symbol.

CHAPTER TWELVE : RACAH'S LEMMA

12.1 The 3jm Symbol

Racah's lemma [4] has proved to be a fundamental result for any application of the descent in symmetry technique. Briefly, for IRs it relates the 3jm symbols of a group G to the 3jm symbols of a subgroup K by a factor or unitary transformation independent of the m values. To use the descent in symmetry technique in grey groups we shall find the lemma of equal importance. However, as the proofs we have sighted for linear groups directly use Schur's lemma [4, 53], and Schur's lemma only holds in a restricted sense for grey groups, we offer a proof based on the orthogonality relations. We may distinguish between two different types of subgroup of a grey group and the lemma will assume a different form for each. These are: a grey subgroup, consisting of both linear and anilinear operators, and a subgroup consisting of linear operators only.

We consider first a grey subgroup K of grey G. We label the ICRs of G by j as usual, and of K by k. Upon restriction to K, each ICR of G may be reduced to a direct sum of ICRs of K by some unitary transformation. Applying the inverse transformation to the ICRs of G, we obtain 'symmetry adapted' ICRs of G - that is, on restriction to K the matrices are already in block diagonal form:

$$j(u) = \begin{pmatrix} k(u) & & 0 \\ & k'(u) & \\ & & \ddots \\ 0 & & & \ddots \end{pmatrix} \tag{12.1}$$

and similarly for the anti-linear operators. We reduce the triple product in G by equation (6.6):

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1} = j_1(u)^{m_1}{}_{n_1}\, j_2(u)^{m_2}{}_{n_2}\, j_3(u)^{m_3}{}_{n_3} \times (j_1 j_2 j_3)^{n_1 n_2 n_3}{}_{r_1} \tag{12.2}$$

If we restrict m_1, m_2, $m_3 \leqslant [k_1], [k_2], [k_3]$ respectively, then $j_1(u)$, $j_2(u)$ and $j_3(u)$ are zero for n_1, n_2, $n_3 > [k_1], [k_2], [k_3]$ respectively, and for $u \in H'$ the linear subgroup of K

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1} = k_1(u)^{m_1}{}_{n_1} k_2(u)^{m_2}{}_{n_2} k_3(u)^{m_3}{}_{n_3} \times (j_1 j_2 j_3)^{n_1 n_2 n_3}{}_{r_1}$$

As this is independent of u, it may be integrated over H' and divided by the volume of H'. Equation (6.8) may then be used to give

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_{r_1} = (k_1 k_2 k_3)^{m_1 m_2 m_3}{}_{r_2} (k_1 k_2 k_3)^{r_2}{}_{n_1 n_2 n_3} \times (j_1 j_2 j_3)^{n_1 n_2 n_3}{}_{r_1} \tag{12.3}$$

Defining the isoscalar by

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{pmatrix}^{r_2}_{r_1} = (k_1 k_2 k_3)^{r_2}{}_{n_1 n_2 n_3} (j_1 j_2 j_3)^{n_1 n_2 n_3}{}_{r_1} \tag{12.4}$$

gives Racah's lemma for the 3jm symbols as

$$(j_1j_2j_3)^{m_1m_2m_3}{}_{r_1} = \begin{pmatrix} j_1j_2j_3 \\ k_1k_2k_3 \end{pmatrix}^{r_2}_{r_1} (k_1k_2k_3)^{m_1m_2m_3}{}_{r_2} \qquad (12.5)$$

If K is a grey subgroup, there are anti-linear forms of these four equations, and as might be expected these give the isoscalar as real. On the other hand if K is a linear subgroup there are no anti-linear equations and hence no reality condition. This is perfectly reasonable as there is a free choice of phase for the 3jm symbol of a linear group whereas there is not in a grey group.

12.2 The Coupling Coefficient

It is a characteristic of the methods of this thesis, that in formulae involving the 3jm symbol the integral equations (6.8) and (6.9) are used. With the coupling coefficient no such equations exist and derivations usually end up with invoking Schur's lemma. Such is the case here. As in the last section we suppose j_1, j_2 and j_3 to be symmetry adapted to the subgroup K and then use equation (8.5)

$$<j_1j_2|j_3>^{m_1m_2}{}_{r_1m_3}\; j_3(u)^{m_3}{}_{n_3} = j_1(u)^{m_1}{}_{n_1}\; j_2(u)^{m_2}{}_{n_2} \times <j_1j_2|j_3>^{n_1n_2}{}_{r_1n_3} \qquad (12.6)$$

Restricting m_1, m_2, $n_3 \leq [k_1]$, $[k_2]$, $[k_3]$ respectively and $u \varepsilon H'$,

$$<j_1j_2|j_3>^{m_1m_2}{}_{r_1m_3}\; k_3(u)^{m_3}{}_{n_3} = k_1(u)^{m_1}{}_{n_1}\; k_2(u)^{m_2}{}_{n_2} \times <j_1j_2|j_3>^{n_1n_2}{}_{r_1n_3}$$

Expanding the product $k_1 \otimes k_2$

$$<j_1j_2|j_3>^{m_1m_2}{}_{r_1m_3}\; k_3(u)^{m_3}{}_{n_3}$$

$$= \sum_{k_4} <k_1k_2|k_4>^{m_1m_2}{}_{r_2m_4}\; k_4(u)^{m_4}{}_{n_4}\; <k_1k_2|k_4>^{r_2n_4}{}_{n_1n_2} \times <j_1j_2|j_3>^{n_1n_2}{}_{r_1n_3} \qquad (12.7)$$

or

$$<k_1k_2|k_4>^{r_2m_4}{}_{m_1m_2}\; <j_1j_2|j_3>^{m_1m_2}{}_{r_1m_3}\; k_3(u)^{m_3}{}_{n_3}$$

$$= k_4(u)^{m_4}{}_{n_4}\; <k_1k_2|k_4>^{r_2n_4}{}_{n_1n_2}\; <j_1j_2|j_3>^{n_1n_2}{}_{r_1n_3} \qquad (12.8)$$

If K is a grey group a similar equation holds for the anti-linear operators. Thus k_4 must equal k_3 and the product of the two coupling coefficients commutes with k_3. We may then write

$$\langle k_1 k_2 | k_2 \rangle^{r_1 m_4}{}_{m_1 m_2} \langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3}$$

$$= \langle j_1 k_1 + j_2 k_2 | j_3 k_3 \rangle^{r_2 m_4}{}_{r_1 m_3} \qquad (12.9)$$

for a coupling coefficient isoscalar $\langle j_1 k_1 + j_2 k_2 | j_3 k_3 \rangle$. Substituting this back into equation (12.7),

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3} k_3(u)^{m_3}{}_{n_3} = \langle k_1 k_2 | k_3 \rangle^{m_1 m_2}{}_{r_2 m_4} k_3(u)^{m_4}{}_{n_4}$$

$$\times \quad \langle j_1 k_1 + j_2 k_2 | j_3 k_3 \rangle^{r_2 n_4}{}_{r_1 n_3}$$

$$= \langle k_1 k_2 | k_3 \rangle^{m_1 m_2}{}_{r_2 m_4} \langle j_1 k_1 + j_2 k_2 | j_3 k_3 \rangle^{r_2 m_4}{}_{r_1 m_3} k_3(u)^{m_3}{}_{n_3}$$

by commutativity. This gives for the lemma

$$\langle j_1 j_2 | j_3 \rangle^{m_1 m_2}{}_{r_1 m_3} = \langle j_1 k_1 + j_2 k_2 | j_3 k_3 \rangle^{r_2 m_4}{}_{r_1 m_3}$$

$$\times \quad \langle k_1 k_2 | k_3 \rangle^{m_1 m_2}{}_{r_2 m_4} \qquad (12.10)$$

This is not quite as simple as the 3jm form, and it is easy to see why. For $\langle j_1 j_2 | j_3 \rangle$ only spans a <u>subspace</u> of the 3jm multiplicity space and $\langle k_1 k_2 | k_3 \rangle$ similarly only spans a subspace of the 3km multiplicity space. A coupling coefficient is needed to map this second subspace into the first.

If K is a linear subgroup of G, then Schur's lemma applied to equation (12.8) shows that there is no m dependence in the isoscalar.

12.3 Properties of the Isoscalars

The orthogonality properties of the 3jm and coupling coefficient tensors may be used to give orthogonality properties for the isoscalars. Before dealing with these, we comment that equations (12.5) and 12.10) only hold for $m_1, m_2, m_3 \leq [k_1], [k_2], [k_3]$ respectively. From equation (12.1) though, it follows that this restriction is only one of simplicity, and that similar relations hold for other m values. For example, if $j_1 = k_1 \oplus k_1' \oplus$......as by equation (12.1),

$$(j_1 j_2 j_3)^{m_1+[k_1]m_2 m_3}_{r_1} = \begin{pmatrix} j_1 & j_2 & j_3 \\ k_1' & k_2 & k_3 \end{pmatrix}^{r_2}_{r_1} \langle k_1' \, k_2 \mid k_3 \rangle^{m_1 m_2}{}_{r_2 m_3}$$

for $m_1, m_2, m_3 \leq [k_1'], [k_2], [k_3]$ respectively. A sum over the m values for the tensors in G will necessarily invoke a sum over the ICRs or IRs k contained in j. It may happen that an ICR or IR k may be contained more than once in j, in which case it will be necessary to attach multiplicity labels to k also.

From these preliminaries it is straightforward to find the orthogonality relations which are

$$\delta_{\dot{r}_3 r_1} = \sum_{\substack{k_1 k_2 k_3 \\ \alpha\,\beta\,\gamma \\ \alpha'\beta'\gamma'}} \begin{pmatrix} j_1 & j_2 & j_3 \\ \alpha k_1 & \beta k_2 & \gamma k_3 \end{pmatrix}^{r_2}_{r_1} \begin{pmatrix} j_1 & j_2 & j_3 \\ \alpha' k_1 & \beta' k_2 & \gamma' k_3 \end{pmatrix}^{\dot{r}_2}_{\dot{r}_3} \delta_{\dot{r}_2 r_2} \qquad (12.11)$$

$$\delta_{\dot{m}_3 m_3}\delta_{\dot{r}_3 r_1} = \sum_{\substack{k_1k_2\\ \alpha\beta\alpha'\beta'}} \langle j_1\alpha k_1 + j_2\beta k_2|j_3\gamma k_3\rangle^{r_2m_2}_{r_1m_3}$$

$$\times \langle j_1\alpha' k_1 + j_2\beta' k_2|j_3\gamma k_3\rangle^{\dot{r}_2\dot{m}_2}_{\dot{r}_3\dot{m}_3}\, \delta_{\dot{r}_2 r_2}\delta_{\dot{m}_2 m_2} \qquad (12.12)$$

The 3jm isoscalar has permutation properties derived from those of the 3jm symbols. Thus if M(12,3) and N(12,3) give transpositions for the 3jm and 3km symbols respectively,

$$\begin{pmatrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{pmatrix}^{r_2}_{r_1} = M(12,3)^{r_3}{}_{r_1}\; N^{-1}(12,3)^{r_2}{}_{r_4} \begin{pmatrix} j_2 & j_1 & j_3 \\ k_2 & k_1 & k_3 \end{pmatrix}^{r_4}_{r_3} \qquad (12.13)$$

CHAPTER THIRTEEN : TRANSFORMATIONS UNDER A CHANGE OF BASIS

13.1 The 3jm Symbol

Transformation properties under a change of basis are always of importance, especially when symmetry adaptation to different subgroup chains is used. Here, as in the rest of this thesis, we find some coefficients possessing the same properties as those in representation theory, while others show some interesting divergences. The most straightforward is the 3jm symbol.

Let P be a unitary change of basis matrix. Then as in equations (2.17) and (2.22) an ICR j transforms according to

$$j(u)^{m'}{}_{n'} \;=\; P^{m'}{}_{m}\, j(u)^{m}{}_{n}\, P^{n}{}_{n'} \tag{13.1}$$

and

$$j(a)^{m'}{}_{\dot{n}'} \;=\; P^{m'}{}_{m}\, j(a)^{m}{}_{\dot{n}}\, P^{\dot{n}}{}_{\dot{n}'} \tag{13.2}$$

with the primes labelling the realization of j in the new basis. Substitution into equation (6.8) yields immediately

$$(j_1j_2j_3)^{m_1'm_2'm_3'}{}_{r'}\,(j_1j_2j_3)^{r'}{}_{n_1'n_2'n_3'}$$

$$= P^{m_1'}_{1\;m_1}\; P^{m_2'}_{2\;m_2}\; P^{m_3'}_{3\;m_2}\; (j_1j_2j_3)^{m_1m_2m_3}{}_{r}$$

$$\times\quad (j_1j_2j_3)^{r}{}_{n_1n_2n_3}\; P^{n_1}_{1\;n_1'}\; P^{n_2}_{2\;n_2'}\; P^{n_3}_{3\;n_3'}$$

or

$$(j_1j_2j_3)^{m_1'm_2'm_3'}{}_{r'} = E^r{}_{r'}\, P_1^{m_1'}{}_{m_1}\, P_2^{m_2'}{}_{m_2}\, P_3^{m_3'}{}_{m_3}\, (j_1j_2j_3)^{m_1m_2m_3}{}_r \tag{13.3}$$

with $$E^r{}_{r'} = (j_1j_2j_3)^r{}_{n_1n_2n_3}\, P_1^{n_1}{}_{n_1'}\, P_2^{n_2}{}_{n_2'}\, P_3^{n_3}{}_{n_3'}\, (j_1j_2j_3)^{n_1'n_2'n_3'}{}_r \tag{13.4}$$

as an orthogonal tensor.

A special case of this occurs when each P_i is an element of the commutator algebra of j :

$$(j_1j_2j_3)^{m_1'm_2'm_3'}{}_{r'} = E^r{}_{r'} \langle j_1\underset{\sim}{0}|j_1\rangle^{m_1'0}{}_{m_1} \langle j_2\underset{\sim}{0}|j_2\rangle^{m_2'0}{}_{m_2}$$
$$\times \langle j_3\underset{\sim}{0}|j_3\rangle^{m_3'0}{}_{m_3} (j_1j_2j_3)^{m_1m_2m_3}{}_r$$

Since now $j_1(u)^{m_1'}{}_{n'} = \delta^{m_1'}{}_{m_4}\, j_1(u)^{m_4}{}_{n_4}\, \delta^{n_4}{}_{n_1'}$ etc., this reads

$$(j_1j_2j_3)^{m_4m_5m_6}{}_{r'} = E^r{}_{r'} \langle j_1\underset{\sim}{0}|j_1\rangle^{m_4\underset{\sim}{0}}{}_{m_1} \langle j_2\underset{\sim}{0}|j_2\rangle^{m_5\underset{\sim}{0}}{}_{m_2}$$
$$\times \langle j_3\underset{\sim}{0}|j_3\rangle^{m_6\underset{\sim}{0}}{}_{m_3} (j_1j_2j_3)^{m_1m_2m_3}{}_r \tag{13.5}$$

showing that E cannot be taken as simultaneously diaganal for all transformations and hence is a general rank two orthogonal tensor.

13.2 The Coupling Coefficient

Applying these same transformations to the coupling coefficient equation (8.1) gives

$$j_1(u)^{m'_1}{}_{n'_1} \; j_2(u)^{m'_2}{}_{n'_2}$$

$$= \sum_{j_3} \langle j_1 j_2 | j_3\rangle^{m'_1 m'_2}{}_{r'm'_3} \; j_3(u)^{m'_3}{}_{n'_3} \; \langle j_1 j_2 | j_3\rangle^{r'n'_3}{}_{n'_1 n'_2} \tag{13.6}$$

$$= \sum_{j_4} P_1^{m'_1}{}_{m_1} \; P_2^{m'_2}{}_{m_2} \; \langle j_1 j_2 | j_4\rangle^{m_1 m_2}{}_{rm_4} \; P_4^{m_4}{}_{m'_4} \; j_4(u)^{m'_4}{}_{n'_4}$$

$$\times \quad P_4^{n'_4}{}_{n_4} \; \langle j_1 j_2 | j_4\rangle^{rn_4}{}_{n_1 n_2} \; P_1^{n_1}{}_{n'_1} \; P_2^{n_2}{}_{n'_2} \tag{13.7}$$

Orthogonality of the P and of the coupling coefficients may be used to turn this and the corresponding anti-linear equation into an intertwining relation between j_3 and j_4, from which

$$P_1^{m'_1}{}_{m_1} \; P_2^{m'_2}{}_{m_2} \; P_3^{m_4}{}_{m'_4} \; \langle j_1 j_2 | j_3\rangle^{m_1 m_2}{}_{rm_4} \; \langle j_1 j_2 | j_3\rangle^{r'm'_3}{}_{m'_1 m'_2}$$

$$= F^{r'm'_3}{}_{rm'_4} \tag{13.8}$$

This may be inserted back into these equations to yield

$$\langle j_1 j_2 | j_3\rangle^{m'_1 m'_2}{}_{r'm'_3}$$

$$= F^{rm'_4}{}_{r'm'_3} \; P_1^{m'_1}{}_{m_1} \; P_2^{m'_2}{}_{m_2} \; P_3^{m_3}{}_{m'_4} \; \langle j_1 j_2 | j_3\rangle^{m_1 m_2}{}_{rm_3} \tag{13.9}$$

with

$$F^{rm'_4}{}_{r'm'_3} = \langle j_1 j_2 | j_3\rangle^{rm_3}{}_{m_1 m_2} \; P_1^{m_1}{}_{m'_1} \; P_2^{m_2}{}_{m'_2} \; P_3^{m'_4}{}_{m_3} \langle j_1 j_2 | j_3\rangle^{m'_1 m'_2}{}_{r'm'_3} \tag{13.10}$$

CHAPTER FOURTEEN : SPECIAL n-jm SYMBOLS

14.1 Introduction

At many stages in this thesis it has proved useful to consider the relation between G and its linear subgroup H. Thus, ICRs are constructed in a definite manner from the IRs of H, and the permutational properties of a 3jm tensor in G are determined by those of the 3jm tensors in H. Thus it would appear highly likely that all n-jm tensors in G can be found from those in H - indeed this has already been done for the 1jm symbol in chapter three. It should be evident from the two defining equations

$$(j_1 j_2 j_3)^{r_1}{}_{m_1 m_2 m_3}\ j_1(u)^{m_1}{}_{n_1}\ j_2(u)^{m_2}{}_{n_2}\ j_3(u)^{m_3}{}_{n_3} = (j_1 j_2 j_3)^{r_1}{}_{n_1 n_2 n_3}$$

and

$$(j_1 j_2 j_3)^{r_1}{}_{m_1 m_2 m_3}\ j_1(\theta)^{m_1}{}_{\dot{n}_1}\ j_2(\theta)^{m_2}{}_{\dot{n}_2}\ j_3(\theta)^{m_3}{}_{\dot{n}_3} = (j_1 j_2 j_3)^{r_1}{}_{\dot{n}_1 \dot{n}_2 \dot{n}_3}$$

that such constructions will be heavily dependent both on the types of ICRs occurring and on the matrix realizations of θ. In order to simplify the following material we firstly drop the tensor notation and replace it with a simpler matrix notation and secondly choose basis vectors in H in order that θ assumes a simple form. This does not cause any loss of generality for transformations to other bases may be made by equation (13.3).

14.2 THE 1jm SYMBOL

The 1jm symbol is defined to be the matrix which transforms an ICR to its complex conjugate. It was shown in chapter five that although this matrix is not necessarily unique, the time reversal operator may always be used:

$$j(u) = j(\theta)\ j(u)^{*}\ j(\theta)^{-1} \text{ and } j(a) = j(\theta)\ j(a)^{*}\ j(\theta)^{-1*}$$

Thus the problem reduces to finding a matrix form of $j(\theta)$. Some discussion earlier was given to block diagonalizing $j(\theta)$ but for our purpose here this is not particularly useful. It is better to find the 1jm symbol in H and extend it to G. This may be done explicitly for IRs of the first and second kinds by a suitable standardization of the IR matrices.

An IR of the first kind is equivalent to its conjugate and also to a real representation. Choose this real form:

$$k(u) = k(u)^{*}$$

An irreducible representation of the second kind is equivalent to its conjugate but not to a real representation. A suitable form is

$$k(u) = \begin{bmatrix} k_1(u) & k_2(u) \\ -k_2(u)^{*} & k_1(u)^{*} \end{bmatrix} \tag{14.1}$$

with

$$k(u) = \begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix} k(u)^{*} \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix} \tag{14.2}$$

This may be obtained from the form given by Wybourne [75] by a permutation of the basis vectors.

No standardization need be made for an IR of the third kind, apart from the obvious one that the matrices of k^* are the complex conjugates of thase of k.

Combining these with the results of chapter three yields the following for the various types of ICR:

$$\underline{\text{type}}\ (A) \qquad j(u) = k(u) \quad \text{and} \quad j(\theta) = I \tag{14.3}$$

$$\underline{\text{type}}\ (B) \qquad j(u) = \begin{pmatrix} k(u) & 0 \\ 0 & k(u) \end{pmatrix} \quad \text{and} \quad j(\theta) = \begin{pmatrix} 0 & 0 & 0 & I \\ 0 & 0 & -I & 0 \\ 0 & -I & 0 & 0 \\ I & 0 & 0 & 0 \end{pmatrix} \tag{14.4}$$

$$\underline{\text{type}}\ (C) \qquad j(u) = \begin{pmatrix} k(u) & 0 \\ 0 & k(u)^* \end{pmatrix} \quad \text{and} \quad j(\theta) = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \tag{14.5}$$

$$\underline{\text{type}}\ (D) \qquad j(u) = k(u) \ \text{and}\ j(\theta) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{14.6}$$

$$\underline{\text{type}}\ (E) \qquad j(u) = \begin{pmatrix} k(u) & 0 \\ 0 & k(u) \end{pmatrix} \quad \text{and} \quad j(\theta) = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{14.7}$$

$$\underline{\text{type}}\ (F) \qquad j(u) = \begin{pmatrix} k(u) & 0 \\ 0 & k(u)^* \end{pmatrix} \quad \text{and} \quad j(\theta) = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \tag{14.8}$$

in which the 1jm symbol is given explicitly. The 1j symmetry ϕ_j upon interchange of the m values in trivially found from this.

14.3 THE 2jm SYMBOL

Using the conventions of the previous section the 2jm symbol may easily be constructed by use of equation (8.16)

$$(j\ \underset{\sim}{0}\ j)^{r}{}_{m_1 0\, m_2} = [j]^{-\frac{1}{2}}\delta_{m_4 \dot{m}_3}\begin{bmatrix}\dot{m}_3 & \\ & m_2\end{bmatrix}\langle j\ \underset{\sim}{0} \mid j\rangle^{rm_4}{}_{m_1 0}$$

where $\begin{bmatrix}\dot{m}_3 & \\ & m_2\end{bmatrix} = j(\theta)^{-1}$ and the coupling coefficients are given in section 3.4. Whereas in representation theory there is essentially only one coupling coefficient, in corepresentation theory there may be up to four. Each row of the 2jm symbol may be found separately and after checking orthogonality between rows the complete symbol may be found. It turns out to be already in symmetrized form.

<u>type</u> (A) $\quad \langle j\ \underset{\sim}{0} \mid j\rangle = I$ and $j(\theta)^{-1} = I$

Now $\langle j\ \underset{\sim}{0} \mid j\rangle$ is a $[j]$ by $[j]$ matrix, as is $j(\theta)^{-1}$, whereas $(j\ \underset{\sim}{0}\ j)$ is one by $[j]^2$. The δ term 'smears' the product on the right as we illustrate by the case $[j] = 2$. We have

$$(j\ \underset{\sim}{0}\ j)^{1}{}_{1\,0\,1} = 2^{-\frac{1}{2}}\ \langle j\ \underset{\sim}{0} \mid j\rangle^{m_4}{}_{10}\,\delta_{m_4 \dot{m}_3}\begin{bmatrix}\dot{m}_3 & \\ & 1\end{bmatrix}$$

The only non-vanishing term is when $\dot{m} = m = 1$ so

$$(j\ \underset{\sim}{0}\ j)_{1\,0\,1} = 2^{-\frac{1}{2}}$$

Similarly

$$(j \underset{\sim}{0} j)_{201} = (j \underset{\sim}{0} j)_{102} = 0$$

and

$$(j \underset{\sim}{0} j)_{202} = 2^{-\frac{1}{2}}$$

Thus $(j \underset{\sim}{0} j)_{m\,0\,m} = 2^{-\frac{1}{2}} (1\,0\,0\,1)_{m\,0\,m}$

The general case when $[j] = n$ follows similarly as

$$(j \underset{\sim}{0} j)_{m_1 0 m_2} = [j]^{-\frac{1}{2}} (E_1\ E_2\ \ldots . E_n)_{m_1 0 m_2} \qquad (14.9)$$

where E_i is an elementary row vector with n columns and one in the ith position and zeroes elsewhere

<u>type</u> (B) $\langle j \underset{\sim}{0} | j \rangle = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \begin{bmatrix} -iI & 0 & 0 & 0 \\ 0 & -iI & 0 & 0 \\ 0 & 0 & iI & 0 \\ 0 & 0 & 0 & iI \end{bmatrix},$

$$\begin{bmatrix} 0 & 0 & -iI & 0 \\ 0 & 0 & 0 & -iI \\ -iI & 0 & 0 & 0 \\ 0 & -iI & 0 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 0 & 0 & -I & 0 \\ 0 & 0 & 0 & -I \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix}$$

and $j(\theta)^{-1} = \begin{bmatrix} 0 & 0 & 0 & I \\ 0 & 0 & -I & 0 \\ 0 & -I & 0 & 0 \\ I & 0 & 0 & 0 \end{bmatrix}$

where the coupling coefficients drawn from section 3.4 have had their block size adjusted to conform to the block size of $j(\theta)^{-1}$. A similar construction as the last applied to each of the four coupling coefficients gives the four rows of the 2jm symbol which may be combined as

$$
(j\ \underset{\sim}{0}\ j)_{m_1 0 m_2} =
$$

$$
\begin{matrix}
 & 101 & 102 & 103 & 104 & 201 & 202 & 203 & 204 & 301 & 302 & 303 & 304 & 401 & 402 & 403 & 404 \\
[j]^{-\frac{1}{2}} & \left(\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right. &
\begin{matrix} -E \\ iE \\ 0 \\ 0 \end{matrix} &
\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} &
\begin{matrix} 0 \\ 0 \\ -iE \\ E \end{matrix} &
\begin{matrix} E \\ -iE \\ 0 \\ 0 \end{matrix} &
\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} &
\begin{matrix} 0 \\ 0 \\ iE \\ -E \end{matrix} &
\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} &
\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} &
\begin{matrix} 0 \\ 0 \\ -iE \\ -E \end{matrix} &
\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} &
\begin{matrix} -E \\ -iE \\ 0 \\ 0 \end{matrix} &
\begin{matrix} 0 \\ 0 \\ iE \\ E \end{matrix} &
\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix} &
\begin{matrix} E \\ iE \\ 0 \\ 0 \end{matrix} &
\left.\begin{matrix} 0 \\ 0 \\ 0 \\ 0 \end{matrix}\right)_{m_1 0 m_2}
\end{matrix}
\qquad (14.10)
$$

with E as the row vector $(E_1, E_2 \ldots E_n)$. The block structure of the m values is shown along the top from which it is found that the first three rows possess $\{1^2\}$ symmetry and the last $\{2\}$ symmetry as required.

<u>type</u> (C)
$$\langle j\, \underset{\sim}{Q} \,|\, j\rangle = \begin{pmatrix} I & 0 \\ I & I \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} -iI & 0 \\ 0 & iI \end{pmatrix}$$

and

$$j(\theta)^{-1} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

to give

$$(j\ \underset{\sim}{Q}\ j)^r_{m_1 0 m_2} = [j]^{-\frac{1}{2}} \begin{pmatrix} \overset{101}{0} & \overset{102}{-iE} & \overset{201}{iE} & \overset{202}{0} \\ 0 & E & E & 0 \end{pmatrix}^r_{m_1 0 m_2} \qquad (14.11)$$

with the first row anti-symmetric and the second symmetric.

<u>type</u> (D)
$$\langle j\, \underset{\sim}{Q} \,|\, j\rangle = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \quad \text{and} \quad j(\theta)^{-1} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

giving one anti-symmetric row

$$(j\ \underset{\sim}{Q}\ j)^1_{m_1 0 m_2} = [j]^{-\frac{1}{2}} \begin{pmatrix} \overset{101}{0} & \overset{102}{-E} & \overset{201}{E} & \overset{202}{0} \end{pmatrix}^1_{m_1 0 m_2}$$

(14.12)

<u>type</u> (E)
$$\langle j\, \underset{\sim}{Q} \,|\, j\rangle = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}, \begin{pmatrix} -iI & 0 \\ 0 & iI \end{pmatrix}, \begin{pmatrix} 0 & -iI \\ -iI & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

$$\text{and} \quad j(\theta)^{-1} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}$$

yielding

$$(j \underset{\sim}{0} j)^{r}_{m_1 0 m_2} = [j]^{-\frac{1}{2}} \begin{pmatrix} & 101 & 102 & 201 & 202 \\ & 0 & -E & E & 0 \\ & 0 & iE & iE & 0 \\ & -iE & 0 & 0 & iE \\ & E & 0 & 0 & E \end{pmatrix}^{r}_{m_1 0 m_2} \qquad (14.13)$$

The first row is anti-symmetric and the other three symmetric

<u>type</u> (F) $\quad \langle j \underset{\sim}{0} | j \rangle = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \quad \begin{pmatrix} -iI & 0 \\ 0 & iI \end{pmatrix}$

from which

$$(j \underset{\sim}{0} j)^{r}_{m_1 0 m_2} = [j]^{-\frac{1}{2}} \begin{pmatrix} & 101 & 102 & 201 & 202 \\ & 0 & -iE & -iE & 0 \\ & 0 & E & -E & 0 \end{pmatrix}^{r}_{m_1 0 m_2} \qquad (14.14)$$

where the first row is symmetric and the second anti-symmetric.

14.4 THE 3jm SYMBOL

For the 2jm symbols we were able to explicity find the symbols using the standardizations of the IRs and the time reversal operator, but now we have a far more messy problem in that there are a large number (twenty six) types of 3jm symbol. Further, whilst a general unitary transformation in the multiplicity label may be applied to the 3jm symbols of the linear subgroup, only orthogonal transformations are allowed in the grey group because of the anti-linear operators. This means that only a fairly restricted set of 3jm symbols of the linear subgroup may be used in construction of those of the grey group. In this section we give a number of examples to illustrate these restrictions and to show how under these the 3jm symbols may be constructed. As in the last section, the standardized ICRs of section 2 are used.

example 1 j_1, j_2 and j_3 all of type (A)

This is the simplest case, for $j(\theta) = I$ and on descent to the linear subgroup H there is no branching. Suppose P reduces the triple product in G:

$$j_1 \otimes j_2 \otimes j_3(u) = P^{*T} P \oplus \ldots..$$

and

$$j_1 \otimes j_2 \otimes j_3(a) = P^{*T} P^{*} \oplus \ldots.$$

or more conveniently

$$P(j_1 \otimes j_2 \otimes j_3)(u) = P \qquad (14.15)$$

and

$$P(j_1 \otimes j_2 \otimes j_3)\ (a) = P^* \qquad (14.16)$$

where the number of rows of P equals the multiplicity of $\underset{\sim}{Q}$ in $j_1 \otimes j_2 \otimes j_3$ and the number of columns is $[j_1] \times [j_2] \times [j_3]$. From equation (14.9) since $j_1(u) = k_1(u)$ etc., P must also reduce $k_1 \otimes k_2 \otimes k_3$ and hence is a 3jm symbol of H. By setting $a = \theta$ in the second equation we find $P = P^*$. Thus any <u>real</u> 3jm symbol of H is also a 3jm symbol of G.

<u>example 2</u> j_1 of type (A), j_2 and j_3 of type (C)

Here

$$j_1 \otimes j_2 \otimes j_3\ (u) = k_1(u) \otimes \begin{pmatrix} k_2(u) & 0 \\ 0 & k_2(u)^* \end{pmatrix} \otimes \begin{pmatrix} k_3(u) & \\ 0 & k_3(u)^* \end{pmatrix}$$

$$= \begin{pmatrix} k_1 \otimes k_2 \otimes k_3(u) & 0 & 0 & 0 \\ 0 & k_1 \otimes k_2 \otimes k_3^*(u) & 0 & 0 \\ 0 & 0 & k_1 \otimes k_2^* \otimes k_3(u) & 0 \\ 0 & 0 & 0 & k_1 \otimes k_2^* \otimes k_3^*(u) \end{pmatrix} \qquad (14.17)$$

and

$$j_1 \otimes j_2 \otimes j_3(\theta) = \begin{pmatrix} 0 & 0 & 0 & I \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ I & 0 & 0 & 0 \end{pmatrix} \qquad (14.18)$$

There are two relevant 3jm symbols of H: P_1 which reduces $k_1 \otimes k_2 \otimes k_3$ (with P_1^* reducing $k_1 \times k_2^* \times k_3^*$) and P_2 which reduces $k_1 \otimes k_2 \otimes k_3^*$ (with P_2^* reducing $k_1 \otimes k_2^* \otimes k_3$). From the linear equation a trial value for P would be

$$P = \begin{pmatrix} P_1 & & & 0 \\ & P_2 & & \\ & & P_2^* & \\ 0 & & & P_1^* \end{pmatrix}$$

but equation (14.18) mixes the first and last rows and also the second and third rows, and a little experimentation shows that P may be taken as

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} P_1 & 0 & 0 & P_1^* \\ 0 & P_2 & P_2^* & 0 \\ 0 & -iP_2 & iP_2^* & 0 \\ iP_1 & 0 & 0 & -iP_1^* \end{pmatrix} \tag{14.19}$$

Note that there are no restrictions on the choice of P_1 and P_2 so this is one of the easiest cases to construct.

<u>example 3</u> j_1 and j_2 of type (A), j_3 of type (B).

we have

$$j_1 \otimes j_2 \otimes j_3(u) = \begin{pmatrix} k_1 \otimes k_2 \otimes k_3(u) & 0 \\ 0 & k_1 \otimes k_2 \otimes k_3(u) \end{pmatrix} \tag{14.20}$$

and $$j_1 \otimes j_2 \otimes j_3(\theta) = \begin{pmatrix} 0 & J \\ -J & 0 \end{pmatrix} \tag{14.21}$$

where $$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{14.22}$$

Now the multiplicity of $\underset{\sim}{0}$ in $k_1 \otimes k_2 \otimes k_3$ is even [49] and [k] is even, so the matrix P_1 which reduces $k_1 \otimes k_2 \otimes k_3$ may be written as

$$P_1 = \begin{pmatrix} P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} p_1 & p_2 \\ p_3 & p_4 \end{pmatrix}$$

and with the standardization of k_3 we also have

$$k_1 \otimes k_2 \otimes k_3(u) = \begin{pmatrix} k_4(u) & k_5(u) \\ -k_5(u)^* & k_4(u)^* \end{pmatrix}$$

It follows that P_1 may be taken as

$$P_1 = \begin{pmatrix} p_1 & p_2 \\ -p_2^* & p_1^* \end{pmatrix} \tag{14.23}$$

or that

$$P_2 J = P_3^* \quad \text{and} \quad P_3 J = -P_2^* \tag{14.24}$$

The reason for this particular choice is as follows: from table one the multiplicity of j_3 in $j_1 \otimes j_2$ is exactly one-half the multiplicity of k_3 in $k_1 \otimes k_2$ and the multiplicity of $\underset{\sim}{0}$ in $j_3 \otimes j_3$ is four. Equation (8.21) which gives the 3jm symbol in terms of the two sets of coupling coefficients implies that we should look for a 4 x 2 block structure of the 3jm symbol rather than a 2 x 2 block structure as suggested by equation (14.20) and that each block row should be made up of P_2 and P_3. As the 3jm symbol must also reduce equation (14.21) a relation between P_2, P_3 and J is required, of the form of equations (14.24) After these preliminaries it is straightforward to verify that P may be taken as

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} P_2 & P_3 \\ P_3 & -P_2 \\ iP_2 & -iP_3 \\ iP_3 & iP_2 \end{pmatrix} \qquad (14.25)$$

This comparative complexity seems to be shared by all couplings involving an IR of the second kind due to the form of the matrix which gives equivalence to the conjugate. We conclude this section with probably the most extreme case:

<u>example 4</u> $j_1 = j_2 = j_3$ of type (B)

Here

$$j \otimes j \otimes j\ (u) = \begin{pmatrix} k \otimes k \otimes k\ (u) & & 0 \\ & k \otimes k \otimes k\ (u) & \\ & & \ddots \\ 0 & & & k \otimes k \otimes k\ (u) \end{pmatrix} \qquad (14.26)$$

as an 8 x 8 block matrix, and

$$j\otimes j\otimes j\ (\theta) = \begin{pmatrix} & & & & & & & J\otimes J\otimes J \\ & & & & & & -J\otimes J\otimes J & \\ & & 0 & & & -J\otimes J\otimes J & & \\ & & & & J\otimes J\otimes J & & & \\ & & & -J\otimes J\otimes J & & & & \\ & & J\otimes J\otimes J & & & & & \\ & J\otimes J\otimes J & & & & & & \\ -J\otimes J\otimes J & & & & & 0 & & \end{pmatrix} \qquad (14.27)$$

Again the multiplicity of $\underset{\sim}{0}$ in $k \otimes k \otimes k$ is even and now $[k] \otimes [k] \otimes [k]$ is divisible by eight. Thus P which reduces $k \otimes k \otimes k$ may be written as

$$P_1 = \begin{pmatrix} P_2 \\ P_3 \end{pmatrix} = \begin{pmatrix} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \\ p_9 & p_{10} & p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \end{pmatrix}$$

It is straight forward but tedious to verify that, similar to the last example, we may choose P_3 to satisfy

$$P_2(J \otimes J \otimes J) = P_3^* \quad \text{and} \quad P_3(J \otimes J \otimes J) = -P_2^* \qquad (14.28)$$

If each block row is to be constructed from P_2 and P_3 we shall need sixteen such rows, with eight block columns. There is an additional complication, for from section 7.3 $\Gamma_j = (4\{3\} \oplus 2\{21\}) \otimes \Gamma_k$ where Γ_j and Γ_k are the S_3 symmetries of the 3jm symbols in each group. Taking these into account and using the permutation matrices for {21} of Butler [48] or Hamermesh [76] gives, with the block labelling at the top and the symmetry structure down the side

$$
P = \begin{array}{c}
\begin{array}{cccccccc}
111 & 112 & 121 & 122 & 211 & 212 & 221 & 222
\end{array} \\
\left(\begin{array}{cccccccc}
P_2/\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & P_3/\sqrt{2} \\
P_3/\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & -P_2/\sqrt{2} \\
P_2/\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & -iP_3/\sqrt{2} \\
iP_3/\sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 & iP_2/\sqrt{2} \\
0 & P_2/\sqrt{6} & P_2/\sqrt{6} & -P_3/\sqrt{6} & P_2/\sqrt{6} & -P_3/\sqrt{6} & -P_3/\sqrt{6} & 0 \\
0 & P_3/\;6 & P_3/\sqrt{6} & P_2/\sqrt{6} & P_3/\sqrt{6} & P_2/\sqrt{6} & P_2/\sqrt{6} & 0 \\
0 & iP_2/\sqrt{6} & iP_2/\sqrt{6} & iP_3/\sqrt{6} & iP_2/\sqrt{6} & iP_3/\sqrt{6} & iP_3/\sqrt{6} & 0 \\
0 & iP_3/\sqrt{6} & iP_3/\sqrt{6} & -iP_2/\sqrt{6} & iP_3/\sqrt{6} & -iP_2/\sqrt{6} & -iP_2/\sqrt{6} & 0 \\
0 & P_2/\sqrt{3} & -P_2/2\sqrt{3} & P_3/2\sqrt{3} & -P_2/2\sqrt{3} & P_3/2\sqrt{3} & -P_3/\sqrt{3} & 0 \\
0 & P_3/\sqrt{3} & -P_3/2\sqrt{3} & -P_2/2\sqrt{3} & -P_3/2\sqrt{3} & -P_2/2\sqrt{3} & P_2/\sqrt{3} & 0 \\
0 & 0 & P_2/2 & P_3/2 & -P_2/2 & -P_3/2 & 0 & 0 \\
0 & 0 & P_3/2 & -P_2/2 & -P_3/2 & -P_2/2 & 0 & 0 \\
0 & iP_2/\sqrt{3} & -iP_2/2\sqrt{3} & -iP_3/2\sqrt{3} & -iP_2/2\sqrt{3} & -iP_3/2\sqrt{3} & iP_3/\sqrt{3} & 0 \\
0 & iP_3/\sqrt{3} & -iP_3/2\sqrt{3} & iP_2/2\sqrt{3} & -iP_3/2\sqrt{3} & iP_2/2\sqrt{3} & -iP_2/\sqrt{3} & 0 \\
0 & 0 & iP_2/2 & -iP_3/2 & -iP_2/2 & iP_3/2 & 0 & 0 \\
0 & 0 & iP_3/2 & iP_2/2 & -iP_3/2 & -iP_2/2 & 0 & 0
\end{array}\right)
\begin{array}{l}
\\ \{3\} \otimes \Gamma_k \\ \\ \{3\} \otimes \Gamma_k \\ \\ \{3\} \otimes \Gamma_k \\ \\ \{3\} \otimes \Gamma_k \\ \\ \\ \{21\} \otimes \Gamma_k \\ \\ \\ \\ \{21\} \otimes \Gamma_k \\ \\
\end{array}
\end{array}
\qquad (14.29)
$$

These methods have been used to deal with all types of 3jm symbols, and the results are presented in the appendix. Permutational symmetry when two or more of j_1, j_2 and j_3 are equal has been ignored, but this is easily found.

CHAPTER FIFTEEN : CONCLUSION

We have seen in the preceding chapters the changes and modifications required to construct the Racah algebra for an arbitrary grey compact group. Some of the results found are stronger than the corresponding results for linear groups but some, lamentably, are very much weaker. To summarize briefly:

(a) By considering the reduction of the triple product to the identity ICR, a perfectly satisfactory 3jm symbol may be found. This symbol possesses permutation and completeness properties as in linear groups.

(b) The Wigner-Eckart theorem holds for these groups and uses the 3jm symbol. This allows the powerful methods of tensor operators to be used freely.

(c) Racah's lemma for the 3jm symbols holds just as for linear groups. Combining this with the Wigner-Eckart theorem means that operators classified under one group can be used to find matrix elements of a subgroup. Again the methods used in linear groups can be equally used in grey groups.

(d) On the other hand, the reduction of two ICRs to a third gives rise to some annoying problems. The weak form of Schur's lemma causes incompleteness, non-uniqueness and some orthogonality difficulties.

(e) The 3jm symbol is not simply related to the coupling coefficient through a Wigner tensor and a transformation in the multiplicity label. An additional coupling coefficient tensor $\langle j\, \underset{\sim}{Q} | j \rangle$ is required, and for ICRs of types (b) and (c) this is non-trivial. This has the potential to cause grave problems in the symmetry properties of the coupling coefficients as we find ourselves relating spaces in the multiplicity labels which have different dimensions.

(f) This possible hiatus in the symmetries of the coupling coefficient could be ignored if it were not for the fact that the coefficient occurs in a number of areas, particularly in the n-j symbols. Here we had to introduce three different symbols of varying dimensions in the multiplicities with different properties. No discussion was given to the Biedenharn identity or Racah back-coupling rule for the simple reason that they depend on special properties of the 3jm symbols and coupling coefficients which at best will only hold for special choeces, and at worst may not hold at all.

The basic aim of this thesis has has been accomplished: to examine the form of the Racah algebra for grey groups and to find the properties and domain of use of the various symbols. The initial expectation from the theory for linear groups that the n-jm symbols would be sufficient to explain all aspects of the Racah algebra is in doubt in the area of the n-j symbols. In the second part of this thesis the grey double point groups are considered in detail, and for these groups symmetrized coupling coefficients and 3jm symbols orthogonal in a sum over only two m-values may be found. Whether this is accidental to these groups, or can be done for all groups remains at present an unsolved problem.

On a wider front, this thesis leads to optimism towards the development of Racah algebra techniques for other group theoretic methods which do not involve simple homomorphic mappings onto multiplicative matrix groups. These include projective representations, corepresentations of magnetic groups not including the operation of time reversal, and projective corepresentations. At present, extensions into these areas seem singularly lacking (although see Altmann and Palacio [77] for projective representations).

Applications of this theory have not been dealt with at all. In view of the validity of the Wigner-Eckart theorem, such applications must be along the same lines as for linear groups (see for example references [53] and [75]). Two immediate areas are chemical calculations for non-magnetic, paramagnetic and diamagnetic crystals, and elementary particle calculations where time reversal is a symmetry operation. Previously most such calculations were performed in the linear subgroup and then time reversal symmetry added in, or by the physically indefensible step of restricting the base field to the reals. Cracknell [17] has pointed out that it is easy to make mistakes using this first method and where applicable, non-linear symmetries should be considered equally with the linear symmetries.

APPENDIX : 3jm Symbols of a Grey Group G from the 3km Symbols of the Linear Subgroup H.

In chapter fourteen section four some examples were given of constructions of 3jm symbols for a grey group G from the 3km symbols of the linear subgroup H. In this appendix a complete list of such constructions is given.

A 3km symbol for any particular triad $(k_1k_2k_3)$ is indeterminate to within a unitary transformation in the multiplicity label. Such a freedom, while useful for symmetry adaptation etcetera, is rather a nuisance for the induction process as the form of a 3jm symbol is heavily dependent on the properties of the 3km symbols under complex conjugation. Without loss of generality though these properties may be standardised in order that simple and definite constructions may be given. A unitary transformation can always be applied after. We take then the following:

(a) k_1, k_2 and k_3 all of the first kind

$$Q\left[k_1 \otimes k_2 \otimes k_3\ (u)\right] = Q$$

and $$Q^*\left[I_1 \otimes I_2 \otimes I_3\right] = Q$$

where I_i is the unit matrix of dimension $[k_i]$

(b) k_1 and k_2 of the first kind, k_3 of the second

$$Q\left[k_1 \otimes k_2 \otimes k_3\ (u)\right] = Q$$

and $$J_M\, Q^*\left[I_1 \otimes I_2 \otimes J_3\right] = Q$$

Here J_M is the matrix $\begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ of dimension

equal to the (even) multiplicity of $\underset{\sim}{Q}$ in $k_1 \otimes k_2 \otimes k_3$, and $J_3 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ has dimension $[k_3]$. Each block is square.

As an alternative formulation of this, we may set

$Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$ to give

$$Q_1^* [I_1 \otimes I_2 \otimes J_3] = Q_2$$

and $$Q_2^* [I_1 \otimes I_2 \otimes J_3] = -Q_1^*$$

(c) k_1 and k_2 of the first kind, k_3 of the third

$$Q\ [k_1 \otimes k_2 \otimes k_3\ (u)] = Q$$

and $$Q^* [k_1 \otimes k_2 \otimes k_3^*\ (u)] = Q^*$$

(d) k_1 of the first kind, k_2 and k_3 of the second

$$Q\ [k_1 \otimes k_2 \otimes k_3\ (u)] = Q$$

and $$Q^* [I_1 \otimes J_2 \otimes J_3] = Q$$

(e) k_1 of the first kind, k_2 of the second and k_3 of the third

$$Q\ [k_1 \otimes k_2 \otimes k_3\ (u)] = Q$$

and $$Q'\ [k_1 \otimes k_2 \otimes k_3^*\ (u)] = Q'$$

with $$Q^* [I_1 \otimes J_2 \otimes I_3] = Q'$$

and $$Q'^* [I_1 \otimes J_2 \otimes I_3] = -Q$$

(f) k_1 of the first kind, k_2 and k_3 of the third

$$Q\ [k_1 \otimes k_2 \otimes k_3\ (u)] = Q$$

and $$Q^* [k_1 \otimes k_2^* \otimes k_3^*\ (u)] = Q^*$$

In addition there may also be non-zero 3km symbols Q' and Q'^* for the triads $(k_1 k_2 k_3^*)$ and $(k_1 k_2^* k_3)$ respectively satisfying similar equations.

(g) k_1, k_2 and k_3 all of the second kind.

$$Q \left[k_1 \otimes k_2 \otimes k_3 (u)\right] = Q$$

and $J_M Q^* \left[J_1 \otimes J_2 \otimes J_3\right] = Q$

Alternatively, with $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$

$$Q_1^* \left[J_1 \otimes J_2 \otimes J_3\right] = Q_2$$

and $Q_2^* \left[J_1 \otimes J_2 \otimes J_3\right] = -Q_1$

(h) k_1 and k_2 of the second kind, k_3 of the third.

$$Q \left[k_1 \otimes k_2 \otimes k_3 (u)\right] = Q$$

and $Q' \left[k_1 \otimes k_2 \otimes k_3^* (u)\right] = Q'$

with $Q^* \left[J_1 \otimes J_2 \otimes I_3\right] = Q'$

and $Q'^* \left[J_1 \otimes J_2 \otimes I_3\right] = Q$

(i) k_1 of the second kind, k_2 and k_3 of the third.

$$Q \left[k_1 \otimes k_2 \otimes k_3 (u)\right] = Q$$

and $Q' \left[k_1 \otimes k_2^* \otimes k_3^* (u)\right] = Q'$

with $Q^* \left[J_1 \otimes I_2 \otimes I_3\right] = Q'$

and $Q'^* \left[J_1 \otimes I_2 \otimes I_3\right] = -Q$

In addition there may be non-zero Q" and Q'''
obeying similar relations reducing the triads
$(k_1 k_2 k_3^*)$ and $(k_1 k_2^* k_3)$ respectively.

(j) k_1, k_2 and k_3 all of the third kind.

Q reduces $(k_1 k_2 k_3)$, Q^* reduces $(k_1^* k_2^* k_3^*)$;

Q' reduces $(k_1 k_2 k_3^*)$, Q'^* reduces $(k_1^* k_2^* k_3)$ etcetera.

The proofs of all these are fairly straightforward and depend on two results:

Lemma One

If M is a symmetric unitary matrix, then

$$M = A D A^T$$

Where A is orthogonal and D unitary diagonal. This has been shown by Wigner [6]

Lemma Two

If M is an anti-symmetric unitary matrix then

$$M = A J D A^T = - A D J A^T$$

with A orthogonal, D unitary diagonal and $J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ as before.

Proof

If $\underline{w}$ is any eigenvector of M, then

$$M \underline{w} = \lambda \underline{w}$$

and $$M \underline{w}^* = -\lambda \underline{w}^*$$

by unitarity and anti-symmetry. Setting

$$\underline{w} = \underline{u} + i \underline{v}$$

with $\underline{u}$, $\underline{v}$ real, this gives

$$M \underline{u} = i \lambda \underline{v}$$

and $$M \underline{v} = -i \lambda \underline{u}$$

Decomposing each eigenvector $\underline{w}_n$ in this way, we now set

$$A = (\underline{u}_1, \underline{u}_2, \ldots, \underline{v}_1, \underline{v}_2, \ldots)$$

and $D = \text{diag.} (i\lambda_1, i\lambda_2, \ldots, i\lambda_1, i\lambda_2, \ldots)$

The result easily follows.

To show how these are used consider first case (a) . From

$$Q\left[k_1 \otimes k_2 \otimes k_3\ (u)\right] = Q$$

and $k_i^* = k_i$,

$$Q^*\left[k_1 \otimes k_2 \otimes k_3\ (u)\right] = Q^*$$

Hence

$$Q^* = MQ$$

where M is a unitary matrix in the multiplicity label.

Let $M = A D A^T$ as above, C be one of the diagonal unitary square roots of D, and Q' by

$$Q = A C^* Q'$$

Substituting and cancelling,

$$Q'^* = Q'$$

as required.

Next consider case (b). Here

$$Q\left[k_1 \otimes k_2 \otimes k_3\ (u)\right] = Q$$

gives $Q^*\left[k_1 \otimes k_2 \otimes (J\ k_3\ J^{-1})\ (u)\right] = Q^*$

or $Q^*\left[I_1 \otimes I_2 \otimes J_3\right]\left[k_1 \otimes k_2 \otimes k_3\ (u)\right] = Q^*\left[I_1 \otimes I_2 \otimes J_3\right]$

Hence $Q^*\left[I_1 \otimes I_2 \otimes J_3\right] = M Q$

where M is again a unitary matrix in the multiplicity label.

Let $M = A J_M D A^T$ as in lemma two, and C be a diagonal unitary square root of D satisfying

$$C J_M = -J_M C$$

This is always possible due to the special form of D. Now define Q' by

$$Q = A J_M C^* Q'$$

Substituting and cancelling,

$$J_M\, Q'^* \left[I_1 \otimes I_2 \otimes J_3 \right] = Q'$$

as required.

The other cases follow similarly. We remark that simpler proofs do exist, but that the ones above give a prescriptive methods for finding standardised 3km symbols from arbitrary ones.

With these 3km symbols, the 3jm symbols for grey G are found by the methods of chapter fourteen section four. We set

$$P \left[j_1 \otimes j_2 \otimes j_3 \ (u) \right] = P$$

and
$$P \left[j_1 \otimes j_2 \otimes j_3 \ (\theta) \right] = P^*$$

Using the appropriate 3km symbols of cases (a) - (j) gives:

(1) j_1, j_2 and j_3 of type (A).

$$P = Q$$

(2) j_1 and j_2 of type (A), j_3 of type (B)

$$P = 1/\sqrt{2} \begin{pmatrix} Q_1 & Q_2 \\ iQ_1 & -iQ_2 \\ Q_2 & -Q_1 \\ iQ_2 & iQ_1 \end{pmatrix}$$

(3) j_1 and j_2 of type (A), j_3 of type (c)

$$P = 1/\sqrt{2} \begin{pmatrix} Q & Q^* \\ iQ & -iQ^* \end{pmatrix}$$

(4) j_1 of type (A), j_2 and j_3 of type (B)

$$P = 1/\sqrt{2} \begin{pmatrix} Q & 0 & 0 & Q \\ iQ & 0 & 0 & -iQ \\ 0 & Q & -Q & 0 \\ 0 & iQ & iQ & 0 \end{pmatrix}$$

(5) j_1 of type (A), j_2 of type (B), j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & Q' \\ iQ & 0 & 0 & -iQ' \\ 0 & Q' & -Q & 0 \\ 0 & iQ' & iQ & 0 \end{pmatrix}$$

(6) j_1 of type (A), j_2 and j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & Q^* \\ iQ & 0 & 0 & -iQ^* \\ 0 & Q' & Q'^* & 0 \\ 0 & iQ' & -iQ'^* & 0 \end{pmatrix}$$

(7) j_1, j_2 and j_3 all of type (B).

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & Q_2^* \\ iQ_1 & 0 & 0 & 0 & 0 & 0 & 0 & -iQ_2^* \\ Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_1^* \\ iQ_2 & 0 & 0 & 0 & 0 & 0 & 0 & iQ_1^* \\ 0 & Q_1 & 0 & 0 & 0 & 0 & -Q_2^* & 0 \\ 0 & iQ_1 & 0 & 0 & 0 & 0 & iQ_2^* & 0 \\ 0 & Q_2 & 0 & 0 & 0 & 0 & Q_1^* & 0 \\ 0 & iQ_2 & 0 & 0 & 0 & 0 & -iQ_1^* & 0 \\ 0 & 0 & Q_1 & 0 & 0 & -Q_2^* & 0 & 0 \\ 0 & 0 & iQ_1 & 0 & 0 & iQ_2^* & 0 & 0 \\ 0 & 0 & Q_2 & 0 & 0 & Q_1^* & 0 & 0 \\ 0 & 0 & iQ_2 & 0 & 0 & -iQ_1^* & 0 & 0 \\ 0 & 0 & 0 & Q_1 & Q_2^* & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ_1 & -iQ_2^* & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_2 & -Q_1^* & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ_2 & iQ_1^* & 0 & 0 & 0 \end{pmatrix}$$

(8) j_1 and j_2 of type (B), j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & Q' \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & -iQ' \\ 0 & Q' & 0 & 0 & 0 & 0 & Q & 0 \\ 0 & iQ' & 0 & 0 & 0 & 0 & -iQ & 0 \\ 0 & 0 & Q & 0 & 0 & Q' & 0 & 0 \\ 0 & 0 & iQ & 0 & 0 & -iQ' & 0 & 0 \\ 0 & 0 & 0 & Q' & Q & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ' & -iQ & 0 & 0 & 0 \end{pmatrix}$$

(9) j_1 of type (B), j_2 and j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & Q' \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & -iQ' \\ 0 & 0 & 0 & Q' & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ' & iQ & 0 & 0 & 0 \\ 0 & Q'' & 0 & 0 & 0 & 0 & Q''' & 0 \\ 0 & iQ'' & 0 & 0 & 0 & 0 & -iQ''' & 0 \\ 0 & 0 & Q''' & 0 & 0 & -Q'' & 0 & 0 \\ 0 & 0 & iQ''' & 0 & 0 & iQ'' & 0 & 0 \end{pmatrix}$$

where Q, Q' reduce $(k_1\ k_2\ k_3)$ and $(k_1\ k_2^*\ k_3^*)$ respectively and Q'', Q''' (if non-zero) reduce $(k_1\ k_2\ k_3^*)$ and $(k_1\ k_2^*\ k_3)$ respectively.

(10) j_1, j_2 and j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & Q^* \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & -iQ^* \\ 0 & Q' & 0 & 0 & 0 & 0 & Q'^* & 0 \\ 0 & iQ' & 0 & 0 & 0 & 0 & -iQ'^* & 0 \\ 0 & 0 & Q'' & 0 & 0 & Q''^* & 0 & 0 \\ 0 & 0 & iQ'' & 0 & 0 & -iQ''^* & 0 & 0 \\ 0 & 0 & 0 & Q''' & Q'''^* & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ''' & -iQ'''^* & 0 & 0 & 0 \end{pmatrix}$$

where Q reduces $(k_1\ k_2\ k_3)$, Q' reduces $(k_1\ k_2\ k_3^*)$, Q" reduces $(k_1\ k_2^*\ k_3)$ and Q''' reduces $(k_1\ k_2^*\ k_3^*)$

(11) j_1 and j_2 of type (D), j_3 of type (A)

$$P = Q$$

(12) j_1 and j_2 of type (D), j_3 of type (B)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q_1 & Q_2 \\ iQ_1 & -iQ_2 \\ Q_2 & -Q_1 \\ iQ_2 & iQ_1 \end{pmatrix}$$

(13) j_1 and j_2 of type (D), j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & Q' \\ iQ & -iQ' \end{pmatrix}$$

(14) j_1 of type (D), j_2 of type (E), j_3 of type (A)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q_1 & Q_2 \\ iQ_1 & -iQ_2 \\ Q_2 & -Q_1 \\ iQ_2 & iQ_1 \end{pmatrix}$$

(15) j_1 of type (D), j_2 of type (E), j_3 of type (B)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & Q \\ iQ & 0 & 0 & -iQ \\ 0 & Q & -Q & 0 \\ 0 & iQ & iQ & 0 \end{pmatrix}$$

(16) j_1 of type (D), j_2 of type (E), j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & Q' \\ iQ & 0 & 0 & -iQ' \\ 0 & Q' & -Q & 0 \\ 0 & iQ' & iQ & 0 \end{pmatrix}$$

(17) j_1 and j_2 of type (E), j_3 of type (A)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & Q \\ iQ & 0 & 0 & -iQ \\ 0 & Q & -Q & 0 \\ 0 & iQ & iQ & 0 \end{pmatrix}$$

(18) j_1 and j_2 of type (E), j_3 of type (B)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q_1 & 0 & 0 & 0 & 0 & 0 & 0 & Q_2 \\ iQ_1 & 0 & 0 & 0 & 0 & 0 & 0 & -iQ_2 \\ Q_2 & 0 & 0 & 0 & 0 & 0 & 0 & -Q_1 \\ iQ_2 & 0 & 0 & 0 & 0 & 0 & 0 & iQ_1 \\ 0 & Q_1 & 0 & 0 & 0 & 0 & -Q_2 & 0 \\ 0 & iQ_1 & 0 & 0 & 0 & 0 & iQ_2 & 0 \\ 0 & Q_2 & 0 & 0 & 0 & 0 & Q_1 & 0 \\ 0 & iQ_2 & 0 & 0 & 0 & 0 & -iQ_1 & 0 \\ 0 & 0 & Q_1 & 0 & 0 & -Q_2 & 0 & 0 \\ 0 & 0 & iQ_1 & 0 & 0 & iQ_2 & 0 & 0 \\ 0 & 0 & Q_2 & 0 & 0 & Q_1 & 0 & 0 \\ 0 & 0 & iQ_2 & 0 & 0 & -iQ_1 & 0 & 0 \\ 0 & 0 & 0 & Q_1 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ_1 & -iQ_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_2 & -Q_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ_2 & iQ_1 & 0 & 0 & 0 \end{pmatrix}$$

(19) j_1 and j_2 of type (E), j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & Q^* \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & -iQ^* \\ 0 & Q^* & 0 & 0 & 0 & 0 & Q & 0 \\ 0 & iQ^* & 0 & 0 & 0 & 0 & -iQ & 0 \\ 0 & 0 & Q & 0 & 0 & -Q^* & 0 & 0 \\ 0 & 0 & iQ & 0 & 0 & iQ^* & 0 & 0 \\ 0 & 0 & 0 & Q^* & -Q & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ^* & iQ & 0 & 0 & 0 \end{pmatrix}$$

(20) j_1 of type (E), j_2 of type (F), j_3 of type (A)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & -Q^* \\ iQ & 0 & 0 & iQ^* \\ 0 & Q^* & Q & 0 \\ 0 & iQ^* & -iQ & 0 \end{pmatrix}$$

(21) j_1 of type (E), j_2 of type (F), j_3 of type (B)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & -Q' \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & iQ' \\ 0 & Q & 0 & 0 & 0 & 0 & Q' & 0 \\ 0 & iQ & 0 & 0 & 0 & 0 & -iQ' & 0 \\ 0 & 0 & Q' & 0 & 0 & -Q & 0 & 0 \\ 0 & 0 & iQ' & 0 & 0 & iQ & 0 & 0 \\ 0 & 0 & 0 & Q' & Q & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ' & -iQ & 0 & 0 & 0 \end{pmatrix}$$

(22) j_1 of type (E), j_2 of type (F), j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & -Q^* \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & iQ^* \\ 0 & 0 & 0 & Q^* & Q & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ^* & -iQ & 0 & 0 & 0 \\ 0 & Q' & 0 & 0 & 0 & 0 & -Q'^* & 0 \\ 0 & iQ' & 0 & 0 & 0 & 0 & iQ'^* & 0 \\ 0 & 0 & Q'^* & 0 & 0 & Q' & 0 & 0 \\ 0 & 0 & iQ'^* & 0 & 0 & -iQ'^* & 0 & 0 \end{pmatrix}$$

(23) j_1 and j_2 of type (F), j_3 of type (A)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & Q^{*} \\ iQ & 0 & 0 & -iQ^{*} \\ 0 & Q' & -Q'^{*} & 0 \\ 0 & iQ' & iQ'^{*} & 0 \end{pmatrix}$$

(24) j_1 and j_2 of type (F), j_3 of type (B)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & Q' \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & -iQ' \\ 0 & Q & 0 & 0 & 0 & 0 & -Q' & 0 \\ 0 & iQ & 0 & 0 & 0 & 0 & iQ' & 0 \\ 0 & 0 & Q'' & 0 & 0 & -Q''' & 0 & 0 \\ 0 & 0 & iQ'' & 0 & 0 & iQ''' & 0 & 0 \\ 0 & 0 & 0 & Q'' & Q''' & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ'' & -iQ''' & 0 & 0 & 0 \end{pmatrix}$$

(25) j_1 and j_2 of type (F), j_3 of type (C)

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} Q & 0 & 0 & 0 & 0 & 0 & 0 & Q^{*} \\ iQ & 0 & 0 & 0 & 0 & 0 & 0 & -iQ^{*} \\ 0 & Q' & 0 & 0 & 0 & 0 & Q'^{*} & 0 \\ 0 & iQ' & 0 & 0 & 0 & 0 & -iQ'^{*} & 0 \\ 0 & 0 & Q'' & 0 & 0 & -Q''^{*} & 0 & 0 \\ 0 & 0 & iQ'' & 0 & 0 & iQ''^{*} & 0 & 0 \\ 0 & 0 & 0 & Q''' & -Q'''^{*} & 0 & 0 & 0 \\ 0 & 0 & 0 & iQ''' & iQ'''^{*} & 0 & 0 & 0 \end{pmatrix}$$

where Q reduces $(k_1\ k_2\ k_3)$, Q' reduces $(k_1\ k_2\ k_3^{*})$, Q'' reduces $(k_1\ k_2^{*}\ k_3)$, and Q''' reduces $(k_1\ k_2^{*}\ k_3^{*})$.

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THE RACAH ALGEBRA FOR GROUPS WITH TIME REVERSAL SYMMETRY

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in the

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PART TWO:

TABLES

All references to chapters and equations
are to the chapters and equations of
Part One of this thesis

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ONE : INTRODUCTION

1.1 Introduction

Many single crystals in chemistry are fairly regular in structure and have the symmetry (to a good approximation) of a point group. This symmetry restricts the possible forms of the space-part of the quantum mechanical energy functions to those which form irreducible basis vectors for an irreducible representation of the symmetry group. Such groups are the point groups K, O, T, D_n and C_n and if reflections and space inversions are also included, O_h, D_{nd}, C_{nv} etc. In cases where the spin-orbit coupling is larger than the crystal field it is customary to use spinor wave functions, and the geometrical symmetry still applies though we then have to use the 'spinor' representations of the double groups K^*, O^*, D, T^*, D_n^*, and C_n^* etc.

In addition to the well-known operations of rotation, reflection and inversion, for crystals which do have a zero magnetic moment the anti-linear operator θ also commutes with the Hamiltonian and is thus an element of the symmetry group of the Hamiltonian. Thus for single non-magnetic, paramagnetic and diamagnetic crystals the theory developed earlier is applicable.

In these tables the 3jm symbols for the grey double point groups K^*, O^*, D_n^* and C_n^* are given together with other tables useful for these groups. Grey T^* is not given as there is no branching on descent from grey O^*.

1.2 Notation

For purposes of establishing the Racah algebra the tensor notation has proved invaluable. However, for the purposes of tables and use it is not necessary to use such a formal notation and it may be replaced by the simpler notations more common in physics and chemistry. To some extent this is not an easy task due to the variety of notations-and even worse-phase conventions that exist, each with their own advocates through historical, practical or theoretical reasons. We defer phase conventions to later sections. Here we give the equivalences between the tensor notation and a notation common in chemistry through the work of Griffith [1]. In the forthcoming book by Butler [2] we expect to see extensive tables of 3jm symbols for the non-grey point groups and since a standardisation needs to be adopted in this area these tables will probably be modified to allow an easy 'descent in symmetry' to his tables.

(a) The Basis Vectors

The basis vector $e^{j}{}_{(m)} = |jm\rangle$ and $e^{j}_{(\dot{m})} = \langle jm|$. These should not be confused with the basis vectors of SU(2) which will henceforth be denoted by capitals as $|JM\rangle$. A more familiar notation to chemists would be that of Bethe:.

$$e^{\Gamma}{}_{(c)} = |\Gamma_c\rangle$$

(b) The 3jm Symbol

Basically for historical reasons–namely the book by Griffith, we adopt the $\bar{V}$ notation to give

$$(j_1 j_2 j_3)^{m_1 m_2 m_3}{}_r = \bar{V}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}_r \tag{1}$$

with

$$(j_1 j_2 j_3)^r{}_{m_1 m_2 m_3} = \bar{V}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}^{r^*} \tag{2}$$

or in the Bethe notation

$$(\Gamma_1 \Gamma_2 \Gamma_3)^{abc}{}_r = \bar{V}\begin{pmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 \\ a & b & c \end{pmatrix}_r$$

(c) The Coupling Coefficient

We write

$$<j_1 j_2 | j_3>^{m_1 m_2}{}_{rm_3} = <j_1 j_2 m_1 m_2 | j_1 j_2 j_3 m_3>_r \tag{3}$$

with

$$<j_1 j_2 | j_3>^{rm_3}{}_{m_1 m_2} = <j_1 j_2 j_3 m_3 | j_1 j_2 m_1 m_2>^r \tag{4}$$

All other coefficients may be left unchanged.

1.3 Computation of the Coefficients

At least three methods are available for computation of the 3jm symbols. The defining equations (6.1) and (6.2) and the variants which follow form a good starting point for a theoretical analysis but not for practical computations. The considerations of chapter fourteen give a simple method of construction if the ICR matrices are in standard form but geometrical considerations in the linear subgroup rarely give this form. This leaves descent in symmetry from grey SU(2). This method was chosen because of our prior experience with it, the available tables of 3j coefficients for SU(2) [3] and the comparative ease of writing computer programmes. The 6j method of Butler and Wybourne [4] is only with hindsight available for these grey groups due to their special nature.

The first part of the use of the descent in symmetry technique is to establish the basis vectors of the ICRs of the grey double point groups in terms of the vectors $|JM>$ of grey SU(2). Whereas this may be performed by the use of projection operators in linear groups, such a technique is not applicable to the grey groups. For a type (b) ICR it is easily shown by example with grey C_3^* that there are just not enough orthogonal operators to project out basis vectors for such an ICR. Clearly such a lack of operators is connected to the commuting matrices of an ICR of type (b). (This deficiency in the projection operators causes an essential error in the proof by Rudra [5] that basis vectors for this type of ICR are not orthogonal). For these low dimensional groups it is an easy matter to take basis vectors for the linear subgroup [6] and juggle them around to produce basis vectors for the grey group.

This leads to the basis vectors given in the tables.

Racah algebra methods have always been plagued with problems of phase conventions and uniqueness of choice. Here is an appropriate place to start such a discussion for the grey groups. For an ICR of type (a) the anti-linearity of θ fixes the phase of a set of basis vectors to within sign. For a type (b) ICR the specific matrix realizations given in chapter four divide the basis vectors into two subspaces of equal dimension left invariant under the linear operators, with time reversal mixing these subspaces. The matrices commuting with such an ICR show that a basis set may then be transformed by

$$\begin{pmatrix} z_1 I & z_2 I \\ -z_2^* I & z_1^* I \end{pmatrix}$$

and still form a basis for the ICR. Thus there is a large degree of arbitrariness in the choice of the vectors. This choice may even be exploited to the extent that some 3jm symbols do not even appear in the descent in symmetry method as we found by accident in going from grey O^* to grey T^* and from grey D_n^* to grey C_n^*. In calculations based on the descent in symmetry such a result is highly desirable as it means that many of the isoscalars and reduced matrix elements may be automatically set to zero, but for calculations not involving this descent the full set of 3jm symbols may be necessary. Thus the basis vectors have been chosen to give a full set of 3jm symbols.

Similar reasoning applies to an ICR of type (c), where the division into two sets of basis vectors under the linear operators is indeterminate by the matrix transformation

$$\begin{pmatrix} zI & 0 \\ 0 & z^*I \end{pmatrix}$$

Again a choice is made to give all 3jm symbols.

The 3jm symbols and isoscalars now follow from equations (12.5) and (13.3). As SU(2) is multiplicity free the tensor $E^{r}{}_{r'}$ of equation (13.4) is either plus or minus one and as continuity requires the positive sign, the equations may be combined as

$$\begin{pmatrix} J_1 & J_2 & J_3 \\ k_1 & k_2 & k_3 \end{pmatrix}^{r_2}_{1} \qquad \bar{V} \begin{pmatrix} k_1 & k_2 & k_3 \\ m_1' & m_2' & m_3' \end{pmatrix}_{r_2}$$

$$= P_1{}^{m_1'}{}_{M_1} P_2{}^{m_2'}{}_{M_2} P_3{}^{m_3'}{}_{M_3} \quad \bar{V} \begin{pmatrix} J_1 & J_2 & J_3 \\ M_1 & M_2 & M_3 \end{pmatrix} \qquad (5)$$

where the $\bar{V}$ coefficient on the right is a $\bar{V}$ coefficient of SU(2) and the matrices P_i are given by the basis vectors of the grey double point group. Contrary to the recent statements by Pooler [7] this is an excellent method for computing the left-hand side either by hand or by computer. Both methods were used to calculate the left-hand side of this equation for many of the groups considered here.

At this point multiplicity and phase problems raise their head. We may assume that an orthogonal transformation has been performed in the 3km multiplicity space so that the isoscalar is non-zero for only one r_2. Where the group is not multiplicity free some searching is required in the J values in order that the total 3jm tensor be orthogonal between multiplicity labels.

In view of the comments concerning basis vectors, we can only regard the success of this search as accidental to these groups.

Normalization of any column of the 3jm symbol fixes the 3jm symbols and hence the isoscalar to within phase. As the isoscalar now is effectively a scalar and is real, the 3jm symbols are determined to within sign. However, as this is precisely the phase freedom allowed by equation (6.2), it means the sign is arbitrary.

In addition to the basic definitions, one of the basic motivations for constructing 3jm symbols is permutational symmetry. This restricts the phase freedom according to equations (7.11) and (12.13). When two or three of the ICRs are equivalent we have again what can only be regarded as a fortuitous coincidence: for the lowest values of J which give a particular column of a 3jm tensor of the subgroup, the symmetry properties of the 3jm symbol in SU(2) are the same as those of the 3jm symbol in the subgroup. This does not hold in general for higher J values as the case $J_1 = 5$, $J_2 = 4$, $J_3 = 2$ and $k_1 = k_2 = k_3 = E$ in the octahedral group shows. Also for the higher J values when the 3jm symbol is not MF, the descent in symmetry does not give a symmetrized column of the 3jm symbol. Thus the 3jm symbols calculated from the lower J values should be used to calculate these higher isoscalars which are not now non-zero for only one multiplicity label. This has been carried out for the octahedral and icosahedral groups.

The requirement of permutational symmetry when two or three of ICRs are equal leaves only an arbitrary sign among the whole set of permuted isoscalars and 3jm tensors as the mixed symmetry term [21] does not occur for these groups. This was generally fixed as positive for the isoscalar in the defining cases. When none of the ICRs of the grey group are equivalent the permutation phase of the 3jm tensor is arbitrary. In conformity with earlier work [6] this was taken to be the symmetry of the corresponding $\bar{V}$ coefficient of grey SU(2) so that the defining isoscalars are invariant under permutations. This choice is questionable especially as higher J isoscalars are not invariant but as we do not at present have access to a computer we have kept to this choice here.

This completes the discussion of the 3jm tensor and the 3jm isoscalar. We now turn to the coupling coefficient. These have not been calculated as they may all be found for these groups from the 3jm symbols tabulated. To see this, consider first the case where the multiplicity of the coupling coefficient is only one, no matter what the multiplicity of the 3jm symbol. Then in equation (12.10) the m dependant part of the isoscalar may be absorbed into the coupling coefficient as in equation (8.10). The coupling coefficients and $\bar{V}$ symbols of SU(2) are simply related as this group is multiplicity free and so equations (12.5) and (12.10) may be readily equated. Choosing the isoscalars as above, equation (8.16) follows

$$< k_1k_2m_1m_2|k_1k_2k_3\dot{m}_3>_{(r)} = \pm [k_3]^{\frac{1}{2}} \begin{bmatrix} \dot{m}_3 & \\ & m_4 \end{bmatrix}^{\bar{V}} \begin{pmatrix} k_1 & k_2 & k_3 \\ m_1 & m_2 & m_3 \end{pmatrix}_r \qquad (6)$$

When the coupling coefficient is not multiplicity free, its multiplicity by inspection of the tables is equal to the multiplicity of the 3jm tensor. Hence the above equation holds again. As all $\bar{V}$ coefficients of SU(2) and the grey double point groups are simple phase, the coupling coefficients are also simple phase even though the multiplicities of permuted orders may be different. The sign relating two permuted symbols is partially fixed by permutations.

1. Properties of the Coefficients and Examples

The permutational equation (7.11) becomes in this notation

$$\bar{V}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}_{r_1} = M(12,3)^{r_2}{}_{r_1}\ \bar{V}\begin{pmatrix} j_2 & j_1 & j_3 \\ m_2 & m_1 & m_3 \end{pmatrix}_{r_2}$$

etcetera. All the groups here are simple phase and all M are taken in diagonal form. Every cyclic permutation leaves the sign unchanged. The parity of all transpositions is taken to be the same within one multiplicity label and is given in the tables. For example, from table 2.4,

$$\bar{V}\begin{pmatrix} T_2 & E & T_1 \\ 0 & \varepsilon & 0 \end{pmatrix} = -\frac{1}{\sqrt{3}}$$

and since the parity is odd,

$$\bar{V}\begin{pmatrix} E & T_2 & T_1 \\ \varepsilon & 0 & 0 \end{pmatrix} = \bar{V}\begin{pmatrix} T_2 & T_1 & E \\ 0 & 0 & \varepsilon \end{pmatrix} = \bar{V}\begin{pmatrix} T_1 & E & T_2 \\ 0 & \varepsilon & 0 \end{pmatrix} = \frac{1}{\sqrt{3}}$$

whereas

$$\bar{V}\begin{pmatrix} T_1 & T_2 & E \\ 0 & 0 & \varepsilon \end{pmatrix} = V\begin{pmatrix} E & T_1 & T_2 \\ \varepsilon & 0 & 0 \end{pmatrix} = -\frac{1}{\sqrt{3}}$$

The conjugation equation (6.20) becomes

$$\bar{V}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} = \begin{pmatrix} m_1 & \\ & n_1 \end{pmatrix}\begin{pmatrix} m_2 & \\ & n_2 \end{pmatrix}\begin{pmatrix} m_3 & \\ & n_3 \end{pmatrix}\bar{V}\begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix}^{*}$$

Thus for example from tables 3.2 and 3.3, since

$$\bar{V}\begin{pmatrix} V & V & T_1 \\ 1 & -1 & 0 \end{pmatrix} = \frac{i}{\sqrt{30}}$$

$$\frac{i}{\sqrt{30}} = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}\begin{pmatrix} -1 & \\ & 1 \end{pmatrix}\begin{pmatrix} 0 & \\ & 0 \end{pmatrix} \bar{V}\begin{pmatrix} V & V & T_1 \\ -1 & 1 & 0 \end{pmatrix}^*$$

$$= 1 \times 1 \times 1 \times \bar{V}\begin{pmatrix} V & V & T_1 \\ -1 & 1 & 0 \end{pmatrix}^*$$

Hence

$$\bar{V}\begin{pmatrix} V & V & T_1 \\ -1 & 1 & 0 \end{pmatrix} = \frac{-i}{\sqrt{30}}$$

which agrees with the permutational result.

TWO : Octahedral Group

2.1 Introduction

$\bar{V}$ coefficients are given for this group in two configurations : the standard one with the z-axis a four-fold axis and the [111] axis as three-fold, and in tetrahedral configuration with the z-axis now a three-fold axis, the four-fold axis lying in the y-z plane making an angle of $\sin^{-1}(\sqrt{2}/\sqrt{3})$ with the positive z-axis. Basis vectors in standard configuration are the same as in Griffith [1] and Golding [8] from which the Wigner tensors may be readily calculated by

$$\theta\,|JM\rangle = (-1)^{J+M}\,|J-M\rangle$$

This is given in table 2.1

In tetrahedral configuration the basis vectors are given to J = 4 in table 2.2 and the Wigner tensor in table 2.3.

The $\bar{V}$ coefficients are given in tables 2.4 and 2.5. A serial order was adapted for the ICRs of

$$A_1 < T_1 < E < T_2 < A_2 < E' < U' < E''$$

The tables are ordered with $j_1 \geqslant j_2 \geqslant j_3$ and $j_3 = [A_1 \ldots\ldots j_2]$, $j_2 = [A_1 \ldots\ldots j_1]$ and $j_1 = [A_1 \ldots\ldots E'']$. Table 2.2 was adapted from Golding [8] and both 2.2 and 2.3 were later checked by computer.

In addition to these, isoscalar factors for grey $SU(2) \supset$ grey O^* were also calculated using a computer programme to generate the $\bar{V}$ coefficients of SU(2). A serial order was adapted as above for the multiplicity free cases for each triple of ICRs of grey O^* and a similar order was used for the J-values of SU(2). For the non-multiplicity free cases a single isoscalar is given from normalization of the corresponding $\bar{V}$ coefficient and then the resolution of this symbol into the standard $\bar{V}$ coefficients of table 2.4. The isoscalars form table 2.6.

$\bar{W}$ coefficients are given in table 2.7. They were calculated by means of equation (9.17)

Table 2.1 : The Wigner Tensor $\begin{pmatrix} m & \\ & \dot{n} \end{pmatrix}$ for Grey O^* in

Standard configuration

j	m	$\dot{n}$	value
A_1	a_1	a_1	1
T_1	1	-1	1
	0	0	-1
	-1	1	1
E	θ	θ	1
	ε	ε	1
T_2	1	-1	1
	0	0	-1
	-1	1	1
A_2	a_2	a_2	1
E'	α	β	1
	β	α	-1
U'	κ	ν	1
	λ	μ	-1
	μ	λ	1
	ν	κ	-1
E"	α	β	1
	β	α	-1

Table 2.2: Basis Vectors for Grey O^* in Tetrahedral Configuration

The table has been separated into integer and spinor ICRs. Along the top is given the possible M-values. Down the side are the J-values and the ICR basis labelling. The value of a coefficient of $|JM\rangle$ is given by an entry in the table. Blank spaces correpond to zero. For example, from line five

$$|2Ex\rangle = \frac{1}{\sqrt{3}}\,|22\rangle + \frac{i\sqrt{2}}{\sqrt{3}}\,|2{-1}\rangle$$

J	km	4	3	2	1	0	-1	-2	-3	-4
0	$A_1 a_1$					1				
	$T_1 1$				1					
1	$T_1 0$					1				
	$T_1 {-1}$						1			
	Ex			$1/\sqrt{3}$			$i\sqrt{2}/\sqrt{3}$			
	Ey				$i\sqrt{2}/\sqrt{3}$			$1/\sqrt{3}$		
2	$T_2 1$				$-i/\sqrt{3}$			$\sqrt{2}/\sqrt{3}$		
	$T_2 0$					i				
	$T_2 {-1}$			$\sqrt{2}/\sqrt{3}$			$-i/\sqrt{3}$			
	$A_2 a_2$		$-\sqrt{2}/\sqrt{3}$			$i\sqrt{5}/3$			$-\sqrt{2}/\sqrt{3}$	
	$T_1 1$				$-1/\sqrt{6}$			$i\sqrt{5}/\sqrt{6}$		
	$T_1 0$		$-i\sqrt{5}/3\sqrt{2}$			$2/3$			$-i\sqrt{5}/3\sqrt{2}$	
3	$T_1 {-1}$			$1/\sqrt{6}$			$-i\sqrt{5}/\sqrt{6}$			
	$T_2 1$				$-i\sqrt{5}/\sqrt{6}$			$1/\sqrt{6}$		
	$T_2 0$		$-1/\sqrt{2}$						$1/\sqrt{2}$	
	$T_2 {-1}$			$-1/\sqrt{6}$			$i\sqrt{5}/\sqrt{6}$			

J	Km	4	3	2	1	0	-1	-2	-3	-4
4	A_1a_1		$i\sqrt{10}/3\sqrt{3}$			$-\sqrt{7}/3\sqrt{3}$			$i\sqrt{10}/3\sqrt{3}$	
	Ex	$\sqrt{7}/3\sqrt{3}$			$2i/3\sqrt{3}$			$-4/3\sqrt{3}$		
	Ey			$-4/3\sqrt{3}$			$2i/3\sqrt{3}$			$\sqrt{7}/3\sqrt{3}$
	$T_1 1$	$-i\sqrt{2}/3$			$\sqrt{7}/3\sqrt{2}$			$-i\sqrt{7}/3\sqrt{2}$		
	$T_1 0$		$i/\sqrt{2}$						$-i/\sqrt{2}$	
	$T_1 -1$			$i\sqrt{7}/3\sqrt{2}$			$-\sqrt{7}/3\sqrt{2}$			$i\sqrt{2}/\sqrt{3}$
	$T_2 1$	$-\sqrt{14}/3\sqrt{3}$			$5i/3\sqrt{6}$			$-1/3\sqrt{6}$		
	$T_2 0$		$-\sqrt{7}/3\sqrt{6}$			$i2\sqrt{5}/3\sqrt{3}$			$-\sqrt{7}/3\sqrt{6}$	
	$T_2 -1$			$-1/3\sqrt{6}$			$5i/3\sqrt{6}$			

J	Km	7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2
1/2	E'α				1				
	E'β					1			
3/2	U'κ			$1/\sqrt{3}$			$-i\sqrt{2}/\sqrt{3}$		
	U'λ				1				
	U'μ					1			
	U'ν			$-i\sqrt{2}/\sqrt{3}$			$1/\sqrt{3}$		
5/2	E''α				$i\sqrt{5}/\sqrt{3}$			$-2/3$	
	E''β		$2/3$			$-i\sqrt{5}/3$			
	U'κ			$i2\sqrt{2}/3$			$1/3$		
	U'λ				$-2i/3$			$-\sqrt{5}/3$	
	U'μ		$-\sqrt{5}/3$			$-2i/3$			
	U'ν			$1/3$			$i2\sqrt{2}/3$		

J	Km	7/2	5/2	3/2	1/2	-1/2	-3/2	-5/2	-7/2
7/2	E'α	$-i\sqrt{5}/3\sqrt{6}$			$\sqrt{7}/3\sqrt{3}$			$-i\sqrt{35}/3\sqrt{6}$	
	E'β		$i\sqrt{35}/3\sqrt{6}$			$-\sqrt{7}/3\sqrt{3}$			$i\sqrt{5}/3\sqrt{6}$
	E"α	$-\sqrt{7}/3\sqrt{2}$			$i\sqrt{5}/3$			$-i/3\sqrt{2}$	
	E"β		$-1/3\sqrt{2}$			$i\sqrt{5}/3$			$-\sqrt{7}/3\sqrt{2}$
	U'κ			$-1/3$			$-i2\sqrt{2}/3$		
	U'λ	$i\sqrt{14}/3\sqrt{3}$			$-\sqrt{5}/3\sqrt{3}$			$-i2\sqrt{2}/3\sqrt{3}$	
	U'μ		$-i2\sqrt{2}/3\sqrt{3}$			$-\sqrt{5}/3\sqrt{3}$			$i\sqrt{14}/3\sqrt{3}$
	U'ν			$-i2\sqrt{2}/3$			$1/3$		

Table 2.3: The Wigner Tensor $\begin{pmatrix} m & \\ & \dot{m} \end{pmatrix}$ for grey O^* in Tetrahedral Configuration.

j	m	$\dot{n}$	value
A_1	a_1	a_1	1
T_1	1	-1	1
	0	0	-1
	-1	1	1
E	x	y	1
	y	x	1
T_2	1	-1	1
	0	0	-1
	-1	1	1
A_2	a_2	a_2	1
E'	α	β	1
	β	α	-1
U'	κ	ν	1
	λ	μ	-1
	μ	λ	1
	ν	κ	-1
E"	α	β	1
	β	α	-1

Table 2.4: The coefficients $\bar{V}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ for grey O^* in Standard Configuration

The ICRs are ordered with $j_1 \geqslant j_2 \geqslant j_3$ and $j_3 = [A_1 \ldots\ldots j_2]$, $j_2 = [A_1 \ldots\ldots j_1]$ and $j_3 = [A_1 \ldots E'']$. The J values for which these were calculated are given on the left from which the parity of each transposition is found. An asterix denotes those parities which are essential. The others are arbitrary.

J_1 J_2 J_3	j_1 j_2 j_3	Multiplicity label	Transposition parity	m_1 m_2 m_3	$\bar{V}$
0 0 0	A_1 A_1 A_1	1	even*	a_1 a_1 a_1	1
1 1 0	T_1 T_1 T_1	1	even*	1 -1 a_1	$1/\sqrt{3}$
				0 0 a_1	$-1/\sqrt{3}$
1 1 1	T_1 T_1 T_1	1	odd*	1 0 -1	$1/\sqrt{6}$
2 1 1	E T_1 T_1	1	even*	θ 0 0	$1/\sqrt{3}$
				θ -1 1	$1/2\sqrt{3}$
				ε 1 1	$1/2\sqrt{3}$
2 2 0	E E A_1	1	even*	θ θ a_1	$1/\sqrt{2}$
				ε ε a_1	$1/\sqrt{2}$
2 2 2	E E E	1	even*	θ θ θ	-1/2
				θ ε ε	1/2
2 1 1	T_2 T_1 T_1	1	even*	1 0 1	$-1/\sqrt{6}$
				0 1 1	$-1/\sqrt{6}$

$J_1\ J_2\ J_3$	$j_1\ j_2\ j_3$	Multiplicity label	Transposition parity	$m_1\ m_2\ m_3$	$\bar{V}$
2 2 1	T_2 E T_1	1	odd	0 ε 0	$-1/\sqrt{3}$
				1 ε -1	$-1/2\sqrt{3}$
				1 θ 1	1/2
2 2 0	T_2 T_2 A_1	1	even*	1 -1 a_1	$1/\sqrt{3}$
				0 0 a_1	$-1/\sqrt{3}$
2 2 1	T_2 T_2 T_1	1	odd*	1 0 -1	$-1/\sqrt{6}$
				1 -1 0	$1/\sqrt{6}$
2 2 2	T_2 T_2 E	1	even*	0 0 θ	$-1/\sqrt{3}$
				-1 1 θ	$-1/2\sqrt{3}$
				1 1 ε	-1/2
2 2 2	T_2 T_2 T_2	1	even*	1 0 1	$-1/\sqrt{6}$
3 2 2	A_2 E E	1	odd*	a_2 ε θ	$1/\sqrt{2}$
3 2 1	A_2 T_2 T_1	1	even	a_2 -1 1	$-1/\sqrt{3}$
				a 0 0	$1/\sqrt{3}$
3 3 0	A_2 A_2 A_1	1	even*	a_2 a_2 a_1	1
$\frac{1}{2}$ $\frac{1}{2}$ 0	E' E' A_1	1	odd*	α β a_1	$-1/\sqrt{2}$
$\frac{1}{2}$ $\frac{1}{2}$ 1	E' E' T_1	1	even*	α α -1	$-1/\sqrt{3}$
				α β 0	$1/\sqrt{6}$
$\frac{3}{2}$ $\frac{1}{2}$ 1	U' E' T	1	odd	κ β -1	-1/2
				λ α -1	$1/2\sqrt{3}$
				κ β 0	$1/\sqrt{6}$
$\frac{3}{2}$ $\frac{1}{2}$ 2	U' E' E	1	even	λ β θ	-1/2
				κ α ε	-1/2

$J_1\ J_2\ J_3$	$j_1\ j_2\ j_3$	Multiplicity label	Transposition parity	$m_1\ m_2\ m_3$	$\bar{V}$
$\frac{3}{2}\ \frac{1}{2}\ 2$	$U'\ E'\ T_2$	1	even	$\kappa\ \alpha\ 0$	$1/\sqrt{6}$
				$\kappa\ \beta\ 1$	$1/2\sqrt{3}$
				$\lambda\ \alpha\ 1$	$1/2$
$\frac{3}{2}\ \frac{3}{2}\ 0$	$U'\ U'\ A_1$	1	odd*	$\kappa\ \nu\ a_1$	$-1/2$
				$\lambda\ \mu\ a_1$	$1/2$
$\frac{3}{2}\ \frac{3}{2}\ 1$	$U'\ U'\ T_1$	1	even*	$\kappa\ \mu\ -1$	$-1/\sqrt{10}$
				$\kappa\ \nu\ 0$	$\sqrt{3}/2\sqrt{5}$
				$\lambda\ \lambda\ -1$	$\sqrt{2}/\sqrt{15}$
				$\lambda\ \mu\ 0$	$-1/2\sqrt{15}$
$\frac{3}{2}\ \frac{3}{2}\ 3$	$U'\ U'\ T_1$	2	even*	$\kappa\ \kappa\ 1$	$\sqrt{5}/2\sqrt{6}$
				$\kappa\ \mu\ -1$	$1/2\sqrt{10}$
				$\kappa\ \nu\ 0$	$1/2\sqrt{15}$
				$\lambda\ \lambda\ -1$	$\sqrt{3}/2\sqrt{10}$
				$\lambda\ \mu\ 0$	$\sqrt{3}/2\sqrt{5}$
$\frac{3}{2}\ \frac{3}{2}\ 2$	$U'\ U'\ E$	1	odd*	$\kappa\ \nu\ \theta$	$-1/2\sqrt{2}$
				$\kappa\ \lambda\ \varepsilon$	$-1/2\sqrt{2}$
				$\lambda\ \mu\ \theta$	$-1/2\sqrt{2}$
$\frac{3}{2}\ \frac{3}{2}\ 2$	$U'\ U'\ T_2$	1	odd*	$\kappa\ \lambda\ 0$	$1/2\sqrt{3}$
				$\kappa\ \mu\ 1$	$1/\sqrt{6}$
$\frac{3}{2}\ \frac{3}{2}\ 3$	$U'\ U'\ T_2$	2	even*	$\kappa\ \kappa\ -1$	$1/2\sqrt{2}$
				$\kappa\ \lambda\ 0$	$1/2\sqrt{3}$
				$\kappa\ \mu\ 1$	$-1/2\sqrt{6}$
				$\lambda\ \lambda\ 1$	$-1/2\sqrt{2}$

J_1 J_2 J_3	j_1 j_2 j_3	Multiplicity label	Transposition parity	m_1 m_2 m_3	$\bar{V}$
$\frac{3}{2}$ $\frac{3}{2}$ 3	U' U' A_2	1	even*	κ λ a_2	-1/2
$\frac{5}{2}$ $\frac{3}{2}$ 2	E" E' T_2	1	even	α α -1 β α 0	-1/√3 1/√6
$\frac{5}{2}$ $\frac{3}{2}$ 3	E" E' A_2	1	odd	β α a_2	1/√2
$\frac{5}{2}$ $\frac{3}{2}$ 1	E" U' T_1	1	odd	α κ 0 β κ 1 α λ 1	1/√6 1/2√3 1/2
$\frac{5}{2}$ $\frac{3}{2}$ 2	E" U' E	1	even	α κ θ β λ ε	1/2 -1/2
$\frac{5}{2}$ $\frac{3}{2}$ 2	E" U' T_2	1	even	β κ -1 α λ -1 β λ 0	-1/2 1/2√3 1/√6
$\frac{5}{2}$ $\frac{5}{2}$ 0	E" E" A_1	1	odd*	α β a_1	-1/√2
$\frac{5}{2}$ $\frac{5}{2}$ 1	E" E" T_1	1	even*	α α -1 α β 0	1/√3 -1/√6

Table 2.5: The Coefficients $\bar{V}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ for Grey O^* in Tetrahedral Configuration

The coefficients are arranged in exactly the same order as in the last table, and have the same transposition symmetry. The two non-MF cases are distinguished here by the J value on T_1 and T_2

j_1	j_2	j_3	m_1	m_2	m_3	$\bar{V}$
A_1	A_1	A_1	a_1	a_1	a_1	1
T_1	T_1	A_1	0	0	a_1	$-1/\sqrt{3}$
			-1	1	a_1	$1/\sqrt{3}$
T_1	T_1	T_1	-1	0	1	$-1/\sqrt{6}$
E	T_1	T_1	x	0	1	$i/\sqrt{6}$
			x	-1	-1	$1/\sqrt{6}$
E	E	A_1	y	x	a_1	1
E	E	E	x	x	x	$1/\sqrt{2}$
T_2	T_1	T_1	1	1	1	$\sqrt{2}/3$
			1	-1	0	$-i/3\sqrt{2}$
			0	0	0	$-i\sqrt{2}/3$
			0	-1	1	$-i/3\sqrt{2}$
T_2	E	T_1	1	x	0	$1/2\sqrt{3}$
			1	y	1	$i/\sqrt{2}\sqrt{3}$
			0	x	1	$1/\sqrt{6}$
T_2	T_2	A_1	0	0	a_1	$-1/\sqrt{3}$
			-1	1	a_1	$1/\sqrt{3}$
T_2	T_2	T_1	0	1	-1	$1/\sqrt{6}$
			-1	1	0	$-1/\sqrt{6}$
T_2	T_2	E	0	1	x	$-i/\sqrt{6}$
			-1	-1	x	$-1/\sqrt{6}$
T_2	T_2	T_2	-1	0	1	$-i/3\sqrt{2}$
			0	0	0	$-i\sqrt{2}/3$
			-1	-1	-1	$\sqrt{2}/3$
A_2	E	E	a_2	y	x	$-i/\sqrt{2}$
A_2	T_2	T_1	a_2	1	-1	$1/\sqrt{3}$
			a_2	0	0	$-1/\sqrt{3}$

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$
A_2	A_2	A_1	a_2	a_2	a_1	1
E'	E'	A_1	β	α	a_1	$1/\sqrt{2}$
E'	E'	T_1	α	α	-1	$-1/\sqrt{3}$
			β	α	0	$1/\sqrt{6}$
U'	E'	T_1	κ	α	1	$i/\sqrt{6}$
			κ	β	-1	$-1/2\sqrt{3}$
			λ	α	-1	$1/2\sqrt{3}$
			λ	β	0	$1/\sqrt{6}$
U'	E'	E	λ	α	x	$-i/2$
U'	E'	T_2	κ	α	1	$\sqrt{5}/3\sqrt{2}$
			κ	β	-1	$-i/2\sqrt{3}$
			λ	α	-1	$i/2\sqrt{3}$
			μ	α	0	$i/\sqrt{6}$
U'	U'	A_1	ν	κ	a_1	1/2
			μ	λ	a_1	1/2
U'	U'	T_1	κ	κ	0	$i\sqrt{2}/\sqrt{3}\sqrt{5}$
			λ	κ	1	$-i/\sqrt{3}\sqrt{5}$
			λ	λ	-1	$\sqrt{2}/\sqrt{3}\sqrt{5}$
			μ	κ	-1	$-1/\sqrt{2}\sqrt{3}\sqrt{5}$
			μ	λ	0	$-1/2\sqrt{3}\sqrt{5}$
			ν	κ	0	$-1/2\sqrt{3}\sqrt{5}$
E"	E'	T_2	α	α	-1	$-1/\sqrt{3}$
			α	β	0	$1/\sqrt{6}$
E"	E'	A_2	α	β	a_2	$-1/\sqrt{2}$
E"	U'	T_1	α	κ	1	$-1/\sqrt{6}$
			α	λ	-1	$i/2\sqrt{3}$
			α	μ	0	$i/\sqrt{6}$
			α	ν	1	$-i/2\sqrt{3}$

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$
U'	U'	E	λ	κ	y	1/2
U'	U'	T_2	λ	κ	-1	$-i/\sqrt{6}$
			μ	λ	0	$-i/2\sqrt{3}$
			ν	κ	0	$-i/2\sqrt{3}$
U'	U'	$3T_2$	κ	κ	0	$-1/\sqrt{6}$
			λ	κ	1	$1/2\sqrt{3}$
			λ	λ	-1	$i/\sqrt{6}$
U'	U'	A_2	μ	λ	a_2	$-i/2$
			ν	κ	a_2	$i/2$
E"	U'	E	α	λ	x	1/2
E"	U'	T_2	α	κ	1	$i/\sqrt{6}$
			α	λ	-1	$1/2\sqrt{3}$
			α	μ	0	$-1/\sqrt{6}$
			α	ν	1	$1/2\sqrt{3}$
E"	E"	A_1	β	α	a_1	$1/\sqrt{2}$
E"	E"	T_1	β	α	0	$-1/\sqrt{6}$
			α	x	-1	$-1/\sqrt{3}$
U'	U'	$3T_1$	κ	κ	0	$i/\sqrt{2}\sqrt{3}\sqrt{5}$
			λ	κ	1	$-i/2\sqrt{3}\sqrt{5}$
			λ	λ	-1	$1/\sqrt{2}\sqrt{3}\sqrt{5}$
			μ	κ	-1	$\sqrt{2}/\sqrt{3}\sqrt{5}$
			μ	λ	0	$1/\sqrt{3}\sqrt{5}$
			ν	κ	0	$1/\sqrt{3}\sqrt{5}$

Table 2.6: The Isoscalars $\begin{pmatrix} j_1 & j_2 & j_3 \\ k_1 & k_2 & k_3 \end{pmatrix}$ for Grey SU(2) ⊃ Grey O^*

Each table gives the isoscalars for a triad (k_1 k_2 k_3) ordered as in the previous two tables. Internally each is then ordered by the triad (J_1 J_2 J_3) with J_3 = [0......J_2], J_2 = [0,....J_1] and J_1 = [0.....6]. In cases where the branching SU(2) ⟶ O^* is not multiplicity free, the additional labelling a, b,... is required. The serial order b > a is adopted. For example, for (k_1 k_2 k_3) = (U'U'T_1) and (J_1 J_2 J_3) = (9/2, 9/2, 5) the six entries are in order

$$\begin{pmatrix} 9/2 & 9/2 & 5 \\ aU' & aU' & aT_1 \end{pmatrix}, \quad \begin{pmatrix} 9/2 & 9/2 & 5 \\ aU' & aU' & bT_1 \end{pmatrix},$$

$$\begin{pmatrix} 9/2 & 9/2 & 5 \\ bU' & aU' & aT_1 \end{pmatrix}, \quad \begin{pmatrix} 9/2 & 9/2 & 5 \\ bU' & aU' & bT_1 \end{pmatrix},$$

$$\begin{pmatrix} 9/2 & 9/2 & 5 \\ bU' & bU' & aT_1 \end{pmatrix}, \quad \begin{pmatrix} 9/2 & 9/2 & 5 \\ bU' & bU' & bT_1 \end{pmatrix}$$

For the multiplicity free cases each line consists of the triad (J_1 J_2 J_3) which is followed by the sign and the powers of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$, $\sqrt{13}$, $\sqrt{17}$, $\sqrt{19}$. This in some cases is followed by a further number which multiplies the whole coefficient. Thus in the first table for (A_1 A_1 A_1) the third line is

4 4 4 + 1 -3 0 2 -1 -1 0 0

gives the isoscalar

$$\begin{pmatrix} 4 & 4 & 4 \\ A_1 & A_1 & A_1 \end{pmatrix} = \frac{7\sqrt{2}}{3\sqrt{3}\sqrt{11}\sqrt{13}}$$

The non-MF triads ($U'U'T_1$) and ($U'U'T_2$) are given separately at the end. Here additional information is required as there are two isoscalars. Following (J_1 J_2 J_3) is the value of the isoscalar derived by normalization of the $\bar{V}$ coefficient from this coupling. Following the sign are the power of $\sqrt{2}$ to $\sqrt{19}$ and then in many cases the square of another number. Two further entries on the right resolve this isoscalar to give the standard $\bar{V}$ coefficients of table 2.4. Each of these entries gives the sign, the powers of $\sqrt{2}$ to $\sqrt{19}$ and then the ratio $a/\sqrt{b}$. For example the complete entry for 7/2U', 5/2U', $5bT_1$ is

```
7/2   7/2   5  +  -2  -1  -1  0  -1  0  0  0  113 -  -1  0  -1 00000 31/113
                                                 +  -1  0  -1 00200 1/113
```

giving

$$\begin{pmatrix} 7/2 & 7/2 & 5 \\ U' & U' & bT_1 \end{pmatrix}_1 = \frac{\sqrt{113}}{2\sqrt{3}\sqrt{5}\sqrt{11}} \times \frac{-\ 31}{\sqrt{2}\sqrt{5}\sqrt{113}}$$

and

$$\begin{pmatrix} 7/2 & 7/2 & 5 \\ U' & U' & bT_1 \end{pmatrix}_2 = \frac{\sqrt{113}}{2\sqrt{3}\sqrt{5}\sqrt{11}} \times \frac{13}{\sqrt{2}\sqrt{5}\sqrt{113}}$$

		A1		A1			A1				
0	0	0	+	0	0	0	0	0	0	0	0
4	4	0	+	0	-2	0	0	0	0	0	0
4	4	4	+	1	-3	0	2	-1	-1	0	0
6	4	4	-	3	-2	1	0	-1	-1	0	0
6	6	0	+	0	0	0	0	0	-1	0	0
6	6	4	-	-2	1	0	2	-1	-1	-1	0
6	6	6	+	3	0	0	0	-1	-1	-1	-1

		T1		T1			A1				
1	1	0	+	0	0	0	0	0	0	0	0
3	1	4	−	0	−2	0	0	0	0	0	0
3	3	0	+	0	1	0	−1	0	0	0	0
3	3	4	+	−1	0	0	0	−1	0	0	0
3	3	6	−	−1	0	2	−1	−1	−1	0	0
4	1	4	−	0	−2	0	0	0	0	0	0
4	3	4	−	−1	−2	0	1	−1	0	0	0
4	3	6	+	−1	1	1	0	−1	−1	0	0
4	4	0	+	0	−1	0	0	0	0	0	0
4	4	4	+	−1	−2	0	2	−1	−1	0	0
4	4	6	+	−1	−1	−1	0	−1	−1	0	0
5	1	4	+	−2	−2	1	1	−1	0	0	0
5	1	6	−	−2	2	0	0	−1	−1	0	0
5	3	4	−	0	0	1	0	−1	−1	0	0
5	3	6	+	−3	0	0	1	−1	−1	0	0
5	4	4	+	0	−2	1	0	−1	−1	0	0
5	4	6	−	−4	1	0	2	−1	−1	0	0
5	5	0	+	0	1	0	0	−1	0	0	0
5	5	4	+	−1	0	0	1	−1	−1	0	0
5	5	6	−	1	0	1	0	−1	−1	−1	0
5	1	4	+	−2	0	0	0	−1	0	0	0
5	1	6	+	−2	0	1	1	−1	−1	0	0
5	3	4	+	0	0	0	1	−1	−1	0	0
5	3	6	+	−3	2	1	0	−1	−1	0	0
5	4	4	+	0	0	0	1	−1	−1	0	0
5	4	6	+	−4	1	−1	1	−1	−1	0	0
5	5	0		0							

5	5	4	+	-1	0	1	0	-1	-1	0	0
5	5	6	-	-1	2	0	1	-1	-1	-1	0
5	5	0	+	0	1	0	0	-1	0	0	0
5	5	4	-	-1	0	0	1	-1	-1	0	0
5	5	6	-	3	2	-1	0	-1	-1	-1	0
6	1	6	+	0	0	0	0	0	-1	0	0
6	3	4	-	0	0	2	-1	-1	-1	0	0
6	3	6	-	-3	3	0	0	-1	-1	0	0
6	4	4	+	0	0	0	1	-1	-1	0	0
6	4	6	+	-3	1	1	1	-1	-1	-1	0
6	5	4	-	-3	0	1	1	-1	-1	0	0
6	5	6	-	-3	5	0	0	-1	-1	-1	0
6	5	4	+	-3	0	0	0	-1	-1	0	0
6	5	6	-	-3	1	1	1	-1	-1	-1	0
6	6	0	+	0	1	0	0	0	-1	0	0
6	6	4	-	6	0	0	0	-1	-1	-1	0
6	6	6	+	1	1	0	0	-1	-1	-1	-1

		T1		T1			T1					
1	1	1	+	0	0	0	0	0	0	0	0	
3	3	1	−	−3	2	0	−1	0	0	0	0	
3	3	3	−	0	0	0	−1	0	0	0	0	
4	3	1	+	−3	−1	1	0	0	0	0	0	
4	3	3		0								
4	4	1	+	−3	−1	−1	0	0	0	0	0	
4	4	3	−	0	−1	−1	1	−1	0	0	0	
4	4	4		0								
5	3	3	+	−5	0	2	−1	−1	0	0	0	
5	4	1	−	−1	−1	0	1	−1	0	0	0	
5	4	3	+	−5	3	0	0	−1	−1	0	0	
5	4	4	+	−5	−1	0	0	−1	−1	0	0	37
5	5	1	+	−6	2	1	0	−1	0	0	0	
5	5	3	+	−2	0	1	1	−1	−1	0	0	
5	5	4		0								
5	5	5	+	−9	0	0	0	−1	−1	0	0	43
5	3	3	−	−5	2	1	0	−1	0	0	0	
5	4	1	−	−1	1	−1	0	−1	0	0	0	
5	4	3	+	−5	1	−1	3	−1	−1	0	0	
5	4	4	−	−5	1	1	1	−1	−1	0	0	
5	5	1	+	−6	2	0	1	−1	0	0	0	
5	5	3	−	−2	2	0	0	−1	−1	0	0	
5	5	4	+	0	1	0	0	−1	−1	0	0	
5	5	5	+	−9	2	1	1	−1	−1	0	0	
5	5	1	−	−6	0	−1	0	1	0	0	0	
5	5	3	+	−2	2	−1	1	−1	−1	0	0	
5	5	4		0								

5	5	5	+	-9	2	0	0	1	-1	0	0
5	5	5	+	-9	4	1	1	-1	-1	0	0
6	3	3		0							
6	4	3	-	1	1	1	-1	-1	-1	0	0
6	4	4		0							
6	5	1	-	-4	2	0	1	-1	-1	0	0
6	5	3	-	-3	4	0	0	-1	-1	0	0
6	5	4	+	-4	1	0	1	-1	-1	0	0
6	5	5		0							
6	5	1	+	-4	0	1	2	-1	-1	0	0
6	5	3	-	-3	2	1	-1	-1	-1	0	0
6	5	4	+	-4	3	1	0	-1	-1	0	0
6	5	5	-	-3	2	0	2	-1	-1	-1	0
6	5	5		0							
6	6	1	+	-2	0	0	-1	0	-1	0	0
6	6	3	+	-1	3	0	-1	-1	-1	0	0
6	6	4		0							
6	6	5	-	-5	3	0	-1	-1	-1	1	0
6	6	5	-	-5	3	1	0	-1	-1	-1	0
6	6	6		0							

		E		T1			T1				
2	1	1	+	1	0	-1	0	0	0	0	0
2	3	1	-	0	2	-1	-1	0	0	0	0
2	3	3	+	2	0	-1	-1	0	0	0	0
2	4	3		0							
2	4	4	-	2	-1	-1	1	-1	0	0	0
2	5	3	-	1	0	1	-1	-1	0	0	0
2	5	4		0							
2	5	5	+	1	0	1	0	-1	-1	0	0
2	5	3		0							
2	5	4	-	1	1	-1	0	-1	0	0	0
2	5	5		0							
2	5	5	-	1	2	-1	0	-1	-1	0	0
2	6	4	-	0	1	0	0	-1	-1	0	0
2	6	5		0							
2	6	5	-	5	1	0	-1	-1	-1	0	0
2	6	6	-	1	1	-1	-1	-1	-1	0	0
4	3	1	-	0	-2	1	-1	0	0	0	0
4	3	3	+	-1	0	1	-1	-1	0	0	0
4	4	1	+	0	-2	-1	1	0	0	0	0
4	4	3	+	-1	-2	-1	2	-1	0	0	0
4	4	4	+	-1	-2	1	1	-1	-1	0	0
4	5	1	+	-2	-2	2	0	-1	0	0	0
4	5	3	-	0	0	2	-1	-1	-1	0	0
4	5	4	-	0	-2	0	1	-1	-1	0	0
4	5	5	+	-1	0	1	0	-1	-1	0	0
4	5	1	-	-2	0	-1	1	-1	0	0	0
4	5	3	-	0	0	-1	2	-1	-1	0	0

4	5	4	+	0	0	1	0	-1	-1	0	0
4	5	5	-	-1	0	0	1	-1	-1	0	0
4	5	5	-	-1	0	1	0	-1	-1	0	0
4	6	3	+	0	0	1	0	-1	-1	0	0
4	6	4	+	0	0	1	0	-1	-1	0	0
4	6	5	+	-3	0	0	2	-1	-1	0	0
4	6	5	+	-3	0	1	-1	-1	-1	0	0
4	6	6	-	6	0	1	-1	-1	-1	-1	0
5	3	3		0							
5	4	1	+	0	1	-1	0	-1	0	0	0
5	4	3	+	2	1	-1	1	-1	-1	0	0
5	4	4		0							
5	5	1		0							
5	5	3		0							
5	5	4	+	1	1	0	0	-1	-1	0	0
5	5	5		0							
5	5	1	+	3	0	-1	0	-1	0	0	0
5	5	3	-	1	2	-1	-1	-1	-1	0	0
5	5	4		0							
5	5	5	-	0	2	0	0	-1	-1	0	0
5	5	5		0							
5	6	1	-	1	0	1	0	-1	-1	0	0
5	6	3	-	0	2	1	-1	-1	-1	0	0
5	6	4		0							
5	6	5	+	2	2	0	0	-1	-1	-1	0
5	6	5		0							
5	6	6		0							
6	3	3	+	-1	0	2	0	-1	-1	0	0
6	4	3	+	-1	1	1	-1	-1	-1	0	0
6	4	4	-	-1	-1	-1	1	-1	-1	0	0

6	5	1	+	-2	2	0	1	-1	-1	0	0
6	5	3	-	-3	0	0	2	-1	-1	0	0
6	5	4	-	-4	1	0	1	-1	-1	0	0
6	5	5	+	1	0	1	1	-1	-1	-1	0
6	5	1	+	-2	0	1	0	-1	-1	0	0
6	5	3	+	-3	2	1	-1	-1	-1	0	0
6	5	4	-	-4	1	-1	2	-1	-1	0	0
6	5	5	-	-1	2	0	0	-1	-1	-1	0
6	5	5	+	3	2	-1	1	-1	-1	-1	0
6	6	1	+	0	0	0	-1	0	-1	0	0
6	6	3	-	-3	3	0	-1	-1	-1	0	0
6	6	4	-	-3	1	1	2	-1	-1	-1	0
6	6	5	-	-3	5	0	-1	-1	-1	-1	0
6	6	5	+	-3	1	1	2	-1	-1	-1	0
6	6	6	-	1	1	0	1	-1	-1	-1	-1

		E		E			A1				
2	2	0	+	1	0	-1	0	0	0	0	0
2	2	4	+	0	-1	-1	0	0	0	0	0
4	2	4	+	3	-2	0	0	-1	0	0	0
4	2	6	-	0	1	0	0	-1	-1	0	0
4	4	0	+	1	-2	0	0	0	0	0	0
4	4	4	+	2	-3	0	0	-1	-1	0	0
4	4	6	+	8	-2	-1	0	-1	-1	0	0
5	2	4	-	0	-1	0	0	-1	0	0	0
5	2	6	-	3	0	0	0	-1	-1	0	0
5	4	4	+	3	0	0	0	-1	-1	0	0
5	4	6	+	-1	1	-1	0	-1	-1	0	0
5	5	0	+	1	0	0	0	-1	0	0	0
5	5	4	-	0	-1	0	1	-1	-1	0	0
5	5	6	-	4	1	-1	0	-1	-1	-1	0
6	2	4	+	2	-1	1	0	-1	-1	0	0
6	2	6	-	2	0	-1	0	-1	-1	0	0
6	4	4	+	4	-2	0	0	-1	-1	0	0
6	4	6	-	-1	3	1	0	-1	-1	-1	0
6	5	4	+	1	-1	0	0	-1	-1	0	0
6	5	6	-	1	0	1	1	-1	-1	-1	0
6	6	0	+	1	0	0	0	0	-1	0	0
6	6	4	+	-1	-1	0	0	-1	-1	-1	2
6	6	6	-	4	2	0	0	-1	-1	-1	-1

E E E

2	2	2	+	3	0	-1	-1	0	0	0	0
4	2	2	+	1	-1	0	-1	0	0	0	0
4	4	2	-	6	-2	-1	-1	-1	0	0	0
4	4	4	+	9	-3	1	-1	-1	-1	0	0
5	4	2	-	1	-1	-1	1	-1	0	0	0
5	4	4		0							
5	5	2	-	3	1	-1	0	-1	-1	0	0
5	5	4	-	1	-1	1	0	-1	-1	0	0
5	5	5		0							
6	4	2	+	1	-1	0	1	-1	-1	0	0
6	4	4	+	5	-2	-1	1	-1	-1	0	0
6	5	2	-	4	0	0	-1	-1	-1	0	0
6	5	4	-	0	-1	-1	-1	1	-1	0	0
6	5	5	+	5	1	-1	1	-1	-1	-1	0
6	6	2	+	5	2	-1	-1	-1	-1	0	0
6	6	4	-	2	-1	1	-1	-1	-1	-1	0
6	6	5		0							
6	6	6	+	7	0	0	1	-1	-1	-1	-1

		T2		T1			T1				
2	1	1	+	0	1	-1	0	0	0	0	0
2	3	1	+	1	1	-1	-1	0	0	0	0
2	3	3	-	-3	1	-1	-1	0	0	0	0
2	4	3	-	-3	0	0	0	0	0	0	0
2	4	4	-	-3	0	-1	1	-1	0	0	0
2	5	3	-	-2	1	1	-1	-1	0	0	0
2	5	4	-	-2	0	0	1	-1	0	0	0
2	5	5	+	-6	1	1	0	-1	-1	0	0
2	5	3	-	-2	1	0	0	-1	0	0	0
2	5	4	+	-2	0	-1	0	-1	0	0	0
2	5	5	+	-6	1	0	1	-1	-1	0	0
2	5	5	+	-6	3	-1	0	-1	1	0	0
2	6	4	+	1	0	0	0	-1	-1	0	0
2	6	5	+	-4	2	1	0	-1	-1	0	0
2	6	5	-	-4	2	0	-1	-1	-1	0	0
2	6	6	-	-2	2	-1	-1	-1	-1	2	0
3	3	1	+	-3	1	1	-1	0	0	0	0
3	3	3		0							
3	4	1	-	-3	0	0	0	0	0	0	0
3	4	3	+	0	0	0	0	-1	0	0	0
3	4	4		0							
3	5	3	-	-5	3	1	-1	-1	0	0	0
3	5	4	+	-5	0	1	0	-1	-1	0	0
3	5	5		0							
3	5	3	+	-5	1	0	0	-1	0	0	0
3	5	4	+	-5	0	2	1	-1	-1	0	0
3	5	5	+	0	1	1	0	-1	-1	0	0

3	5	5		0							
3	6	3	+	-1	1	1	0	-1	-1	0	0
3	6	4	-	-1	0	0	-1	1	-1	0	0
3	6	5	+	-3	1	1	0	-1	-1	0	0
3	6	5	-	-3	5	0	-1	-1	-1	0	0
3	6	6		0							
4	3	1	-	-3	-1	1	-1	0	0	0	0
4	3	3	-	0	1	1	-1	-1	0	0	0
4	4	1	+	-3	-1	-1	1	0	0	0	0
4	4	3	-	2	-1	-1	0	-1	0	0	0
4	4	4	+	0	-1	1	1	-1	-1	0	0
4	5	1	-	-1	-1	0	0	-1	0	0	0
4	5	3	-	-5	1	0	-1	-1	-1	2	0
4	5	4	+	-5	-1	0	1	-1	-1	0	0
4	5	5	+	-2	1	1	0	-1	-1	0	0
4	5	1	+	-1	1	-1	1	-1	0	0	0
4	5	3	+	-5	1	-1	0	1	-1	0	0
4	5	4	-	-5	3	1	0	-1	-1	0	0
4	5	5	-	-2	1	0	1	-1	-1	0	0
4	5	5	-	-2	1	1	0	-1	-1	0	0
4	6	3	+	-1	1	1	0	-1	-1	0	0
4	6	4	-	-1	1	-1	0	-1	-1	0	0
4	6	5	+	-4	1	0	0	-1	-1	0	0
4	6	5	-	-4	1	-1	-1	-1	-1	0	0
4	6	6	-	-1	1	3	-1	-1	-1	-1	0
5	3	3		0							
5	4	1	-	3	0	-1	0	-1	0	0	0
5	4	3	+	1	0	-1	1	-1	-1	0	0
5	4	4		0							
5	5	1	-	-4	1	0	1	-1	0	0	0

5	5	3	-	-2	3	0	0	-1	-1	0	0	
5	5	4	-	-2	0	0	0	-1	-1	0	0	
5	5	5		0								
5	5	1	+	-4	1	-1	0	-1	0	0	0	
5	5	3	-	-2	1	-1	-1	-1	-1	2	0	
5	5	4	-	-2	0	1	1	-1	-1	0	0	
5	5	5	+	-3	1	0	0	-1	-1	0	0	
5	5	5		0								
5	6	1	-	-2	1	1	0	-1	-1	0	0	
5	6	3	-	-1	1	1	-1	-1	-1	0	0	
5	6	4	+	-2	0	-1	0	1	-1	0	0	
5	6	5	-	-5	1	0	0	-1	-1	-1	0	37
5	6	5	-	-5	5	-1	1	-1	-1	-1	0	
5	6	6		0								
6	3	3	+	-6	1	1	0	-1	-1	0	0	
6	4	3	-	-6	0	0	-1	-1	-1	0	0	29
6	4	4	-	-6	0	0	1	-1	1	0	0	
6	5	1	+	-7	1	1	1	-1	-1	0	0	
6	5	3	+	-6	1	3	0	-1	-1	0	0	
6	5	4	-	-7	0	1	1	-1	-1	0	0	
6	5	5	+	-10	1	2	1	-1	-1	-1	2	
6	5	1	+	-7	1	0	0	-1	-1	0	0	23
6	5	3	+	-6	1	0	-1	1	-1	0	0	
6	5	4	+	-7	0	0	0	-1	-1	0	0	
6	5	5	-	-10	1	1	0	-1	-1	-1	0	23
6	5	5	-	-10	3	0	1	-1	-1	1	0	
6	6	1	-	-5	1	3	-1	0	-1	0	0	
6	6	3	+	-2	2	1	-1	-1	-1	0	0	
6	6	4	+	-4	0	0	0	-1	-1	-1	2	
6	6	5	+	-6	2	1	-1	-1	-1	-1	2	

6	6	5	+	−6	2	0	0	−1	1	−1	0
6	6	6	+	−6	2	1	3	−1	−1	−1	−1
6	3	3	−	−6	1	2	0	0	−1	0	0
6	4	3	−	−6	2	1	−1	0	−1	0	0
6	4	4	+	−6	0	−1	1	0	−1	0	0
6	5	1	+	−7	3	0	1	0	−1	0	0
6	5	3	−	−6	1	0	0	0	−1	0	0
6	5	4	−	−7	2	0	1	0	−1	0	0
6	5	5	+	−10	1	1	1	0	−1	−1	0
6	5	1	+	−7	1	1	0	0	−1	0	0
6	5	3	−	−6	3	1	−1	0	−1	0	0
6	5	4	+	−7	2	−1	0	0	−1	0	0
6	5	5	+	−10	3	0	0	0	−1	1	0
6	5	5	−	−10	3	−1	1	2	−1	−1	0
6	6	1	+	−5	1	0	−1	1	−1	0	0
6	6	3	−	−2	2	0	−1	0	−1	0	0
6	6	4	−	−4	2	1	0	0	−1	−1	0
6	6	5	+	−6	2	0	−1	0	1	−1	0
6	6	5	−	−6	2	1	0	0	−1	−1	0
6	6	6	+	−6	2	0	1	0	1	−1	−1

		T2		E			T1				
2	2	1	+	1	0	-1	0	0	0	0	0
2	2	3	+	-1	1	-1	-1	0	0	0	0
2	2	4	+	-1	-1	0	0	0	0	0	0
2	4	3	+	0	0	0	-1	0	0	0	0
2	4	4	+	0	0	-1	0	-1	0	0	0
2	4	5	+	-1	0	0	0	-1	0	0	0
2	4	5	+	-1	0	-1	1	-1	0	0	0
2	4	6	+	0	0	0	1	-1	-1	0	0
2	5	3	+	-1	0	0	0	-1	0	0	0
2	5	4	-	-1	-1	-1	0	-1	0	0	0
2	5	5	+	1	0	0	1	-1	-1	0	0
2	5	5	-	1	2	-1	0	-1	-1	0	0
2	5	6	-	1	3	0	-1	-1	-1	0	0
2	6	4	-	5	-1	0	0	-1	-1	0	0
2	6	5	+	-2	1	1	0	-1	-1	0	0
2	6	5	+	-2	1	0	1	-1	-1	0	0
2	6	6	+	0	1	-1	-1	-1	-1	0	0
3	2	1	+	0	0	0	-1	0	0	0	0
3	2	3	+	0	0	0	-1	0	0	0	0
3	2	4	-	0	-1	-1	0	0	0	0	0
3	2	5	+	-1	0	0	-1	-1	0	0	0
3	2	5	+	-1	2	-1	0	-1	0	0	0
3	4	1	-	0	0	0	-1	0	0	0	0
3	4	3	-	-1	0	0	-1	-1	0	0	0
3	4	4	-	-1	0	0	0	-1	0	0	0
3	4	5	+	4	0	1	-1	-1	-1	0	0
3	4	5	-	2	0	0	0	-1	-1	0	0

3	4	6	+	0	0	0	0	-1	-1	0	0
3	5	3	-	0	0	0	0	-1	0	0	0
3	5	4	+	0	-1	0	1	-1	-1	0	0
3	5	5	+	-1	0	1	0	-1	-1	0	0
3	5	5	+	-1	4	0	-1	-1	-1	0	0
3	5	6	+	0	2	0	-1	-1	-1	0	0
3	6	3	-	-1	0	1	0	-1	-1	0	0
3	6	4	-	-1	-1	0	-1	-1	-1	0	2
3	6	5	-	-3	0	1	0	-1	-1	0	0
3	6	5	+	-3	4	0	-1	-1	-1	0	0
3	6	6	-	-3	1	3	-1	-1	-1	0	0
4	2	3	+	0	0	0	-1	0	0	0	0
4	2	4	+	0	0	-1	0	-1	0	0	0
4	2	5	+	-1	0	0	0	-1	0	0	0
4	2	5	-	-1	0	-1	1	-1	0	0	0
4	2	6	+	0	0	0	1	-1	-1	0	0
4	4	1	+	0	-1	-1	0	0	0	0	0
4	4	3	-	-1	-1	-1	-1	-1	2	0	0
4	4	4	+	-1	1	1	0	-1	-1	0	0
4	4	5	+	2	-1	0	0	-1	-1	0	0
4	4	5		0							
4	4	6	-	0	1	-1	1	-1	-1	0	0
4	5	1	+	0	0	-1	1	-1	0	0	0
4	5	3	-	0	0	-1	0	-1	-1	0	0
4	5	4	-	0	0	1	0	-1	-1	0	0
4	5	5	-	-1	0	0	1	-1	-1	0	0
4	5	5	-	-1	0	1	0	-1	-1	0	0
4	5	6	+	5	0	-1	-1	-1	-1	0	0
4	6	3	+	-1	0	1	0	-1	-1	0	0
4	6	4	-	-1	2	-1	0	-1	-1	0	0

4	6	5	−	−4	0	2	0	−1	−1	0	0
4	6	5	+	−4	0	−1	1	1	−1	0	0
4	6	6	−	−3	0	1	−1	−1	−1	−1	2
5	2	3	+	−1	0	0	0	−1	0	0	0
5	2	4	−	−1	−1	−1	0	−1	0	0	0
5	2	5	+	1	0	0	1	−1	−1	0	0
5	2	5	+	1	2	−1	0	−1	−1	0	0
5	2	6	−	1	3	0	−1	−1	−1	0	0
5	4	1	+	0	0	−1	1	−1	0	0	0
5	4	3	−	0	0	−1	0	−1	−1	0	0
5	4	4	−	0	0	1	0	−1	−1	0	0
5	4	5	−	−1	0	0	1	−1	−1	0	0
5	4	5	+	−1	0	1	0	−1	−1	0	0
5	4	6	+	5	0	−1	−1	−1	−1	0	0
5	5	1	−	1	0	−1	0	−1	0	0	0
5	5	3	+	−1	0	−1	−1	−1	−1	0	0
5	5	4	+	−1	−1	1	1	−1	−1	0	0
5	5	5	−	2	0	0	0	−1	−1	0	0
5	5	5		0							
5	5	6	−	2	2	−1	1	−1	−1	−1	0
5	6	1	−	0	0	1	0	−1	−1	0	0
5	6	3	+	−1	0	1	−1	−1	−1	0	0
5	6	4	−	−2	−1	−1	0	−1	−1	2	0
5	6	5	+	−3	0	2	0	−1	−1	−1	0
5	6	5	+	−3	4	−1	1	−1	−1	−1	0
5	6	6	−	1	1	1	0	−1	−1	−1	0
6	2	4	+	−1	−1	−1	0	−1	−1	0	0
6	2	5	+	2	1	0	0	−1	−1	0	0
6	2	5	−	0	3	−1	−1	−1	−1	0	0
6	2	6	+	0	3	0	−1	−1	−1	0	0

4

6	4	3	+	3	0	0	0	-1	-1	0	0	
6	4	4	+	1	0	0	0	-1	-1	0	0	
6	4	5	-	0	0	1	0	-1	-1	0	0	
6	4	5	-	-2	0	0	-1	1	-1	0	0	
6	4	6	+	-3	0	0	-1	-1	-1	-1	0	23
6	5	1	-	4	0	0	0	-1	-1	0	0	
6	5	3	+	3	0	0	-1	-1	-1	0	0	
6	5	4	-	-2	-1	0	2	-1	-1	0	0	
6	5	5	+	1	0	1	0	-1	-1	-1	0	
6	5	5	+	-1	2	0	1	-1	-1	-1	0	
6	5	6	+	-1	3	0	0	-1	-1	-1	0	
6	6	1	+	-3	0	1	-1	0	-1	0	0	
6	6	3	-	-6	1	1	1	-1	-1	0	0	
6	6	4	+	-6	-1	0	0	-1	-1	-1	0	37
6	6	5	+	-4	1	1	-1	-1	1	-1	0	
6	6	5	+	-4	1	0	0	1	-1	-1	0	
6	6	6	-	-4	1	1	1	-1	-1	1	-1	
6	2	4	+	-1	1	0	0	0	-1	0	0	
6	2	5		0								
6	2	5	+	0	1	0	-1	0	-1	0	0	
6	2	6	+	0	1	-1	-1	0	-1	0	0	
6	4	3		0								
6	4	4	+	1	0	-1	0	0	-1	0	0	
6	4	5		0								
6	4	5	+	-2	4	-1	-1	0	-1	0	0	
6	4	6	+	-3	4	1	-1	0	-1	-1	0	
6	5	1		0								
6	5	3		0								
6	5	4	+	-2	1	-1	0	0	-1	0	0	
6	5	5		0								

6	5	5	+	-1	2	-1	1	0	-1	-1	0
6	5	6	+	-1	1	1	0	0	-1	-1	0
6	6	1	+	-3	2	0	-1	1	-1	0	0
6	6	3	+	-6	1	0	-1	2	-1	0	0
6	6	4	+	-6	1	1	0	0	-1	-1	0
6	6	5	+	-4	1	0	-1	2	-1	-1	0
6	6	5	+	-4	1	1	0	0	-1	-1	0
6	6	6	+	-4	1	2	1	0	-1	-1	-1

		T2		T2			A1					
2	2	0	+	0	1	−1	0	0	0	0	0	
2	2	4	−	1	−2	−1	0	0	0	0	0	
3	2	4	+	0	−2	0	0	0	0	0	0	
3	3	0	+	0	1	0	−1	0	0	0	0	
3	3	4	−	−1	−2	0	0	−1	0	0	0	
3	3	6	+	−1	4	0	−1	−1	−1	0	0	
4	2	4	+	0	−1	0	0	−1	0	0	0	
4	2	6	+	3	0	0	0	−1	−1	0	0	
4	3	4	−	−1	−1	1	0	−1	0	0	0	
4	3	6	−	−1	0	0	1	−1	−1	0	0	
4	4	0	+	0	−1	0	0	0	0	0	0	
4	4	4	−	−1	−2	0	0	−1	1	0	0	
4	4	6	−	−1	−1	1	0	−1	−1	0	0	
5	2	4	+	3	−2	0	0	−1	0	0	0	
5	2	6	−	0	1	0	0	−1	−1	0	0	
5	3	4	+	0	−2	1	1	−1	−1	0	0	
5	3	6	−	−1	2	0	0	−1	−1	0	0	
5	4	4	−	0	−1	0	0	−1	−1	0	0	
5	4	6	+	−2	0	1	0	−1	−1	0	0	
5	5	0	+	0	1	0	0	−1	0	0	0	
5	5	4	+	1	−2	0	1	−1	−1	0	0	
5	5	6	+	1	2	1	0	−1	−1	−1	0	
6	2	4	+	−3	−2	0	0	−1	1	0	0	
6	2	6	+	−3	1	2	0	−1	−1	0	0	
6	3	4	+	−1	−2	0	−1	−1	−1	0	0	23
6	3	6	+	−6	1	0	2	−1	−1	0	0	
6	4	4	−	−1	−1	1	0	−1	−1	0	0	

6	4	6	+	-6	0	0	0	-1	-1	-1	0	
6	5	4	-	-4	-2	1	0	-1	-1	2	0	
6	5	6	-	-4	1	0	1	-1	-1	-1	0	
6	6	0	+	0	1	0	0	0	-1	0	0	
6	6	4	-	-4	-2	0	0	-1	-1	-1	0	59
6	6	6	-	-3	1	0	0	-1	-1	-1	-1	31
6	2	4	+	-3	0	1	0	0	-1	0	0	
6	2	6	-	-3	1	-1	0	0	-1	0	0	
6	3	4	-	-1	0	1	-1	0	-1	0	0	
6	3	6	+	-6	1	1	0	0	-1	0	0	
6	4	4	+	-1	-1	0	0	0	-1	0	0	
6	4	6	-	-6	4	1	0	0	-1	-1	0	
6	5	4	-	-4	0	0	0	0	-1	0	0	
6	5	6	+	-4	1	1	1	0	-1	-1	0	
6	6	0		0								
6	6	4	+	-4	0	1	0	0	-1	-1	0	
6	6	6	-	-3	1	1	2	0	-1	-1	-1	
6	6	0	+	0	1	0	0	0	-1	0	0	
6	6	4	+	-4	2	0	0	1	-1	-1	0	
6	6	6	+	-3	1	0	0	1	-1	-1	-1	

		T2		T2			T1				
2	2	1	+	0	0	-1	0	0	0	0	0
2	2	3	-	2	1	-1	-1	0	0	0	0
2	2	4		0							
3	2	1	-	1	0	0	-1	0	0	0	0
3	2	3	+	-3	0	0	-1	0	0	0	0
3	2	4	-	-3	1	-1	0	0	0	0	0
3	2	5	+	-2	0	2	-1	-1	0	0	0
3	2	5	-	-2	2	-1	0	-1	0	0	0
3	3	1	-	-3	0	0	-1	0	0	0	0
3	3	3	-	0	0	0	-1	0	0	0	0
3	3	4		0							
3	3	5	+	-5	0	0	-1	1	0	0	0
3	3	5	+	-5	2	1	0	-1	0	0	0
3	3	6		0							
4	2	3	-	-3	0	0	-1	0	0	0	0
4	2	4	-	-3	0	-1	2	-1	0	0	0
4	2	5	+	-2	0	0	0	-1	0	0	0
4	2	5	+	-2	0	-1	1	-1	0	0	0
4	2	6	+	1	0	0	1	-1	-1	0	0
4	3	1	+	-3	2	0	-1	0	0	0	0
4	3	3	+	2	0	0	-1	-1	0	0	0
4	3	4	+	0	0	0	0	-1	0	0	0
4	3	5	+	-5	0	1	-1	1	-1	0	0
4	3	5	-	-5	0	0	0	-1	-1	0	0
4	3	6	-	1	0	0	0	-1	-1	0	0
4	4	1	-	-3	-1	1	0	0	0	0	0
4	4	3	+	0	-1	1	-1	-1	0	0	0

4	4	4		0								
4	4	5	+	-5	-1	0	0	-1	-1	0	0	43
4	4	5	+	-5	1	1	1	-1	-1	0	0	
4	4	6		0								
5	2	3	+	0	0	0	0	-1	0	0	0	
5	2	4	+	0	1	-1	0	-1	0	0	0	
5	2	5	-	-4	0	0	1	-1	-1	0	0	
5	2	5	+	-4	2	-1	0	1	-1	0	0	
5	2	6	+	-2	3	0	-1	-1	-1	0	0	
5	3	3	-	-1	0	0	0	-1	0	0	0	
5	3	4	+	-1	1	0	1	-1	-1	0	0	
5	3	5	+	-2	0	1	0	-1	-1	0	0	
5	3	5	+	-2	2	0	-1	-1	-1	0	0	
5	3	6	+	-1	4	0	-1	-1	-1	0	0	
5	4	1		0								
5	4	3	+	-1	0	1	0	-1	-1	0	0	
5	4	4	-	-1	0	1	0	-1	-1	0	0	
5	4	5	-	-2	0	0	1	-1	-1	0	0	
5	4	5	+	-2	0	1	0	-1	-1	0	0	
5	4	6	-	-2	0	3	-1	-1	-1	0	0	
5	5	1	+	-2	0	1	0	-1	0	0	0	
5	5	3	+	2	0	1	-1	-1	-1	0	0	
5	5	4		0								
5	5	5	+	-5	0	0	0	-1	1	0	0	
5	5	5	-	-5	2	1	1	-1	-1	0	0	
5	5	6		0								
6	2	4	+	4	1	-1	0	-1	-1	0	0	
6	2	5	+	-7	1	2	0	-1	-1	0	0	
6	2	5	-	-7	3	-1	-1	-1	-1	0	0	47
6	2	6	-	-5	5	0	-1	-1	-1	0	0	

6	3	3	-	-6	0	0	0	-1	1	0	0	
6	3	4	+	-6	5	1	-1	-1	-1	0	0	
6	3	5	-	-6	0	0	0	1	-1	0	0	
6	3	5	+	-6	2	3	-1	-1	-1	0	0	
6	3	6	-	-4	3	2	-1	-1	-1	0	0	
6	4	3	+	-6	0	0	0	-1	-1	0	2	
6	4	4	-	-6	0	0	0	1	-1	0	0	
6	4	5	+	-7	0	1	0	-1	-1	0	0	
6	4	5	-	-7	0	0	-1	-1	-1	2	0	
6	4	6	+	0	0	0	-1	1	-1	-1	0	
6	5	1	-	-5	0	0	0	1	-1	0	0	
6	5	3	-	-4	0	0	-1	-1	-1	0	0	37
6	5	4	-	-5	1	0	0	-1	-1	0	0	
6	5	5	-	-6	0	1	0	-1	-1	-1	2	
6	5	5	-	-6	2	0	3	-1	-1	-1	0	
6	5	6	-	-6	3	0	2	-1	-1	-1	0	
6	6	1	+	-8	0	0	-1	0	-1	0	0	37
6	6	3	+	-7	1	0	-1	-1	-1	0	2	
6	6	4		0								
6	6	5	-	-11	1	0	-1	-1	1	-1	0	59
6	6	5	+	-11	3	1	0	-1	-1	-1	2	
6	6	6		0								
6	2	4		0								
6	2	5	+	-7	3	1	0	0	-1	0	0	
6	2	5	+	-7	1	2	-1	0	-1	0	0	
6	2	6	-	-5	1	-1	-1	2	-1	0	0	
6	3	3	-	-6	2	1	0	0	-1	0	0	
6	3	4	+	-6	3	0	-1	0	-1	0	0	
6	3	5	+	-6	2	1	0	0	-1	0	0	
6	3	5	-	-6	2	0	-1	0	-1	0	0	

6	3	6	-	-4	1	1	-1	0	-1	0	0	
6	4	3	-	-6	2	1	0	0	-1	0	0	
6	4	4	+	-6	0	-1	2	0	-1	0	0	
6	4	5	+	-7	2	0	0	0	-1	0	0	
6	4	5	-	-7	2	-1	-1	0	-1	2	0	
6	4	6	+	0	2	1	-1	0	-1	-1	0	
6	5	1	+	-5	2	1	0	0	-1	0	0	
6	5	3	-	-4	2	1	-1	0	-1	0	0	
6	5	4	-	-5	3	-1	0	0	-1	0	0	
6	5	5	+	-6	2	0	0	0	-1	-1	0	
6	5	5	-	-6	2	-1	1	0	-1	-1	0	
6	5	6	+	-6	1	3	0	0	-1	-1	0	
6	6	1	+	-8	2	1	-1	1	-1	0	0	
6	6	3	-	-7	3	1	-1	0	-1	0	0	
6	6	4	-	-1	3	0	0	0	-1	-1	0	
6	6	5	+	-11	3	3	-1	0	-1	-1	0	
6	6	5	+	-11	1	0	0	0	-1	-1	0	83
6	6	6	-	-3	1	1	1	0	-1	-1	-1	
6	6	1	+	-8	2	0	-1	0	-1	0	0	
6	6	3	-	-7	1	0	-1	1	-1	0	0	
6	6	4		0								
6	6	5	+	-11	1	0	-1	1	-1	-1	0	37
6	6	5	-	-11	3	1	0	1	-1	-1	0	
6	6	6		0								

		T2		T2			E				
2	2	2	+	1	1	-1	-1	0	0	0	0
2	2	4	-	3	-2	0	-1	0	0	0	0
3	2	2	+	-1	1	0	-1	0	0	0	0
3	2	4	+	0	-2	-1	-1	0	0	0	0
3	2	5	-	-1	2	-1	0	-1	0	0	0
3	3	2		0							
3	3	4	+	-1	-2	1	1	-1	0	0	0
3	3	5		0							
3	3	6	-	-1	2	0	0	-1	-1	0	0
4	2	2	+	-1	0	0	-1	0	0	0	0
4	2	4	-	0	-1	-1	-1	1	0	0	0
4	2	5	+	-1	0	-1	1	-1	0	0	0
4	2	6		0							
4	3	2	+	2	0	-1	-1	0	0	0	0
4	3	4	-	-1	-1	0	-1	-1	0	0	0
4	3	5	+	2	0	0	0	-1	-1	0	0
4	3	6	-	-1	2	0	0	-1	-1	0	0
4	4	2	-	4	-1	-1	-1	-1	0	0	0
4	4	4	-	-1	-2	3	-1	-1	-1	0	0
4	4	5		0							
4	4	6	+	-1	-1	-1	3	-1	-1	0	0
5	2	4	-	1	-2	-1	1	-1	0	0	0
5	2	5	-	1	2	-1	0	-1	-1	0	0
5	2	6	+	0	1	2	-1	-1	-1	0	0
5	3	2	+	-1	2	-1	0	-1	0	0	0
5	3	4	+	0	-2	0	0	-1	-1	0	0
5	3	5	-	-1	4	0	-1	-1	-1	0	0

5	3	6	−	−1	4	0	−1	−1	−1	0	0	
5	4	2	+	−1	0	−1	1	−1	0	0	0	
5	4	4	+	0	−1	1	1	−1	−1	0	0	
5	4	5	−	−1	0	1	0	−1	−1	0	0	
5	4	6	+	−2	2	−1	−1	−1	−1	0	0	
5	5	2	−	1	2	−1	0	−1	−1	0	0	
5	5	4	+	3	−2	1	0	−1	−1	0	0	
5	5	5		0								
5	5	6	+	1	2	−1	1	−1	−1	−1	0	
6	2	4	−	−3	−2	−1	1	1	−1	0	0	
6	2	5	+	0	3	−1	−1	−1	−1	0	0	
6	2	6	+	−3	1	0	−1	1	−1	0	0	
6	3	4	−	−1	−2	3	0	−1	−1	0	0	
6	3	5	+	−1	2	1	−1	−1	−1	0	0	
6	3	6	−	−6	1	0	−1	−1	−1	0	0	23
6	4	2	+	3	0	−1	1	−1	−1	0	0	
6	4	4	−	−1	−1	0	1	−1	−1	0	0	
6	4	5	+	−2	0	0	−1	1	−1	0	0	
6	4	6	−	−6	2	0	−1	−1	−1	−1	0	59
6	5	2	+	6	1	−1	−1	−1	−1	0	0	
6	5	4	+	−4	−2	0	−1	−1	−1	0	0	
6	5	5	+	−1	2	0	1	−1	−1	−1	0	
6	5	6	+	−4	1	0	0	−1	−1	1	0	
6	6	2	−	1	1	1	−1	−1	−1	0	0	
6	6	4	−	−4	−2	1	−1	−1	−1	−1	0	109
6	6	5		0								
6	6	6	+	−3	1	0	1	−1	−1	1	−1	
6	2	4	+	−3	0	0	1	0	−1	0	0	
6	2	5	+	0	1	0	−1	0	−1	0	0	
6	2	6	+	−3	1	−1	−1	0	−1	0	0	

6	3	4	−	−1	0	0	0	0	−1	0	0
6	3	5	−	−1	2	0	−1	0	−1	0	0
6	3	6	−	−6	1	1	−1	0	−1	0	0
6	4	2		0							
6	4	4	+	−1	−1	−1	1	0	−1	0	0
6	4	5	+	−2	4	−1	−1	0	−1	0	0
6	4	6	+	−6	4	1	−1	0	−1	−1	0
6	5	2		0							
6	5	4	−	−4	0	−1	1	0	−1	0	0
6	5	5	−	−1	2	−1	1	0	−1	−1	0
6	5	6	−	−4	1	1	0	0	−1	−1	0
6	6	2		0							
6	6	4	+	−4	0	0	1	0	−1	−1	0
6	6	5	+	0	1	0	0	0	−1	−1	0
6	6	6	+	−3	1	1	1	0	−1	−1	−1
6	6	2	+	1	1	−1	−1	1	−1	0	0
6	6	4	−	−4	2	1	−1	1	−1	−1	0
6	6	5		0							
6	6	6	+	−3	1	0	1	1	−1	−1	−1

		T2		T2			T2				
2	2	2	+	0	2	-1	-1	0	0	0	0
3	2	2		0							
3	3	2	-	-3	1	1	-1	0	0	0	0
3	3	3		0							
4	2	2	-	2	-1	0	-1	0	0	0	0
4	3	2	-	-3	-1	-1	1	0	0	0	0
4	3	3	-	0	-1	1	-1	-1	0	0	0
4	4	2	+	-3	0	-1	-1	-1	2	0	0
4	4	3		0							
4	4	4	+	0	-1	1	-1	-1	-1	0	0
5	3	2	+	0	1	-1	0	-1	0	0	0
5	3	3		0							
5	4	2	+	0	-1	-1	1	-1	0	0	0
5	4	3	+	1	-1	2	0	-1	-1	0	0
5	4	4		0							
5	5	2	+	-2	1	-1	0	-1	-1	0	0
5	5	3		0							
5	5	4	+	2	-1	1	0	-1	-1	0	0
5	5	5		0							
6	3	3	-	-6	1	1	2	-1	-1	0	0
6	4	2	+	4	-1	-1	1	-1	-1	0	0
6	4	3	+	-6	-1	1	0	1	-1	0	0
6	4	4	-	-6	0	2	1	-1	-1	0	0
6	5	2	+	-5	4	-1	-1	-1	-1	0	0
6	5	3	-	-4	1	1	-1	-1	-1	0	0
6	5	4	+	-5	-1	2	-1	1	-1	0	0
6	5	5	-	-6	1	2	1	-1	-1	-1	0

6	6	2	-	-8	2	1	1	-1	-1	0	0	
6	6	3		0								
6	6	4	-	-7	-1	3	3	-1	-1	-1	0	
6	6	5		0								
6	6	6	+	-12	2	1	1	-1	-1	-1	-1	41
6	3	3	+	-6	3	0	0	0	-1	0	0	
6	4	2		0								
6	4	3	+	-6	3	0	0	0	-1	0	0	
6	4	4	-	-6	0	-1	3	0	-1	0	0	
6	5	2	+	-5	2	2	-1	0	-1	0	0	
6	5	3	-	-4	3	0	-1	0	-1	0	0	
6	5	4	-	-5	5	-1	-1	0	-1	0	0	
6	5	5	+	-6	5	-1	1	0	-1	-1	0	
6	6	2	+	-8	2	0	-1	2	-1	0	0	
6	6	3	-	-1	2	0	-1	0	-1	0	0	
6	6	4	+	-7	3	0	-1	0	-1	-1	0	
6	6	5	+	-3	2	0	0	0	-1	-1	0	
6	6	6	-	-12	2	0	1	0	1	-1	-1	
6	6	2	+	-8	2	-1	-1	1	-1	0	0	
6	6	3		0								
6	6	4	-	-7	3	1	-1	1	-1	-1	0	
6	6	5		0								
6	6	6	+	-12	2	1	1	1	1	-1	-1	
6	6	6	-	-12	2	0	1	2	-1	-1	-1	

		A2		E			F				
3	2	2	+	0	0	0	-1	0	0	0	0
3	4	2	-	3	-1	-1	-1	0	0	0	0
3	4	4	-	2	0	0	-1	-1	0	0	0
3	5	2	-	0	1	-1	0	-1	0	0	0
3	5	4	+	3	-1	0	0	-1	-1	0	0
3	5	5	+	0	3	0	-1	-1	-1	0	0
3	6	4	+	4	-1	0	0	-1	-1	0	0
3	6	5	-	4	1	0	-1	-1	-1	0	0
3	6	6	+	-1	0	1	-1	-1	-1	0	0
6	4	2	+	0	-1	-1	1	0	-1	0	0
6	4	4		0							
6	5	2	-	3	0	-1	-1	0	-1	0	0
6	5	4	-	-1	-1	2	-1	0	-1	0	0
6	5	5		0							
6	6	2	-	2	0	0	-1	0	-1	0	0
6	6	4	+	-1	-1	0	-1	2	-1	-1	0
6	6	5	-	1	0	0	0	0	-1	-1	0
6	6	6		0							

		A2		T2			T1				
3	2	1	−	0	0	0	−1	0	0	0	0
3	2	3	−	0	0	0	−1	0	0	0	0
3	2	4	−	0	−1	−1	0	0	0	0	0
3	2	5	−	−1	0	0	−1	−1	0	0	0
3	2	5	+	−1	2	−1	0	−1	0	0	0
3	3	1	−	0	0	0	−1	0	0	0	0
3	3	3	+	−1	0	0	−1	0	0	0	0
3	3	4	−	−1	−1	1	0	−1	0	0	0
3	3	5	+	2	0	0	−1	−1	0	0	0
3	3	5		0							
3	3	6	+	0	2	0	0	−1	−1	0	0
3	4	1	+	0	0	0	−1	0	0	0	0
3	4	3	+	−1	0	0	−1	−1	0	0	0
3	4	4	−	−1	0	0	0	−1	0	0	0
3	4	5	−	4	0	1	−1	−1	−1	0	0
3	4	5	−	2	0	0	0	−1	−1	0	0
3	4	6	+	0	0	0	0	−1	−1	0	0
3	5	3	−	0	0	0	0	−1	0	0	0
3	5	4	−	0	−1	0	1	−1	−1	0	0
3	5	5	+	−1	0	1	0	−1	−1	0	0
3	5	5	−	−1	4	0	−1	−1	−1	0	0
3	5	6	−	0	2	0	−1	−1	−1	0	0
3	6	3	+	3	0	0	0	−1	−1	0	0
3	6	4	−	1	−1	1	−1	−1	−1	0	0
3	6	5	+	1	0	0	0	−1	−1	0	0
3	6	5	−	−1	2	1	−1	−1	−1	0	0
3	6	6	−	−3	5	0	−1	−1	−1	0	0

3	6	3		0							
3	6	4	+	1	1	0	-1	0	-1	0	0
3	6	5		0							
3	6	5	-	-1	2	0	-1	0	-1	0	0
3	6	6	+	-3	1	1	-1	0	-1	0	0
6	2	4	+	3	-1	-1	0	0	-1	0	0
6	2	5	-	-2	1	0	0	0	-1	0	0
6	2	5	+	-2	1	-1	-1	0	-1	0	0
6	2	6	+	0	1	0	-1	0	-1	0	0
6	3	3	-	-1	0	0	0	0	-1	0	0
6	3	4	-	-1	-1	1	-1	0	-1	0	0
6	3	5	-	-3	0	0	0	0	-1	0	0
6	3	5	-	-3	2	1	-1	0	-1	0	0
6	3	6	+	-3	1	0	-1	0	-1	0	0
6	4	3	-	-1	0	0	0	0	-1	0	0
6	4	4	+	-1	0	0	0	0	-1	0	0
6	4	5	+	-4	0	1	0	0	-1	0	0
6	4	5	-	-4	0	0	-1	0	-1	0	0
6	4	6	+	-3	0	0	-1	0	-1	1	0
6	5	1	-	0	0	0	0	0	-1	0	0
6	5	3	+	-1	0	0	-1	0	-1	0	0
6	5	4	+	-2	-1	0	0	0	-1	0	0
6	5	5	+	-3	0	1	0	0	-1	-1	0
6	5	5	-	-3	2	0	1	0	-1	-1	0
6	5	6	-	1	1	0	0	0	-1	-1	0
6	6	1	-	-3	0	0	-1	1	-1	0	0
6	6	3	+	-6	1	0	1	0	-1	0	0
6	6	4	+	-6	-1	1	0	0	-1	1	0
6	6	5	-	-4	1	0	-1	0	1	-1	0
6	6	5	+	-4	1	1	0	0	-1	-1	0

6	6	6	−	−4	1	0	1	0	−1	−1	−1
6	6	1	+	−3	2	1	−1	0	−1	0	0
6	6	3	+	−6	1	1	−1	1	−1	0	0
6	6	4	+	−6	1	0	0	1	−1	−1	0
6	6	5	+	−4	1	1	−1	1	−1	−1	0
6	6	5	−	−4	1	0	0	1	−1	−1	0
6	6	6	+	−4	1	1	1	1	−1	−1	−1

		A2		A2			A1				
3	3	0	+	0	0	0	-1	0	0	0	0
3	3	4	-	1	-1	0	0	-1	0	0	0
3	3	6	-	3	1	0	-1	-1	-1	0	0
6	3	4	+	3	-1	0	-1	0	-1	0	0
6	3	6	-	-2	0	0	0	0	-1	0	0
6	6	0	+	0	0	0	0	0	-1	0	0
6	6	4	+	-2	-1	0	0	1	-1	-1	0
6	6	6	+	3	0	0	0	1	-1	-1	-1

		ED		ED			A1				
1/2	1/2	0	+	0	0	0	0	0	0	0	0
7/2	1/2	4	+	0	-2	0	0	0	0	0	0
7/2	7/2	0	+	-2	0	0	0	0	0	0	0
7/2	7/2	4	+	-2	-3	0	2	-1	0	0	0
7/2	7/2	6	-	-1	-1	2	0	-1	-1	0	0
9/2	1/2	4	+	0	-2	0	0	0	0	0	0
9/2	7/2	4	-	0	-3	0	2	-1	-1	0	0
9/2	7/2	6	+	1	-1	0	1	-1	-1	0	0
9/2	9/2	0	+	0	0	-1	0	0	0	0	0
9/2	9/2	4	+	1	-3	-1	3	-1	-1	0	0
9/2	9/2	6	-	6	-1	-1	0	-1	-1	0	0
11/2	1/2	6	-	0	0	0	0	0	-1	0	0
11/2	7/2	4	+	0	-1	2	0	-1	-1	0	0
11/2	7/2	6	+	-4	0	0	2	-1	-1	0	0
11/2	9/2	4	+	2	-1	0	0	-1	-1	0	0
11/2	9/2	6	+	-1	0	0	2	-1	-1	-1	0
11/2	11/2	0	+	-1	-1	0	0	0	0	0	0
11/2	11/2	4	-	-2	-1	0	2	-1	-1	0	0
11/2	11/2	6	+	1	-1	0	0	-1	-1	-1	0

		ED		ED			T1				
1/2	1/2	1	+	0	0	0	0	0	0	0	0
7/2	1/2	3	+	−2	0	0	0	0	0	0	0
7/2	1/2	4	−	−2	−2	1	0	0	0	0	0
7/2	7/2	1	−	−2	−3	0	1	0	0	0	0
7/2	7/2	3	−	0	−2	0	1	−1	0	0	0
7/2	7/2	4		0							
7/2	7/2	5	+	−4	−2	2	1	−1	−1	0	0
7/2	7/2	5	−	−4	0	1	2	−1	−1	0	0
7/2	7/2	6		0							
9/2	1/2	4	+	2	−2	−1	0	0	0	0	0
9/2	1/2	5	+	−2	0	0	1	−1	0	0	0
9/2	1/2	5	+	−2	2	−1	0	−1	0	0	0
9/2	7/2	1	+	2	−3	0	0	0	0	0	0
9/2	7/2	3	−	−2	−2	0	1	−1	0	0	0
9/2	7/2	4	−	−2	1	−1	2	−1	−1	0	0
9/2	7/2	5	+	−2	−2	0	0	−1	1	0	0
9/2	7/2	5	−	−2	0	−1	1	−1	−1	0	0
9/2	7/2	6	−	0	2	0	0	−1	−1	0	0
9/2	9/2	1	+	0	−3	−1	0	1	0	0	0
9/2	9/2	3	+	3	−2	−1	1	−1	−1	0	0
9/2	9/2	4		0							
9/2	9/2	5	+	−1	−2	−1	0	−1	−1	0	2
9/2	9/2	5	+	−1	0	0	1	−1	−1	0	0
9/2	9/2	6		0							
11/2	1/2	5	−	−3	1	0	0	−1	0	0	0
11/2	1/2	5	+	−3	−1	1	1	−1	0	0	0
11/2	1/2	6	−	−1	−1	0	1	0	−1	0	0

11/2	7/2	3	-	0	-1	2	0	-1	-1	0	0
11/2	7/2	4	-	0	-1	1	0	-1	-1	0	0
11/2	7/2	5	-	-2	-1	0	2	-1	-1	0	0
11/2	7/2	5	+	-2	-1	1	1	-1	-1	0	0
11/2	7/2	6	+	-3	-1	0	1	-1	-1	0	0
11/2	9/2	1		0							
11/2	9/2	3	+	0	-1	2	0	-1	-1	0	0
11/2	9/2	4	-	0	-1	-1	0	1	-1	0	0
11/2	9/2	5	-	-4	-1	0	1	-1	-1	0	0
11/2	9/2	5	-	-4	-1	-1	0	-1	-1	0	0
11/2	9/2	6	+	-2	-1	0	1	1	-1	-1	0
11/2	11/2	1	-	-1	-2	0	0	1	-1	0	0
11/2	11/2	3	+	1	-1	0	1	-1	-1	0	0
11/2	11/2	4		0							
11/2	11/2	5	+	-4	-1	0	0	-1	-1	-1	2
11/2	11/2	5	+	-4	1	1	1	-1	-1	-1	0
11/2	11/2	6		0							

	UD		ED			T1				
3/2 1/2	1	+	0	0	0	0	0	0	0	0
3/2 7/2	3	+	−3	−1	0	0	0	0	0	0
3/2 7/2	4	+	−3	−2	0	1	0	0	0	0
3/2 7/2	5	−	−3	−1	1	1	−1	0	0	0
3/2 7/2	5	−	−3	1	0	0	−1	0	0	0
3/2 9/2	3	−	−1	−1	0	0	0	0	0	0
3/2 9/2	4	−	−1	−2	−1	0	1	0	0	0
3/2 9/2	5	+	−2	−1	0	1	−1	0	0	0
3/2 9/2	5	+	−2	1	−1	0	−1	0	0	0
3/2 9/2	6		0							
3/211/2	4		0							
3/211/2	5	−	−4	1	1	0	−1	−1	0	0
3/211/2	5	+	−4	−1	2	1	−1	−1	0	0
3/211/2	6	+	−2	−1	0	0	1	−1	0	0
5/2 1/2	3	+	0	1	0	−1	0	0	0	0
5/2 7/2	1	−	−2	0	0	0	0	0	0	0
5/2 7/2	3	+	−2	−1	0	0	0	0	0	0
5/2 7/2	4	−	−2	−1	0	1	−1	0	0	0
5/2 7/2	5	−	−2	−1	1	0	−1	0	0	0
5/2 7/2	5	+	−2	−1	0	1	−1	0	0	0
5/2 7/2	6	−	−2	−1	2	0	−1	−1	0	0
5/2 9/2	3	+	−1	−1	0	0	−1	0	0	0
5/2 9/2	4	−	−1	−1	−1	1	−1	0	0	0
5/2 9/2	5	−	1	−1	0	1	−1	−1	0	0
5/2 9/2	5	+	1	−1	−1	2	−1	−1	0	0
5/2 9/2	6	−	2	−1	2	−1	−1	−1	0	0
5/211/2	3	+	−2	−1	2	−1	−1	0	0	0

5/211/2	4	-	-2	-1	2	1	-1	-1	0	0
5/211/2	5	-	-3	-1	1	1	-1	-1	0	0
5/211/2	5	-	-3	1	2	0	-1	-1	0	0
5/211/2	6	+	-1	2	0	0	-1	-1	0	0
7/2 1/2	3	-	-2	0	1	-1	0	0	0	0
7/2 1/2	4	-	-2	-2	0	1	0	0	0	0
7/2 7/2	1	-	0	-3	1	0	0	0	0	0
7/2 7/2	3	-	-2	-2	1	0	-1	0	0	0
7/2 7/2	4	-	-2	-1	0	1	-1	0	0	0
7/2 7/2	5	-	-4	-2	1	0	1	-1	0	0
7/2 7/2	5	-	-4	0	0	1	-1	-1	0	0
7/2 7/2	6	-	-2	2	1	0	-1	-1	0	0
7/2 9/2	1	+	0	-3	-1	1	0	0	0	0
7/2 9/2	3	-	-2	-2	-1	0	1	0	0	0
7/2 9/2	4	+	-2	-1	0	1	-1	-1	0	0
7/2 9/2	5	+	-2	-2	1	1	-1	-1	0	0
7/2 9/2	5	-	-2	0	0	2	-1	-1	0	0
7/2 9/2	6	-	2	2	-1	-1	-1	-1	0	0
7/211/2	3	-	-2	-1	1	-1	1	-1	0	0
7/211/2	4	-	-2	-1	0	1	-1	-1	0	0
7/211/2	5	-	0	-1	1	1	-1	-1	0	0
7/211/2	5	-	0	-1	0	0	-1	-1	0	0
7/211/2	6	+	1	-1	1	0	-1	-1	0	0
9/2 1/2	4	-	-1	-2	-1	0	0	0	0	0
9/2 1/2	5	+	-3	0	0	1	-1	0	0	0
9/2 1/2	5	-	-3	4	-1	0	-1	0	0	0
9/2 7/2	1	+	-3	-3	0	0	0	0	0	0
9/2 7/2	3	-	1	-2	0	1	-1	0	0	0
9/2 7/2	4	+	1	-1	-1	2	-1	-1	0	0
9/2 7/2	5	-	-1	-2	0	0	-1	-1	0	0

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9/2 7/2	5	+	3	0	-1	1	-1	-1	0	0
9/2 7/2	6	-	-3	2	0	0	-1	-1	0	0
9/2 9/2	1	-	3	-3	-1	0	-1	0	0	0
9/2 9/2	3	-	-2	-2	-1	1	1	-1	0	0
9/2 9/2	4	+	-2	-1	0	1	-1	-1	0	0
9/2 9/2	5	-	-2	-2	1	0	-1	-1	0	0
9/2 9/2	5	+	-2	0	0	1	-1	-1	0	0
9/2 9/2	6	-	-2	2	-1	1	-1	-1	0	0
9/211/2	1	+	-2	0	0	1	-1	0	0	0
9/211/2	3	+	-3	-1	0	0	1	-1	0	0
9/211/2	4	-	-3	-1	-1	0	-1	-1	0	0
9/211/2	5	-	-3	-1	0	1	-1	-1	0	0
9/211/2	5	-	-3	-1	-1	0	-1	-1	0	0
9/211/2	6	-	-3	-1	0	1	-1	-1	-1	0
9/2 1/2	4	-	-1	-1	-1	1	0	0	0	0
9/2 1/2	5	+	-3	1	0	0	-1	0	0	0
9/2 1/2	5	+	-3	1	-1	1	-1	0	0	0
9/2 7/2	1	+	-3	-2	0	1	0	0	0	0
9/2 7/2	3	+	-1	-1	0	0	-1	0	0	0
9/2 7/2	4	-	-1	0	-1	1	-1	-1	0	0
9/2 7/2	5	+	1	-1	0	1	-1	-1	0	0
9/2 7/2	5	+	-1	3	-1	0	-1	-1	0	0
9/2 7/2	6	+	-3	1	0	1	-1	-1	0	0
9/2 9/2	1	-	3	-2	-1	1	-1	0	0	0
9/2 9/2	3	-	-2	-1	-1	0	-1	-1	2	0
9/2 9/2	4	-	-2	0	2	0	-1	-1	0	0
9/2 9/2	5	+	-2	-1	-1	1	-1	-1	0	0
9/2 9/2	5	-	-2	3	0	0	-1	-1	0	0
9/2 9/2	6	-	-2	1	-1	2	-1	-1	0	0
9/211/2	1	-	-2	-1	0	0	-1	0	0	0

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9/211/2	3	-	-3	0	0	1	-1	-1	0	0
9/211/2	4	-	-3	2	-1	1	-1	-1	0	0
9/211/2	5	+	3	0	0	0	-1	-1	0	0
9/211/2	5	+	-1	0	-1	1	-1	-1	0	0
9/211/2	6	+	-3	0	2	0	-1	-1	-1	0
11/2 1/2	5		0							
11/2 1/2	5	+	2	-1	0	0	-1	0	0	0
11/2 1/2	6	+	0	-1	1	0	0	-1	0	0
11/2 7/2	3	+	-3	-1	1	1	-1	-1	0	0
11/2 7/2	4	+	-3	-1	0	3	-1	-1	0	0
11/2 7/2	5	+	-3	-1	1	1	-1	-1	0	0
11/2 7/2	5	+	-3	1	0	0	-1	-1	0	0
11/2 7/2	6	-	-2	1	1	0	-1	-1	0	0
11/2 9/2	1	+	0	0	-1	0	-1	0	0	0
11/2 9/2	3	+	-1	-1	-1	-1	-1	-1	0	0
11/2 9/2	4	-	-1	-1	0	1	-1	-1	0	0
11/2 9/2	5	+	-3	-1	3	0	-1	-1	0	0
11/2 9/2	5	-	-3	1	0	1	-1	-1	0	0
11/2 9/2	6	-	-1	1	-1	2	-1	-1	-1	0
11/211/2	1	+	2	-2	1	1	-1	-1	0	0
11/211/2	3	-	0	-1	1	0	-1	-1	0	0
11/211/2	4	-	-1	-1	0	1	-1	-1	0	0
11/211/2	5	-	-5	-1	1	1	-1	1	-1	0
11/211/2	5	-	-5	1	0	0	-1	-1	1	0
11/211/2	6	-	-3	2	1	0	-1	-1	-1	0
11/2 1/2	5	+	-3	1	0	1	-1	0	0	0
11/2 1/2	5	+	-3	-1	1	0	-1	0	0	0
11/2 1/2	6	-	-1	-1	0	0	0	-1	0	0
11/2 7/2	3		0							
11/2 7/2	4		0							

11/2	7/2	5	-	-2	1	0	1	-1	-1	0	0	
11/2	7/2	5	-	-2	-1	1	0	-1	-1	0	0	
11/2	7/2	6	+	-3	-1	0	0	1	-1	0	0	
11/2	9/2	1	+	1	0	0	0	-1	0	0	0	
11/2	9/2	3	-	0	3	0	-1	-1	-1	0	0	
11/2	9/2	4	-	0	1	-1	1	-1	-1	0	0	
11/2	9/2	5	+	-4	1	2	0	-1	-1	0	0	
11/2	9/2	5	+	-4	-1	-1	3	-1	-1	0	0	
11/2	9/2	6	-	-2	-1	0	0	-1	-1	-1	0	23
11/2	11/2	1	-	1	-2	0	1	-1	-1	0	0	
11/2	11/2	3	+	-1	1	0	0	-1	-1	0	0	
11/2	11/2	4	+	-2	-1	1	1	-1	-1	0	0	
11/2	11/2	5	-	-4	1	0	1	-1	-1	-1	0	
11/2	11/2	5	-	-4	3	1	0	-1	-1	-1	0	
11/2	11/2	6	-	-2	4	0	0	-1	-1	-1	0	

	UD		ED			E				
3/2 1/2	2	+	1	0	-1	0	0	0	0	0
3/2 7/2	2	+	-2	1	-1	0	0	0	0	0
3/2 7/2	4	+	0	-2	0	0	0	0	0	0
3/2 7/2	5	-	-2	0	0	0	-1	0	0	0
3/2 9/2	4	+	2	-2	-1	1	-1	0	0	0
3/2 9/2	5	-	1	0	-1	0	-1	0	0	0
3/2 9/2	6	+	2	1	0	0	-1	-1	0	0
3/211/2	4	+	-1	0	0	0	-1	0	0	0
3/211/2	5	+	1	0	0	1	-1	-1	0	0
3/211/2	6	+	0	0	0	0	-1	-1	0	0
5/2 1/2	2	+	1	0	-1	0	0	0	0	0
5/2 7/2	2	-	-1	-2	-1	0	0	0	0	0
5/2 7/2	4	-	1	-1	0	0	-1	0	0	0
5/2 7/2	5	+	-1	-2	0	1	-1	0	0	0
5/2 7/2	6	-	2	-2	2	0	-1	-1	0	0
5/2 9/2	2	+	0	-2	0	0	0	0	0	0
5/2 9/2	4	+	4	-1	-1	0	-1	0	0	0
5/2 9/2	5	-	0	-2	-1	0	-1	1	0	0
5/2 9/2	6	+	2	-2	0	1	-1	-1	0	0
5/211/2	4	-	1	-1	0	0	-1	-1	0	0
5/211/2	5	-	2	0	0	0	-1	-1	0	0
5/211/2	6	-	1	-1	0	0	-1	-1	0	0
7/2 1/2	4	+	1	-2	0	0	0	0	0	0
7/2 7/2	2	+	-1	-1	0	0	0	0	0	0
7/2 7/2	4	+	-1	-3	0	0	-1	0	0	0
7/2 7/2	5	+	-1	1	0	1	-1	-1	0	0
7/2 7/2	6	+	0	-1	1	0	-1	-1	0	0

7/2	9/2	2	+	1	-1	-1	0	-1	0	0	0
7/2	9/2	4	+	1	-3	0	0	-1	-1	0	0
7/2	9/2	5	+	1	1	0	0	-1	-1	0	0
7/2	9/2	6	+	2	-1	-1	1	-1	-1	0	0
7/2	11/2	2	+	-1	0	0	0	-1	0	0	0
7/2	11/2	4	-	5	-1	0	0	-1	-1	0	0
7/2	11/2	5	-	-1	0	0	0	-1	-1	0	0
7/2	11/2	6	+	-3	2	1	0	-1	-1	0	0
9/2	1/2	4	-	0	-2	-1	1	0	0	0	0
9/2	1/2	5	+	0	1	-1	0	-1	0	0	0
9/2	7/2	2	+	-2	-1	0	1	-1	0	0	0
9/2	7/2	4	-	6	-3	-1	1	-1	-1	0	0
9/2	7/2	5	+	-2	1	-1	1	-1	-1	0	0
9/2	7/2	6	+	3	-1	0	0	-1	-1	0	0
9/2	9/2	2	-	-1	-1	1	0	-1	0	0	0
9/2	9/2	4	-	1	-3	2	0	-1	-1	0	0
9/2	9/2	5	-	-1	1	0	1	-1	-1	0	0
9/2	9/2	6	-	0	-1	-1	1	-1	-1	0	0
9/2	11/2	2	-	-1	0	0	1	-1	-1	0	0
9/2	11/2	4	-	0	-1	-1	1	-1	-1	0	0
9/2	11/2	5	+	-2	0	-1	0	-1	-1	0	0
9/2	11/2	6	+	1	2	0	1	-1	-1	-1	0
9/2	1/2	4	-	0	-1	-1	0	0	0	0	0
9/2	1/2	5	-	0	0	-1	1	-1	0	0	0
9/2	7/2	2	-	-2	0	0	0	-1	0	0	0
9/2	7/2	4	+	2	-2	-1	0	1	-1	0	0
9/2	7/2	5	+	-2	0	-1	0	-1	1	0	0
9/2	7/2	6		0							
9/2	9/2	2	-	-1	0	-1	1	-1	0	0	0
9/2	9/2	4	+	1	-2	0	1	-1	-1	0	0

9/2	9/2	5	-	-1	0	0	0	-1	-1	0	0
9/2	9/2	6	-	0	2	-1	0	-1	-1	0	0
9/2	11/2	2	+	-1	3	0	0	-1	-1	0	0
9/2	11/2	4	-	0	2	-1	0	-1	-1	0	0
9/2	11/2	5	-	-2	1	-1	1	-1	-1	0	0
9/2	11/2	6	-	3	1	0	0	-1	-1	-1	0
11/2	1/2	5	+	1	0	0	0	-1	0	0	0
11/2	1/2	6		0							
11/2	7/2	2	-	-2	-2	0	0	-1	0	0	0
11/2	7/2	4	+	0	1	0	0	-1	-1	0	0
11/2	7/2	5	+	-2	-2	0	2	-1	-1	0	0
11/2	7/2	6	-	0	-2	1	0	-1	-1	0	0
11/2	9/2	2	+	1	-2	-1	0	-1	1	0	0
11/2	9/2	4	-	2	1	0	0	-1	-1	0	0
11/2	9/2	5	-	2	-2	0	1	-1	-1	0	0
11/2	9/2	6	+	1	-2	-1	0	-1	-1	1	0
11/2	11/2	2	+	1	-1	0	0	-1	-1	0	0
11/2	11/2	4	-	-2	-1	0	0	-1	-1	0	0
11/2	11/2	5	-	2	0	0	0	-1	-1	-1	0
11/2	11/2	6	-	-1	-1	1	2	-1	-1	-1	0
11/2	1/2	5		0							
11/2	1/2	6	+	1	0	0	0	0	-1	0	0
11/2	7/2	2	+	1	-2	1	0	-1	0	0	0
11/2	7/2	4	+	1	-1	1	0	-1	-1	0	0
11/2	7/2	5	-	1	-2	1	0	-1	-1	0	0
11/2	7/2	6	+	-3	-2	0	0	-1	-1	0	2
11/2	9/2	2	+	2	-2	0	0	-1	-1	0	0
11/2	9/2	4	+	3	-1	-1	0	-1	-1	0	0
11/2	9/2	5	-	1	-2	-1	1	-1	-1	0	0
11/2	9/2	6	+	0	-2	0	0	-1	-1	-1	2

11/211/2	2	+	0	-1	1	0	-1	-1	0	0
11/211/2	4	+	-1	1	1	0	-1	-1	0	0
11/211/2	5	-	-1	0	1	2	-1	-1	-1	0
11/211/2	6	+	2	1	0	0	-1	-1	-1	0

	UD		ED			T2					
3/2 1/2	2	+	0	1	−1	0	0	0	0	0	
3/2 7/2	2	−	−1	0	−1	0	0	0	0	0	
3/2 7/2	3	+	−3	−1	1	0	0	0	0	0	
3/2 7/2	4	+	−3	−1	0	0	0	0	0	0	
3/2 7/2	5	+	1	−1	0	0	−1	0	0	0	
3/2 9/2	3	+	−1	−1	−1	0	0	0	0	0	
3/2 9/2	4	+	−1	−1	−1	1	−1	0	0	0	
3/2 9/2	5	+	4	−1	−1	0	−1	0	0	0	
3/2 9/2	6	+	−3	0	−1	0	−1	1	0	0	
3/2 9/2	6	+	−3	2	0	0	0	−1	0	0	
3/211/2	4	−	2	−1	0	0	−1	0	0	0	
3/211/2	5	+	−2	1	0	1	−1	−1	0	0	
3/211/2	6	−	−5	1	3	0	−1	−1	0	0	
3/211/2	6	+	−5	1	0	0	0	−1	0	0	
5/2 1/2	2	−	2	−1	−1	0	0	0	0	0	
5/2 1/2	3	−	0	−1	1	−1	0	0	0	0	
5/2 7/2	2	+	−2	−3	−1	2	0	0	0	0	
5/2 7/2	3	+	−2	−3	1	0	0	0	0	0	
5/2 7/2	4	+	−2	−2	0	2	−1	0	0	0	
5/2 7/2	5	+	−2	−3	0	1	−1	0	0	0	
5/2 7/2	6	+	−3	−3	1	0	−1	−1	0	0	
5/2 7/2	6	+	−3	−1	2	0	0	−1	0	0	
5/2 9/2	2	+	1	−3	0	0	0	0	0	0	
5/2 9/2	3	−	−1	−3	−1	0	−1	2	0	0	
5/2 9/2	4	+	−1	−2	−1	0	−1	0	0	0	
5/2 9/2	5	−	1	−3	−1	0	−1	−1	0	0	29
5/2 9/2	6	−	−1	−3	−1	−1	−1	−1	0	0	107

5/2 9/2	6	+	-1	-1	0	-1	0	-1	0	0
5/211/2	3	+	-2	1	1	-1	-1	0	0	0
5/211/2	4	-	-2	-2	0	0	-1	-1	2	0
5/211/2	5	-	-1	1	0	0	-1	-1	0	0
5/211/2	6	-	-4	0	1	0	-1	-1	0	0
5/211/2	6	+	-4	-2	0	2	0	-1	0	0
7/2 1/2	3	-	-2	2	0	-1	0	0	0	0
7/2 1/2	4	+	-2	-1	0	0	0	0	0	0
7/2 7/2	2		0							
7/2 7/2	3	+	-2	0	0	0	-1	0	0	0
7/2 7/2	4	-	-2	-2	0	2	-1	0	0	0
7/2 7/2	5	+	-2	0	0	1	-1	-1	0	0
7/2 7/2	6	+	-5	4	0	0	-1	-1	0	0
7/2 7/2	6	-	-5	0	1	0	0	-1	0	0
7/2 9/2	2	+	0	2	-1	0	-1	0	0	0
7/2 9/2	3	+	-2	0	0	0	-1	0	0	0
7/2 9/2	4	+	-2	-2	0	0	-1	1	0	0
7/2 9/2	5	-	2	0	0	0	-1	-1	0	0
7/2 9/2	6	-	-3	2	2	-1	-1	-1	0	0
7/2 9/2	6	+	-3	0	-1	-1	2	-1	0	0
7/211/2	2	-	0	-1	0	0	-1	0	0	0
7/211/2	3	-	-2	-1	0	-1	-1	-1	0	2
7/211/2	4	+	-2	2	0	0	-1	-1	0	0
7/211/2	5	+	0	-1	0	0	-1	-1	0	0
7/211/2	6	-	2	-1	0	0	-1	-1	0	0
7/211/2	6		0							
9/2 1/2	4	+	-1	-1	-1	1	0	0	0	0
9/2 1/2	5	+	-1	2	-1	0	-1	0	0	0
9/2 7/2	2	-	-3	0	0	1	-1	0	0	0
9/2 7/2	3		0							

9/2	7/2	4	+	5	-2	-1	1	-1	-1	0	0
9/2	7/2	5	+	-3	2	-1	1	-1	-1	0	0
9/2	7/2	6	-	-2	0	1	0	-1	-1	0	0
9/2	7/2	6	-	-2	0	0	0	0	-1	0	0
9/2	9/2	2	+	0	0	-1	0	-1	0	0	0
9/2	9/2	3	+	-2	2	0	1	-1	-1	0	0
9/2	9/2	4	-	-2	-2	0	0	-1	-1	2	0
9/2	9/2	5		0							
9/2	9/2	6	-	-5	0	2	1	-1	-1	0	0
9/2	9/2	6	+	-5	0	-1	1	0	-1	0	0
9/2	11/2	2	-	-2	-1	0	1	-1	-1	0	0
9/2	11/2	3	+	-3	-1	3	0	-1	-1	0	0
9/2	11/2	4	+	-3	0	-1	1	-1	-1	0	0
9/2	11/2	5	-	-3	-1	-1	0	1	-1	0	0
9/2	11/2	6	+	-4	-1	1	1	-1	-1	-1	0
9/2	11/2	6	-	-4	3	0	1	0	-1	-1	0
9/2	1/2	4	-	-1	0	-1	0	0	0	0	0
9/2	1/2	5	+	-1	1	-1	1	-1	0	0	0
9/2	7/2	2	-	-3	-1	2	0	-1	0	0	0
9/2	7/2	3	+	-1	-1	1	0	-1	0	0	0
9/2	7/2	4	-	-1	1	-1	0	-1	-1	0	0
9/2	7/2	5	+	-3	-1	-1	0	1	-1	0	0
9/2	7/2	6	-	0	-1	1	1	-1	-1	0	0
9/2	7/2	6		0							
9/2	9/2	2	-	0	-1	-1	1	-1	0	0	0
9/2	9/2	3	-	-2	-1	2	0	-1	-1	0	0
9/2	9/2	4	+	-2	1	0	1	-1	-1	0	0
9/2	9/2	5	+	2	-1	0	0	-1	-1	0	0
9/2	9/2	6	-	-5	-1	0	0	-1	-1	0	2
9/2	9/2	6	-	-5	3	-1	0	0	-1	0	0

9/211/2	2	-	-2	0	2	0	-1	-1	0	0
9/211/2	3	+	-3	0	1	1	-1	-1	0	0
9/211/2	4	+	-3	-1	-1	0	-1	-1	2	0
9/211/2	5	-	-3	0	-1	3	-1	-1	0	0
9/211/2	6	+	-2	0	1	0	-1	-1	-1	0
9/211/2	6	-	-2	2	0	0	0	-1	-1	0
11/2 1/2	5	+	0	-1	0	0	-1	0	0	0
11/2 1/2	6	+	3	-1	0	0	0	-1	0	0
11/2 1/2	6		0							
11/2 7/2	2	+	-1	-3	2	0	-1	0	0	0
11/2 7/2	3	-	-3	-3	0	1	-1	-1	2	0
11/2 7/2	4	-	-3	-2	0	0	1	-1	0	0
11/2 7/2	5	+	1	-3	0	0	-1	1	0	0
11/2 7/2	6	-	-5	-3	0	0	1	-1	0	0
11/2 7/2	6	+	-5	-1	1	0	0	-1	0	0
11/2 9/2	2	-	0	-3	-1	0	-1	1	0	0
11/2 9/2	3	+	-1	-3	0	-1	-1	-1	0	0
11/2 9/2	4	-	-1	-2	0	0	-1	-1	0	0
11/2 9/2	5	+	-1	-3	0	3	-1	-1	0	0
11/2 9/2	6	-	-4	-3	4	2	-1	-1	-1	0
11/2 9/2	6	+	-4	-1	-1	0	0	-1	-1	0
11/211/2	2	-	4	0	0	0	-1	-1	0	0
11/211/2	3	+	0	1	0	0	-1	-1	0	0
11/211/2	4	-	-1	-2	0	0	-1	-1	0	0
11/211/2	5	+	-3	1	0	0	-1	-1	-1	0
11/211/2	6	+	-6	2	0	0	-1	1	-1	0
11/211/2	6	+	-6	-2	3	2	0	-1	-1	0
11/2 1/2	5	-	-1	-1	1	0	-1	0	0	0
11/2 1/2	6	+	-4	-1	1	0	0	-1	0	0
11/2 1/2	6	+	-4	1	0	0	1	-1	0	0

11/2	7/2	2	+	2	-3	1	0	-1	0	0	0	
11/2	7/2	3	+	2	-3	1	1	-1	-1	0	0	
11/2	7/2	4	+	2	-2	1	0	-1	-1	0	0	
11/2	7/2	5	+	0	-3	3	0	-1	-1	0	0	
11/2	7/2	6	-	-6	-3	5	0	-1	-1	0	0	
11/2	7/2	6	+	-6	-1	0	2	0	-1	0	0	
11/2	9/2	2	+	1	-3	0	0	1	-1	0	0	
11/2	9/2	3	-	0	-3	3	-1	-1	-1	0	0	
11/2	9/2	4	+	0	-2	-1	0	-1	-1	0	0	
11/2	9/2	5	+	-2	-3	-1	1	-1	-1	0	0	
11/2	9/2	6	+	-5	-3	1	0	-1	-1	-1	0	103
11/2	9/2	6	+	-5	-1	0	0	0	-1	-1	0	23
11/2	11/2	2		0								
11/2	11/2	3	+	-1	1	1	0	-1	-1	0	0	
11/2	11/2	4	+	-2	-2	1	2	-1	-1	0	0	
11/2	11/2	5	+	-2	3	1	0	-1	-1	-1	0	
11/2	11/2	6	+	-5	0	1	0	-1	-1	-1	0	23
11/2	11/2	6	-	-5	-2	0	0	0	-1	1	0	

		UD		UD			A1				
3/2	3/2	0	+	0	0	0	0	0	0	0	0
5/2	3/2	4	−	0	−2	0	0	0	0	0	0
5/2	5/2	0	+	1	−1	0	0	0	0	0	0
5/2	5/2	4	+	0	−3	0	0	0	0	0	0
7/2	3/2	4	+	0	−2	0	0	0	0	0	0
7/2	5/2	4	+	1	−1	0	0	−1	0	0	0
7/2	5/2	6	−	0	0	1	0	−1	−1	0	0
7/2	7/2	0	+	−1	0	0	0	0	0	0	0
7/2	7/2	4	+	−1	−3	0	0	−1	0	0	0
7/2	7/2	6	+	4	−1	0	0	−1	−1	0	0
9/2	3/2	4	+	−1	−2	0	0	−1	0	0	0
9/2	3/2	6	−	−1	1	0	1	−1	−1	0	0
9/2	5/2	4	−	−1	−1	0	1	−1	0	0	0
9/2	5/2	6	−	−1	0	0	0	−1	−1	0	0
9/2	7/2	4	+	2	−3	1	1	−1	−1	0	0
9/2	7/2	6	+	−1	−1	−1	0	−1	1	0	0
9/2	9/2	0	+	1	0	−1	0	0	0	0	0
9/2	9/2	4	−	−2	−3	−1	1	−1	−1	2	0
9/2	9/2	6	+	−1	−1	1	0	−1	−1	0	0
9/2	3/2	4	+	−1	−1	0	1	−1	0	0	0
9/2	3/2	6	+	−1	0	0	0	−1	−1	0	0
9/2	5/2	4	−	−1	−2	0	0	−1	0	0	0
9/2	5/2	6	+	−1	1	0	1	−1	−1	0	0
9/2	7/2	4	−	4	−2	1	0	−1	−1	0	0
9/2	7/2	6	+	−1	0	−1	1	−1	−1	0	0
9/2	9/2	0		0							
9/2	9/2	4	+	−2	−2	3	0	−1	−1	0	0

9/2	9/2	6	+	1	0	-1	1	-1	-1	0	0
9/2	9/2	0	+	1	0	-1	0	0	0	0	0
9/2	9/2	4	+	-2	-1	-1	1	-1	-1	0	0
9/2	9/2	6	+	-1	1	-1	0	-1	-1	0	0
11/2	3/2	4	+	0	-2	0	0	-1	0	0	0
11/2	3/2	6	+	1	0	1	0	-1	-1	0	0
11/2	5/2	4	-	0	-3	0	0	-1	1	0	0
11/2	5/2	6	-	2	-1	1	0	-1	-1	0	0
11/2	7/2	4	-	0	1	0	0	-1	-1	0	0
11/2	7/2	6	-	-2	0	0	0	-1	-1	0	0
11/2	9/2	4	-	-1	-1	1	1	-1	-1	0	0
11/2	9/2	6	-	-2	0	-1	1	-1	-1	-1	0
11/2	9/2	4	-	-1	-2	3	0	-1	-1	0	0
11/2	9/2	6	-	-2	5	-1	0	-1	-1	-1	0
11/2	11/2	0	+	0	-1	0	0	0	0	0	0
11/2	11/2	4	-	1	-3	0	2	-1	-1	0	0
11/2	11/2	6	-	2	1	0	0	-1	-1	-1	0
11/2	3/2	4	+	1	-2	1	0	-1	0	0	0
11/2	3/2	6	-	0	0	0	0	-1	-1	0	0
11/2	5/2	4	+	3	-3	1	0	-1	-1	0	0
11/2	5/2	6	-	1	-1	0	0	-1	-1	0	0
11/2	7/2	4	+	1	-1	1	0	-1	-1	0	0
11/2	7/2	6	-	-3	2	1	0	-1	-1	0	0
11/2	9/2	4		0							
11/2	9/2	6		0							
11/2	9/2	4	-	4	-2	0	0	-1	-1	0	0
11/2	9/2	6	+	1	1	2	0	-1	-1	-1	0
11/2	11/2	0		0							
11/2	11/2	4	+	0	-3	1	0	-1	-1	0	0
11/2	11/2	6	-	-1	-1	1	2	-1	-1	-1	0

		UD		UD			E				
3/2	3/2	2	+	1	0	-1	0	0	0	0	0
5/2	3/2	2	-	2	0	-1	-1	0	0	0	0
5/2	3/2	4	-	1	-2	1	-1	0	0	0	0
5/2	5/2	2	+	5	-1	-1	-1	0	0	0	0
5/2	5/2	4	+	1	-3	1	-1	0	0	0	0
5/2	5/2	5		0							
7/2	3/2	2	+	-1	1	0	-1	0	0	0	0
7/2	3/2	4	-	3	-2	-1	-1	0	0	0	0
7/2	3/2	5	-	-1	0	-1	1	-1	0	0	0
7/2	5/2	2	+	0	-2	0	-1	0	0	0	0
7/2	5/2	4	-	4	-1	-1	-1	-1	0	0	0
7/2	5/2	5	-	0	-2	-1	2	-1	0	0	0
7/2	5/2	6	+	1	-2	1	1	-1	-1	0	0
7/2	7/2	2	-	2	-1	-1	-1	0	0	0	0
7/2	7/2	4	+	6	-3	1	-1	-1	0	0	0
7/2	7/2	5		0							
7/2	7/2	6	+	1	-1	0	1	-1	-1	0	0
9/2	3/2	4	+	0	-2	-1	1	-1	0	0	0
9/2	3/2	5	-	3	0	-1	0	-1	0	0	0
9/2	3/2	6	+	0	1	0	0	-1	-1	0	0
9/2	5/2	2	-	0	-2	0	0	0	0	0	0
9/2	5/2	4	+	0	1	-1	0	-1	0	0	0
9/2	5/2	5	+	0	-2	-1	0	1	-1	0	0
9/2	5/2	6	-	0	-2	0	-1	-1	-1	2	0
9/2	7/2	2	-	-1	-1	1	0	-1	0	0	0
9/2	7/2	4	+	7	-3	0	0	-1	-1	0	0
9/2	7/2	5	-	-1	1	0	0	-1	-1	0	0

9/2	7/2	6	+	0	-1	-1	-1	-1	1	0	0
9/2	9/2	2	-	1	-1	-1	0	-1	0	0	0
9/2	9/2	4	+	-1	-3	0	0	-1	-1	2	0
9/2	9/2	5		0							
9/2	9/2	6	+	0	-1	1	1	-1	-1	0	0
9/2	3/2	4	-	0	-1	1	0	-1	0	0	0
9/2	3/2	5		0							
9/2	3/2	6	+	0	0	0	1	-1	-1	0	0
9/2	5/2	2	-	0	-1	0	-1	0	0	0	0
9/2	5/2	4	-	0	-2	1	-1	-1	0	0	0
9/2	5/2	5	+	0	-1	1	1	-1	-1	0	0
9/2	5/2	6	+	0	-1	0	0	-1	-1	0	0
9/2	7/2	2	+	-1	4	-1	-1	-1	0	0	0
9/2	7/2	4	+	7	-2	0	-1	-1	-1	0	0
9/2	7/2	5	+	-1	0	0	1	-1	-1	0	0
9/2	7/2	6	+	0	0	1	0	-1	-1	0	0
9/2	9/2	2		0							
9/2	9/2	4	+	-1	-2	2	1	-1	-1	0	0
9/2	9/2	5	-	1	0	0	0	-1	-1	0	0
9/2	9/2	6	-	2	0	-1	0	-1	-1	0	0
9/2	9/2	2	-	1	1	-1	0	-1	0	0	0
9/2	9/2	4	-	-1	-1	0	0	-1	-1	0	0
9/2	9/2	5		0							
9/2	9/2	6	+	0	1	-1	1	-1	-1	0	0
11/2	3/2	4	-	1	-2	-1	1	-1	0	0	0
11/2	3/2	5	+	1	2	-1	0	-1	-1	0	0
11/2	3/2	6	+	2	0	1	-1	-1	-1	0	0
11/2	5/2	4	+	1	-3	-1	1	-1	1	0	0
11/2	5/2	5	-	2	0	-1	1	-1	-1	0	0
11/2	5/2	6	-	3	-1	1	-1	-1	-1	0	0

11/2	7/2	2	+	-1	-2	-1	1	-1	0	0	0	
11/2	7/2	4		0								
11/2	7/2	5	+	-1	-2	1	1	-1	-1	0	0	
11/2	7/2	6	+	-1	-2	0	-1	1	-1	0	0	
11/2	9/2	2	+	5	-2	1	0	-1	-1	0	0	
11/2	9/2	4	+	0	-1	0	0	-1	-1	0	0	
11/2	9/2	5	+	2	-2	0	1	-1	-1	0	0	
11/2	9/2	6	-	-1	-2	-1	0	-1	1	-1	0	
11/2	9/2	2	+	3	-1	-1	1	-1	-1	0	0	
11/2	9/2	4	-	0	-2	0	1	-1	-1	0	0	
11/2	9/2	5	+	2	-1	0	0	-1	-1	0	0	
11/2	9/2	6	+	-1	-1	1	-1	-1	-1	-1	0	31
11/2	11/2	2	-	1	1	-1	1	-1	-1	0	0	
11/2	11/2	4	-	2	-3	1	1	-1	-1	0	0	
11/2	11/2	5		0								
11/2	11/2	6	+	3	1	0	1	-1	-1	-1	0	
11/2	3/2	4	+	0	-2	0	1	-1	0	0	0	
11/2	3/2	5	+	2	0	0	0	-1	-1	0	0	
11/2	3/2	6	+	3	2	0	-1	-1	-1	0	0	
11/2	5/2	4	+	2	-3	0	1	-1	-1	0	0	
11/2	5/2	5	+	3	0	0	-1	-1	-1	0	0	
11/2	5/2	6	+	4	1	0	-1	-1	-1	0	0	
11/2	7/2	2	+	0	-2	0	1	-1	0	0	0	
11/2	7/2	4	+	2	-1	0	1	-1	-1	0	0	
11/2	7/2	5	+	0	-2	0	-1	1	-1	0	0	
11/2	7/2	6	-	0	-2	1	-1	-1	-1	0	0	
11/2	9/2	2	-	0	-2	2	0	-1	-1	0	0	
11/2	9/2	4	-	1	-1	1	0	-1	-1	0	0	
11/2	9/2	5	+	-1	-2	1	1	-1	-1	0	0	
11/2	9/2	6	+	2	-2	0	0	-1	-1	-1	0	

11/2	9/2	2	+	0	-1	0	1	-1	-1	0	0
11/2	9/2	4	+	1	-2	-1	1	-1	-1	0	0
11/2	9/2	5	-	-1	-1	-1	0	-1	-1	0	2
11/2	9/2	6	+	2	-1	0	-1	-1	-1	-1	0
11/2	11/2	2	-	2	-1	0	-1	-1	-1	0	0
11/2	11/2	4	-	-1	-3	0	-1	1	-1	0	0
11/2	11/2	5	+	3	0	0	1	-1	-1	-1	0
11/2	11/2	6		0							
11/2	11/2	2	+	3	1	1	-1	-1	-1	0	0
11/2	11/2	4	-	2	-3	1	-1	-1	-1	0	0
11/2	11/2	5		0							
11/2	11/2	6	+	5	-1	0	1	-1	-1	-1	0

		UD		UD			A2				
3/2	3/2	3	+	0	0	0	-1	0	0	0	0
5/2	3/2	3	-	0	0	0	-1	0	0	0	0
5/2	5/2	3	+	0	0	0	-1	0	0	0	0
7/2	3/2	3	-	0	0	0	-1	0	0	0	0
7/2	5/2	3	-	1	-2	0	-1	0	0	0	0
7/2	5/2	6	+	0	-2	0	1	0	-1	0	0
7/2	7/2	3	-	-1	1	1	-1	-1	0	0	0
7/2	7/2	6		0							
9/2	3/2	3	-	-1	0	-1	0	0	0	0	0
9/2	3/2	6	+	-1	1	-1	0	0	-1	0	0
9/2	5/2	3	+	-1	-2	-1	2	-1	0	0	0
9/2	5/2	6	-	-1	-2	-1	-1	0	-1	2	0
9/2	7/2	3		0							
9/2	7/2	6	-	-1	1	0	-1	0	-1	0	0
9/2	9/2	3	-	-2	1	0	1	-1	-1	0	0
9/2	9/2	6		0							
9/2	3/2	3	+	-1	1	-1	-1	0	0	0	0
9/2	3/2	6	+	-1	0	-1	1	0	-1	0	0
9/2	5/2	3	+	-1	-1	-1	-1	-1	2	0	0
9/2	5/2	6	+	-1	-1	-1	0	0	-1	0	0
9/2	7/2	3	-	2	0	0	-1	-1	0	0	0
9/2	7/2	6	+	-1	0	0	0	0	-1	0	0
9/2	9/2	3	-	-2	0	2	0	-1	-1	0	0
9/2	9/2	6	-	-1	0	0	0	0	-1	0	0
9/2	9/2	3	+	-2	1	2	-1	-1	-1	0	0
9/2	9/2	6		0							
11/2	3/2	6	+	1	0	0	-1	0	-1	0	0

11/2 5/2	3	−	0	0	0	0	−1	0	0	0
11/2 5/2	6	−	2	−1	0	−1	0	−1	0	0
11/2 7/2	3	−	0	−2	1	0	−1	−1	0	0
11/2 7/2	6	+	−2	−2	3	−1	0	−1	0	0
11/2 9/2	3	−	−1	−2	4	−1	−1	−1	0	0
11/2 9/2	6	+	−2	−2	0	0	0	−1	1	0
11/2 9/2	3	−	−1	−1	0	0	−1	−1	0	0
11/2 9/2	6	+	−2	−1	0	−1	0	−1	1	0
11/211/2	3	+	0	2	1	−1	−1	−1	0	0
11/211/2	6		0							
11/2 3/2	6	−	0	0	1	−1	0	−1	0	0
11/2 5/2	3		0							
11/2 5/2	6	−	1	−1	1	−1	0	−1	0	0
11/2 7/2	3	+	1	−2	0	2	−1	−1	0	0
11/2 7/2	6	+	−3	−2	0	−1	2	−1	0	0
11/2 9/2	3	−	2	−2	3	−1	−1	−1	0	0
11/2 9/2	6	−	3	−2	1	0	0	−1	−1	0
11/2 9/2	3	+	2	−1	1	0	−1	−1	0	0
11/2 9/2	6	+	1	−1	1	−1	0	−1	−1	0
11/211/2	3	−	3	0	0	−1	−1	−1	0	0
11/211/2	6	−	−1	−1	0	1	0	−1	−1	0
11/211/2	3	+	0	0	1	−1	−1	−1	0	0
11/211/2	6		0							

		EDD		ED			T2					
5/2	1/2	2	+	0	-1	0	0	0	0	0	0	
5/2	1/2	3	-	2	-1	0	-1	0	0	0	0	
5/2	7/2	2	+	2	-3	0	0	0	0	0	0	
5/2	7/2	3	-	-2	-3	0	0	0	0	0	0	
5/2	7/2	4	+	-2	-2	1	0	-1	0	0	0	
5/2	7/2	5	-	0	-3	1	1	-1	0	0	0	
5/2	7/2	6	-	-3	-3	0	0	-1	-1	0	0	37
5/2	7/2	6	-	-3	-1	1	0	0	-1	0	0	
5/2	9/2	2	-	1	-3	-1	0	0	0	0	0	
5/2	9/2	3	+	5	-3	0	0	-1	0	0	0	
5/2	9/2	4	+	3	-2	0	0	-1	0	0	0	
5/2	9/2	5	+	5	-3	0	0	-1	-1	0	0	
5/2	9/2	6	-	-3	-3	0	-1	-1	-1	0	0	31
5/2	9/2	6	+	-3	-1	3	-1	0	-1	0	0	
5/2	11/2	3	+	0	1	0	-1	-1	0	0	0	
5/2	11/2	4	-	0	-2	1	0	-1	-1	0	0	
5/2	11/2	5	-	-1	1	1	0	-1	-1	0	0	
5/2	11/2	6	+	-4	0	0	0	-1	1	0	0	
5/2	11/2	6	+	-4	-2	1	0	0	-1	0	0	
7/2	1/2	3	+	-2	1	0	-1	0	0	0	0	
7/2	1/2	4	+	-2	0	0	0	0	0	0	0	
7/2	7/2	2	+	-2	-1	0	0	0	0	0	0	
7/2	7/2	3	+	2	-1	0	0	-1	0	0	0	
7/2	7/2	4	-	0	-1	0	0	-1	0	0	0	
7/2	7/2	5	-	0	-1	0	1	-1	-1	0	0	
7/2	7/2	6	-	-5	-1	0	0	-1	-1	0	2	
7/2	7/2	6	+	-5	1	1	0	0	-1	0	0	

7/2 9/2	2	-	2	-1	-1	0	-1	0	0	0	
7/2 9/2	3	+	-2	-1	0	0	-1	0	0	0	
7/2 9/2	4	-	-2	-1	0	0	-1	1	0	0	
7/2 9/2	5	+	0	-1	0	0	-1	-1	0	0	
7/2 9/2	6	+	-3	-1	0	-1	-1	-1	0	0	
7/2 9/2	6	+	-3	1	-1	-1	0	-1	0	0	
7/211/2	2	-	0	0	0	0	-1	0	0	0	
7/211/2	3	-	0	0	2	-1	-1	-1	0	0	
7/211/2	4	+	0	-1	0	0	-1	-1	0	0	
7/211/2	5	-	2	0	0	0	-1	-1	0	0	
7/211/2	6	+	-6	0	2	0	-1	-1	0	0	
7/211/2	6	+	-6	2	1	0	0	-1	0	0	
11/2 1/2	5	+	-1	-1	0	0	0	0	0	0	
11/2 1/2	6	-	-4	-1	0	0	1	-1	0	0	
11/2 1/2	6	+	-4	1	1	0	0	-1	0	0	
11/2 7/2	2	+	0	-3	0	0	0	0	0	0	
11/2 7/2	3	+	0	-3	0	1	0	-1	0	0	
11/2 7/2	4	+	0	-2	0	0	0	-1	0	0	
11/2 7/2	5	-	2	-3	0	0	0	-1	0	0	
11/2 7/2	6	+	-6	-3	0	0	0	-1	0	0	37
11/2 7/2	6	+	-6	-1	1	0	1	-1	0	0	
11/2 9/2	2	-	7	-3	-1	0	0	-1	0	0	
11/2 9/2	3	+	0	-3	0	-1	2	-1	0	0	
11/2 9/2	4	+	0	-2	0	0	0	-1	0	0	
11/2 9/2	5	+	-2	-3	0	1	0	-1	0	0	
11/2 9/2	6	-	-5	-3	0	0	0	-1	-1	0	79
11/2 9/2	6	+	-5	-1	-1	0	1	-1	-1	0	
11/211/2	2	+	-1	0	0	0	0	-1	0	0	
11/211/2	3		0								
11/211/2	4	+	0	-2	0	0	0	-1	0	0	

11/211/2	5	+	2	1	0	0	0	-1	-1	0
11/211/2	6	+	-5	0	2	0	0	-1	-1	0
11/211/2	6	+	-5	-2	1	0	1	-1	-1	0

	FDU		ED			A2				
5/2 1/2	3	+	0	0	0	-1	0	0	0	0
5/2 7/2	3	-	0	-2	0	0	0	0	0	0
5/2 7/2	6	+	1	-2	0	0	0	-1	0	0
5/2 9/2	3	-	1	-2	0	0	-1	0	0	0
5/2 9/2	6	+	5	-2	0	-1	0	-1	0	0
5/211/2	3	+	2	0	0	-1	-1	0	0	0
5/211/2	6	+	0	-1	0	0	0	-1	0	0
7/2 1/2	3	+	0	0	0	-1	0	0	0	0
7/2 7/2	3	+	-2	0	0	0	-1	0	0	0
7/2 7/2	6	-	-1	0	0	0	0	-1	0	0
7/2 9/2	3	-	0	0	0	0	-1	0	0	0
7/2 9/2	6	+	1	0	0	-1	0	-1	0	0
7/211/2	3	-	0	1	0	-1	-1	-1	0	0
7/211/2	6	-	-4	1	0	0	0	-1	0	0
11/2 1/2	6	+	0	0	0	0	0	-1	0	0
11/2 7/2	3	-	0	-2	0	1	0	-1	0	0
11/2 7/2	6	+	-4	-2	0	0	1	-1	0	0
11/2 9/2	3	-	2	-2	0	-1	0	-1	0	0
11/2 9/2	6	+	-1	-2	0	0	1	-1	-1	0
11/211/2	3	-	-1	0	0	0	0	-1	0	0
11/211/2	6	-	1	-1	0	0	1	-1	-1	0

	EDD		UD			T1				
5/2 3/2	1	+	0	−1	0	0	0	0	0	0
5/2 3/2	3	+	−1	1	0	−1	0	0	0	0
5/2 3/2	4	+	−1	−2	0	0	0	0	0	0
5/2 5/2	1	−	4	−2	0	−1	0	0	0	0
5/2 5/2	3	+	−1	−3	0	−1	0	0	0	0
5/2 5/2	4	−	−1	−1	0	0	0	0	0	0
5/2 5/2	5	+	1	−3	1	−1	−1	0	0	0
5/2 5/2	5	+	1	−1	0	0	−1	0	0	0
5/2 7/2	1	−	0	0	0	−1	0	0	0	0
5/2 7/2	3	+	−2	−1	0	−1	0	0	0	0
5/2 7/2	4	−	−2	−1	0	0	−1	0	0	0
5/2 7/2	5	+	−2	−1	3	−1	−1	0	0	0
5/2 7/2	5	+	−2	−1	0	0	−1	0	0	0
5/2 7/2	6	+	2	−1	0	1	−1	−1	0	0
5/2 9/2	3	+	−2	−1	1	0	−1	0	0	0
5/2 9/2	4	+	−2	−1	0	1	−1	0	0	0
5/2 9/2	5	+	−2	−1	1	1	−1	−1	0	0
5/2 9/2	5	+	−2	−1	0	0	1	−1	0	0
5/2 9/2	6	−	−1	−1	1	−1	−1	−1	0	0
5/2 9/2	3	+	−2	2	1	−1	−1	0	0	0
5/2 9/2	4	+	−2	−2	0	0	−1	0	0	0
5/2 9/2	5	−	−2	2	1	0	−1	−1	0	0
5/2 9/2	5	+	−2	0	0	1	−1	−1	0	0
5/2 9/2	6	+	−1	0	1	0	−1	−1	0	0
5/211/2	3	−	−1	−3	0	2	−1	0	0	0
5/211/2	4	+	−1	1	0	0	−1	−1	0	0
5/211/2	5	−	0	−3	3	0	−1	−1	0	0

5/211/2	5	+	0	-1	0	-1	-1	1	0	0
5/211/2	6	+	0	0	0	1	-1	-1	0	0
7/2 3/2	3	-	-3	0	1	-1	0	0	0	0
7/2 3/2	4	-	-3	1	-1	0	0	0	0	0
7/2 3/2	5	-	-3	0	0	0	-1	0	0	0
7/2 3/2	5	+	-3	2	-1	1	-1	0	0	0
7/2 5/2	1	-	-2	-1	1	-1	0	0	0	0
7/2 5/2	3	-	-2	0	1	-1	0	0	0	0
7/2 5/2	4	+	-2	-2	-1	0	-1	0	0	0
7/2 5/2	5	-	2	0	0	-1	-1	0	0	0
7/2 5/2	5	+	0	0	-1	0	-1	0	0	0
7/2 5/2	6	-	-2	0	1	1	-1	-1	0	0
7/2 7/2	1	-	0	0	0	-1	0	0	0	0
7/2 7/2	3	+	-2	3	0	-1	-1	0	0	0
7/2 7/2	4	-	-2	0	1	0	-1	0	0	0
7/2 7/2	5	+	-4	3	0	-1	-1	-1	0	0
7/2 7/2	5	-	-4	1	1	0	-1	-1	0	0
7/2 7/2	6	+	-2	1	0	1	-1	-1	0	0
7/2 9/2	1	-	-3	0	-1	1	0	0	0	0
7/2 9/2	3	-	-1	1	-1	0	-1	0	0	0
7/2 9/2	4	+	-1	0	0	1	-1	-1	0	0
7/2 9/2	5	+	-3	1	1	1	-1	-1	0	0
7/2 9/2	5	+	-3	1	0	0	-1	-1	0	0
7/2 9/2	6	-	-3	1	-1	-1	-1	1	0	0
7/2 9/2	1	+	-3	1	-1	0	0	0	0	0
7/2 9/2	3	-	1	0	-1	-1	-1	0	0	0
7/2 9/2	4	-	1	1	0	0	-1	-1	0	0
7/2 9/2	5	-	-3	0	1	0	-1	-1	0	0
7/2 9/2	5	-	-3	2	0	1	-1	-1	0	0
7/2 9/2	6	-	-3	2	-1	0	-1	-1	0	0

7/2	11/2	3	+	-3	0	0	2	-1	-1	0	0	
7/2	11/2	4	+	-3	-2	1	0	-1	-1	0	0	
7/2	11/2	5	+	-3	0	0	0	-1	-1	0	0	
7/2	11/2	5	-	-3	0	1	-1	-1	-1	0	0	
7/2	11/2	6	-	-2	0	0	1	-1	-1	0	0	
11/2	3/2	4	-	2	-2	-1	0	0	0	0	0	
11/2	3/2	5	+	-4	1	0	1	0	-1	0	0	
11/2	3/2	5	-	-4	1	-1	0	0	-1	0	0	
11/2	3/2	6	-	-2	1	1	-1	0	-1	0	0	
11/2	5/2	3	-	-2	-3	1	0	0	0	0	0	
11/2	5/2	4	-	-2	-1	-1	0	0	-1	0	0	
11/2	5/2	5	-	-3	-3	0	0	2	-1	0	0	
11/2	5/2	5	-	-3	-1	-1	-1	0	-1	0	0	23
11/2	5/2	6	+	-1	0	1	-1	0	-1	0	0	
11/2	7/2	3	+	-2	-1	0	0	0	-1	0	0	
11/2	7/2	4	-	-2	-1	1	0	0	-1	0	0	
11/2	7/2	5	-	0	-1	0	0	0	-1	0	0	
11/2	7/2	5	-	0	-1	1	-1	0	-1	0	0	
11/2	7/2	6	-	-1	-1	0	-1	0	-1	0	0	
11/2	9/2	1	+	-2	0	-1	0	0	0	0	0	
11/2	9/2	3	+	-3	-1	-1	-1	0	-1	0	0	
11/2	9/2	4	-	-3	-1	0	1	0	-1	0	0	
11/2	9/2	5	-	-3	-1	1	0	0	-1	0	0	
11/2	9/2	5	+	-3	-1	0	1	0	-1	0	0	
11/2	9/2	6	+	-3	-1	-1	0	0	-1	-1	0	
11/2	9/2	1	+	-2	-1	-1	1	0	0	0	0	
11/2	9/2	3	-	-3	2	-1	0	0	-1	0	0	
11/2	9/2	4	-	-3	-2	0	0	0	-1	0	0	
11/2	9/2	5		0								
11/2	9/2	5	+	-1	0	0	0	0	-1	0	0	

11/2	9/2	6	+	-3	0	-1	-1	0	-1	-1	0	59
11/2	11/2	1	+	2	-2	0	0	0	-1	0	0	
11/2	11/2	3	-	0	-3	0	-1	2	-1	0	0	
11/2	11/2	4	-	-1	-1	1	0	0	-1	0	0	
11/2	11/2	5	+	-5	-3	0	0	0	-1	-1	0	73
11/2	11/2	5	-	-5	-1	1	1	0	-1	-1	0	
11/2	11/2	6	+	-3	0	0	1	0	-1	-1	0	

5/2	3/2	2	+	1	0	0	-1	0	0	0	0
5/2	3/2	4	-	2	-2	0	-1	0	0	0	0
5/2	5/2	2	+	0	-1	0	-1	0	0	0	0
5/2	5/2	4	-	4	-3	0	-1	0	0	0	0
5/2	5/2	5	+	0	0	0	0	-1	0	0	0
5/2	7/2	2	-	1	-2	1	-1	0	0	0	0
5/2	7/2	4	-	1	1	0	-1	-1	0	0	0
5/2	7/2	5	-	1	-2	0	0	-1	0	0	0
5/2	7/2	6	+	2	-2	0	1	-1	-1	0	0
5/2	9/2	2	-	-1	-2	-1	0	0	0	0	0
5/2	9/2	4	+	1	-1	0	0	-1	0	0	0
5/2	9/2	5	+	-1	-2	0	0	1	-1	0	0
5/2	9/2	6	-	3	-2	1	-1	-1	-1	0	0
5/2	9/2	2	+	-1	-1	1	-1	0	0	0	0
5/2	9/2	4	-	1	-2	0	-1	-1	0	0	0
5/2	9/2	5	-	-1	-1	0	1	-1	-1	0	0
5/2	9/2	6	-	3	-1	1	0	-1	-1	0	0
5/2	11/2	4	+	2	-3	0	1	-1	-1	0	0
5/2	11/2	5	+	1	2	0	-1	-1	-1	0	0
5/2	11/2	6	+	4	1	0	-1	-1	-1	0	0
7/2	3/2	2	-	-2	0	0	-1	0	0	0	0
7/2	3/2	4	+	0	1	-1	-1	0	0	0	0
7/2	3/2	5	-	-2	1	-1	1	-1	0	0	0
7/2	5/2	2	+	-1	1	0	-1	0	0	0	0
7/2	5/2	4	-	1	-2	-1	-1	1	0	0	0
7/2	5/2	5	+	-1	1	-1	0	-1	0	0	0
7/2	5/2	6		0							

7/2	7/2	2	+	-1	0	-1	-1	0	0	0	0
7/2	7/2	4	+	-1	0	1	-1	-1	0	0	0
7/2	7/2	5	+	-1	0	1	0	-1	-1	0	0
7/2	7/2	6	-	0	0	0	1	-1	-1	0	0
7/2	9/2	2	+	-2	0	1	0	-1	0	0	0
7/2	9/2	4	+	2	0	0	0	-1	-1	0	0
7/2	9/2	5	+	-2	0	0	0	-1	-1	0	0
7/2	9/2	6	-	1	0	-1	-1	-1	1	0	0
7/2	9/2	2	+	-2	3	-1	-1	-1	0	0	0
7/2	9/2	4	+	4	1	0	-1	-1	-1	0	0
7/2	9/2	5	+	-2	1	0	1	-1	-1	0	0
7/2	9/2	6	+	1	1	-1	0	-1	-1	0	0
7/2	11/2	2	-	-2	1	-1	1	-1	0	0	0
7/2	11/2	4	+	0	-2	1	1	-1	-1	0	0
7/2	11/2	5	+	-2	3	1	-1	-1	-1	0	0
7/2	11/2	6	+	4	1	0	-1	-1	-1	0	0
11/2	3/2	4	-	-1	-2	-1	1	0	0	0	0
11/2	3/2	5	-	1	0	-1	0	0	-1	0	0
11/2	3/2	6	+	0	0	1	-1	0	-1	0	0
11/2	5/2	4	-	1	-3	-1	1	0	-1	0	0
11/2	5/2	5	-	2	0	-1	-1	0	-1	0	0
11/2	5/2	6	+	1	-1	1	-1	0	-1	0	0
11/2	7/2	2	+	-1	-2	-1	1	0	0	0	0
11/2	7/2	4		0							
11/2	7/2	5	-	-1	-2	3	-1	0	-1	0	0
11/2	7/2	6	-	-3	-2	0	-1	2	-1	0	0
11/2	9/2	2	-	-1	-2	1	0	0	-1	0	0
11/2	9/2	4	-	0	-1	0	0	0	-1	0	0
11/2	9/2	5	+	-2	-2	0	1	0	-1	0	0
11/2	9/2	6	+	1	-2	-1	0	0	-1	-1	0

11/2	9/2	2	+	-1	-1	-1	1	0	-1	0	0
11/2	9/2	4	+	0	-2	0	1	0	-1	0	0
11/2	9/2	5	+	-2	-1	0	0	0	-1	0	0
11/2	9/2	6	+	9	-1	-1	-1	0	-1	-1	0
11/2	11/2	2	-	1	-1	-1	-1	0	-1	0	0
11/2	11/2	4	-	-2	-3	3	-1	0	-1	0	0
11/2	11/2	5		0							
11/2	11/2	6	-	-1	-1	0	1	0	-1	-1	0

	EDU		UD			T2					
5/2 3/2	2	+	0	−1	0	−1	0	0	0	0	
5/2 3/2	3	−	−1	−1	1	−1	0	0	0	0	
5/2 3/2	4	−	−1	1	0	−1	0	0	0	0	
5/2 5/2	2	+	1	0	0	−1	0	0	0	0	
5/2 5/2	3	+	−1	−1	1	−1	0	0	0	0	
5/2 5/2	4	−	−1	−2	0	−1	0	0	0	0	
5/2 5/2	5	−	1	−1	0	0	−1	0	0	0	
5/2 7/2	2	+	0	−3	1	−1	0	0	0	0	
5/2 7/2	3	−	−2	−3	3	−1	0	0	0	0	
5/2 7/2	4	+	−2	−2	0	−1	−1	2	0	0	
5/2 7/2	5	+	2	−3	0	0	−1	0	0	0	
5/2 7/2	6	+	−3	−3	1	3	−1	−1	0	0	
5/2 7/2	6	−	−3	−1	0	1	0	−1	0	0	
5/2 9/2	2	+	0	−3	1	0	0	0	0	0	
5/2 9/2	3	+	−2	−3	2	0	−1	0	0	0	
5/2 9/2	4	+	−2	−2	0	0	−1	0	0	0	
5/2 9/2	5	+	8	−3	0	0	−1	−1	0	0	
5/2 9/2	6	−	−4	−3	2	−1	−1	−1	0	0	23
5/2 9/2	6	−	−4	−1	1	1	0	−1	0	0	
5/2 9/2	2	−	0	−2	−1	−1	0	0	0	0	
5/2 9/2	3	−	−2	−2	0	−1	−1	0	0	0	
5/2 9/2	4	+	−2	−1	0	−1	1	0	0	0	
5/2 9/2	5	−	2	−2	0	1	−1	−1	0	0	
5/2 9/2	6	+	−4	−2	0	0	1	−1	0	0	
5/2 9/2	6	−	−4	0	1	0	0	−1	0	0	
5/211/2	3	−	−1	−1	1	0	−1	0	0	0	
5/211/2	4	+	−1	0	0	1	−1	−1	0	0	

5/211/2	5	−	0	−1	0	−1	−1	−1	0	0	
5/211/2	6	−	−1	0	1	−1	−1	−1	0	0	
5/211/2	6	+	−1	−2	0	−1	2	−1	0	0	
7/2 3/2	2	+	−1	1	0	−1	0	0	0	0	
7/2 3/2	3	−	−3	2	0	−1	0	0	0	0	
7/2 3/2	4	−	−3	0	1	−1	0	0	0	0	
7/2 3/2	5		0								
7/2 5/2	2	−	−2	0	0	−1	0	0	0	0	
7/2 5/2	3	−	−2	0	0	−1	0	0	0	0	
7/2 5/2	4	+	−2	−1	1	−1	−1	0	0	0	
7/2 5/2	5	+	−2	0	1	0	−1	0	0	0	
7/2 5/2	6	+	1	0	0	1	−1	−1	0	0	
7/2 5/2	6		0								
7/2 7/2	2	−	4	−1	−1	−1	0	0	0	0	
7/2 7/2	3	−	−2	−1	1	−1	−1	0	0	0	
7/2 7/2	4	−	−2	−1	1	−1	−1	0	0	0	
7/2 7/2	5	+	−2	−1	3	0	−1	−1	0	0	
7/2 7/2	6	+	−5	−1	1	1	−1	−1	0	0	
7/2 7/2	6	+	−5	1	0	1	0	−1	0	0	
7/2 9/2	2	−	−3	−1	−1	0	−1	0	0	0	
7/2 9/2	3	−	−1	−1	0	0	−1	0	0	0	
7/2 9/2	4	+	−1	−1	0	0	−1	−1	0	0	
7/2 9/2	5	−	−3	−1	0	0	−1	1	0	0	
7/2 9/2	6	+	−4	−1	0	−1	−1	−1	0	0	43
7/2 9/2	6	+	−4	1	−1	1	0	−1	0	0	
7/2 9/2	2	+	−3	0	−1	−1	−1	0	2	0	
7/2 9/2	3	−	3	0	0	−1	−1	0	0	0	
7/2 9/2	4	+	3	0	0	−1	−1	−1	0	0	
7/2 9/2	5	−	−3	0	0	1	−1	−1	0	0	
7/2 9/2	6	+	−4	0	0	0	−1	−1	0	0	

7/2 9/2	6	-	-4	4	-1	0	0	-1	0	0	
7/2 11/2	2	-	-1	0	-1	1	-1	0	0	0	
7/2 11/2	3	+	-3	0	1	0	-1	-1	0	0	
7/2 11/2	4	+	-3	-1	1	1	-1	-1	0	0	
7/2 11/2	5	-	3	0	1	-1	-1	-1	0	0	
7/2 11/2	6	+	-5	0	1	-1	-1	-1	2	0	
7/2 11/2	6	+	-5	2	0	-1	0	-1	0	0	
11/2 3/2	4		0								
11/2 3/2	5	+	-2	-1	1	0	0	-1	0	0	
11/2 3/2	6	-	-5	-1	0	-1	2	-1	0	0	
11/2 3/2	6	+	-5	1	1	-1	1	-1	0	0	
11/2 5/2	3	+	-2	-1	0	0	0	0	0	0	
11/2 5/2	4	-	-2	-2	1	1	0	-1	0	0	
11/2 5/2	5	-	-1	-1	1	-1	0	-1	0	0	
11/2 5/2	6	+	-4	0	0	1	0	-1	0	0	
11/2 5/2	6	+	-4	0	1	-1	1	-1	0	0	
11/2 7/2	2	+	0	-3	-1	1	0	0	0	0	
11/2 7/2	3	+	-2	-3	1	0	0	-1	0	0	
11/2 7/2	4	+	-2	-2	1	1	0	-1	0	0	
11/2 7/2	5	-	0	-3	1	-1	0	-1	0	0	
11/2 7/2	6	+	-4	-3	1	1	0	-1	0	0	
11/2 7/2	6	-	-4	-1	0	-1	1	-1	0	0	
11/2 9/2	2	-	-2	-3	-1	0	0	-1	0	2	
11/2 9/2	3	-	-3	-3	0	-1	0	-1	0	0	29
11/2 9/2	4	+	-3	-2	2	0	0	-1	0	0	
11/2 9/2	5	-	-3	-3	2	1	0	-1	0	0	
11/2 9/2	6	-	-4	-3	0	0	0	-1	-1	0	41
11/2 9/2	6	+	-4	-1	-1	0	3	-1	-1	0	
11/2 9/2	2	-	-2	-2	-1	1	0	-1	0	0	
11/2 9/2	3	+	-3	-2	0	2	0	-1	0	0	

11/2	9/2	4	+	-3	-1	0	1	0	-1	0	0	
11/2	9/2	5	+	-3	-2	0	0	0	-1	0	0	
11/2	9/2	6	+	-2	-2	0	-1	0	-1	-1	0	23
11/2	9/2	6	+	-2	0	-1	-1	1	-1	-1	0	
11/2	11/2	2	-	4	0	-1	-1	0	-1	0	0	
11/2	11/2	3	+	0	-1	1	-1	0	-1	0	0	
11/2	11/2	4	-	-1	0	1	-1	0	-1	0	0	
11/2	11/2	5	-	-3	-1	1	1	0	-1	-1	0	
11/2	11/2	6	+	-6	0	1	1	0	-1	-1	0	
11/2	11/2	6	-	-6	-2	0	3	1	-1	-1	0	

	EDD		EDD			A1				
5/2 5/2	0	+	0	-1	0	0	0	0	0	0
5/2 5/2	4	-	1	-3	0	0	0	0	0	0
7/2 5/2	4	+	0	-2	1	0	-1	0	0	0
7/2 5/2	6	+	1	1	0	0	-1	-1	0	0
7/2 7/2	0	+	-2	0	0	0	0	0	0	0
7/2 7/2	4	-	-2	1	0	0	-1	0	0	0
7/2 7/2	6	-	-1	1	0	0	-1	-1	0	0
11/2 5/2	4	+	2	-3	1	0	0	-1	0	0
11/2 5/2	6	-	0	-1	0	0	0	-1	0	0
11/2 7/2	4	+	0	-2	0	0	0	-1	0	0
11/2 7/2	6	-	-4	1	0	0	0	-1	0	0
11/211/2	0	+	-1	-1	0	0	0	0	0	0
11/211/2	4	+	-2	-3	0	0	1	-1	0	0
11/211/2	6	+	1	-1	0	0	1	-1	-1	0

EDD FDD T1

5/2	5/2	1	-	0	-2	1	-1	0	0	0	0	
5/2	5/2	3	+	3	-3	1	-1	0	0	0	0	
5/2	5/2	4		0								
5/2	5/2	5	+	-1	-3	0	-1	-1	2	0	0	
5/2	5/2	5	-	-1	-1	1	0	-1	0	0	0	
7/2	5/2	1	+	2	-1	0	-1	0	0	0	0	
7/2	5/2	3	+	-2	0	0	-1	0	0	0	0	
7/2	5/2	4	+	-2	-2	0	2	-1	0	0	0	
7/2	5/2	5	-	-2	0	1	-1	-1	0	0	0	
7/2	5/2	5	-	-2	0	0	0	-1	0	0	0	
7/2	5/2	6	-	0	0	0	1	-1	-1	0	0	
7/2	7/2	1	+	-2	1	0	-1	0	0	0	0	
7/2	7/2	3	-	0	0	0	-1	-1	0	0	0	
7/2	7/2	4		0								
7/2	7/2	5	-	-4	0	0	-1	-1	-1	0	0	43
7/2	7/2	5	-	-4	2	1	0	-1	-1	0	0	
7/2	7/2	6		0								
11/2	5/2	3	-	0	-3	0	0	0	0	0	0	
11/2	5/2	4	-	0	-1	0	0	0	-1	0	0	
11/2	5/2	5	+	-3	-3	1	0	0	-1	0	0	
11/2	5/2	5	-	-3	-1	0	-1	2	-1	0	0	
11/2	5/2	6	-	-1	0	0	-1	0	-1	0	0	
11/2	7/2	3	+	0	0	0	0	0	-1	0	0	
11/2	7/2	4	-	0	-2	1	0	0	-1	0	0	
11/2	7/2	5	-	-2	0	0	0	0	-1	0	0	
11/2	7/2	5	+	-2	0	1	-1	0	-1	0	0	
11/2	7/2	6	-	-3	0	2	-1	0	-1	0	0	

104

11/211/2	1	-	-1	-2	0	0	1	-1	0	0
11/211/2	3	-	1	-3	0	-1	1	-1	0	0
11/211/2	4		0							
11/211/2	5	-	-4	-3	0	0	1	1	-1	0
11/211/2	5	+	-4	-1	1	1	1	-1	-1	0
11/211/2	6		0							

105

								UD				UD	T1									
3/2	3/2	1	+	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
3/2	3/2	3	+	0	1	0	-1	0	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
5/2	3/2	1	+	1	-1	0	0	0	0	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
5/2	3/2	3	+	-1	1	0	-1	0	0	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													+	-1	0	0	0	0	0	0	0	1/ 1
5/2	3/2	4	+	-1	-2	1	0	0	0	0	0		+	-1	2	-1	0	0	0	0	0	1/ 1
													+	-1	0	-1	0	0	0	0	0	1/ 1
5/2	5/2	1	+	1	-2	0	-1	0	1	0	0		-	0	2	0	0	0	-1	0	0	1/ 1
													-	2	0	0	0	0	-1	0	0	1/ 1
5/2	5/2	3	+	3	-3	1	-1	0	0	0	0		-	-1	2	-1	0	0	0	0	0	1/ 1
													+	-1	0	-1	0	0	0	0	0	1/ 1
5/2	5/2	4		0																		
5/2	5/2	5	+	-1	-3	2	-1	-1	0	0	0	37	+	-1	2	-1	0	0	0	0	0	1/ 37
													-	-1	0	-1	0	0	0	0	2	1/ 37
5/2	5/2	5	+	-1	-1	1	0	-1	0	0	0		-	-1	2	-1	0	0	0	0	0	1/ 1
													-	-1	0	-1	0	0	0	0	0	1/ 1
7/2	3/2	3	+	-2	-1	2	-1	0	0	0	0		+	4	0	-2	0	0	0	0	0	1/ 1
													+	0	2	-2	0	0	0	0	0	1/ 1
7/2	3/2	4	+	-2	-2	-1	0	0	0	0	0	29	-	6	0	-1	0	0	0	0	0	1/ 29
													+	0	4	-1	0	0	0	0	0	1/ 29
7/2	3/2	5	+	-2	-1	0	0	-1	0	1	0		+	0	0	-1	2	0	0	-1	0	1/ 1
													-	2	2	-1	0	0	0	-1	0	1/ 1
7/2	3/2	5	+	-2	1	-1	1	-1	0	0	0		+	0	0	-1	0	0	0	0	0	1/ 1
													+	2	0	-1	0	0	0	0	0	1/ 1

7/2	5/2	1	+	-1	0	1	-1	0	0	0	0		-	2	0	-1	0	0	0	0	0	1/ 1
													+	0	0	-1	0	0	0	0	0	1/ 1
7/2	5/2	3	+	-1	-1	1	-1	0	0	0	0		-	0	0	-1	0	0	0	0	0	1/ 1
													-	2	0	-1	0	0	0	0	0	1/ 1
7/2	5/2	4	+	-1	-1	-1	2	-1	0	0	0		+	0	0	-1	0	0	0	0	0	1/ 1
													+	2	0	-1	0	0	0	0	0	1/ 1
7/2	5/2	5	+	-2	-1	0	-1	-1	0	0	0	29	+	-1	0	-1	0	0	0	0	0	1/ 29
													+	-1	0	-1	0	0	0	2	0	1/ 29
7/2	5/2	5	+	-2	-1	-1	0	-1	0	0	0	113	-	-1	0	-1	0	0	0	0	0	31/113
													+	-1	0	-1	0	0	2	0	0	1/113
7/2	5/2	6	+	-1	-1	1	1	-1	-1	0	0		+	0	0	-1	0	0	0	0	0	1/ 1
													+	2	0	-1	0	0	0	0	0	1/ 1
7/2	7/2	1	+	-1	-3	1	-1	0	1	0	0		-	0	0	-2	0	0	-1	0	0	1/ 1
													+	2	4	-2	0	0	-1	0	0	1/ 1
7/2	7/2	3	+	2	-2	2	-1	-1	0	0	0		-	-1	0	-3	0	0	2	0	0	1/ 1
													+	-1	4	-3	0	0	0	0	0	1/ 1
7/2	7/2	4		0																		
7/2	7/2	5	+	-4	-2	0	-1	-1	1	0	0	37	+	-1	0	-1	0	0	0	0	2	1/ 37
													+	-1	2	-1	0	0	0	0	0	1/ 37
7/2	7/2	5	+	-3	0	1	0	-1	-1	0	0	29	-	0	0	-1	0	0	0	0	0	1/ 29
													-	4	2	-1	0	0	0	0	0	1/ 29
7/2	7/2	6		0																		
9/2	3/2	3	+	-2	-1	0	0	0	0	0	0		+	-1	0	-1	0	0	0	0	0	1/ 1
													-	-1	2	-1	0	0	0	0	0	1/ 1
9/2	3/2	4	+	-2	-2	-1	0	-1	0	0	0	397	+	-1	0	-1	0	0	0	0	0	1/397
													-	-1	4	-1	2	0	0	0	0	1/397
9/2	3/2	5	+	-3	-1	0	1	-1	0	0	0		-	-1	0	-1	0	0	0	0	0	1/ 1
													+	-1	2	-1	0	0	0	0	0	1/ 1
9/2	3/2	5	+	-3	1	-1	0	-1	0	1	0		+	-1	0	-1	0	2	0	-1	0	1/ 1
													+	-1	0	-1	2	0	0	-1	0	1/ 1

9/2	3/2	6	+	-1	2	1	0	-1	-1	0	0		-	-1	0	-2	2	0	0	0	0	1/ 1
													+	-1	0	-2	0	0	0	0	0	1/ 1
9/2	5/2	3	+	-1	-1	0	0	-1	1	0	0		+	6	0	-1	0	0	-1	0	0	1/ 1
													+	0	0	-1	0	0	-1	0	0	1/ 1
9/2	5/2	4	+	-1	-1	-1	1	-1	0	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
9/2	5/2	5	+	-2	-1	0	1	-1	-1	0	0	29	+	-1	0	-1	0	0	2	0	0	1/ 29
													+	-1	0	-1	0	2	0	0	0	1/ 29
9/2	5/2	5	+	-2	-1	-1	0	-1	0	0	0	29	-	-1	0	-1	4	0	-1	0	0	1/ 29
													+	-1	0	-1	0	0	-1	0	0	37/ 29
9/2	5/2	6	+	2	0	0	-1	-1	-1	0	0		-	-2	-1	-1	0	0	2	0	0	1/ 1
													+	0	-1	-1	0	0	0	2	0	1/ 1
9/2	7/2	1	+	-2	-3	-1	1	0	0	1	0		+	2	0	-1	0	0	0	-1	0	1/ 1
													+	0	4	-1	0	0	0	-1	0	1/ 1
9/2	7/2	3	+	0	-2	-1	0	-1	0	1	0		+	2	0	-1	0	0	0	-1	0	1/ 1
													+	0	4	-1	0	0	0	-1	0	1/ 1
9/2	7/2	4	+	0	1	0	1	-1	-1	0	0		-	2	0	-1	0	0	0	0	0	1/ 1
													+	0	0	-1	0	0	0	0	0	1/ 1
9/2	7/2	5	+	-3	-2	3	1	-1	-1	0	0		-	-1	0	-3	0	0	2	0	0	1/ 1
													+	-1	4	-3	0	0	0	0	0	1/ 1
9/2	7/2	5	+	-3	0	0	0	-1	0	0	0		+	-1	0	-1	2	0	-1	0	0	1/ 1
													+	-1	4	-1	0	0	-1	0	0	1/ 1
9/2	7/2	6	+	-2	2	2	-1	-1	-1	0	0		+	0	2	-2	0	0	0	0	0	1/ 1
													+	4	0	-2	0	0	0	0	0	1/ 1
9/2	9/2	1	+	1	-3	2	0	-1	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	9/2	3	+	0	-2	2	1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	9/2	4		0																		
9/2	9/2	5	+	-3	-2	-1	0	-1	-1	0	0	97	+	-1	0	-1	0	0	0	0	0	43/ 97

					- -1 6 -1 2 0 0 0 0	1/ 97
9/2	9/2	5	+ -3 0 0 1 -1 -1 0 0		+ -1 0 -1 0 0 0 0 0	1/ 1
					- -1 2 -1 0 0 0 0 0	1/ 1
9/2	9/2	6	0			
9/2	3/2	3	+ -2 0 1 -1 0 0 0 0		+ -1 0 -2 2 0 0 0 0	1/ 1
					- -1 0 -2 0 0 0 0 0	1/ 1
9/2	3/2	4	+ -2 -1 -1 1 -1 0 0 0		+ -1 0 -1 0 0 0 0 0	1/ 1
					- -1 2 -1 0 0 0 0 0	1/ 1
9/2	3/2	5	+ -3 0 0 0 -1 0 1 0		+ -1 0 -1 0 2 0 -1 0	1/ 1
					+ -1 0 -1 2 0 0 -1 0	1/ 1
9/2	3/2	5	+ -3 2 -1 1 -1 0 0 0		+ -1 0 -1 0 0 0 0 0	1/ 1
					- -1 2 -1 0 0 0 0 0	1/ 1
9/2	3/2	6	+ -1 1 0 1 -1 -1 0 0		+ -1 0 -1 0 0 0 0 0	1/ 1
					- -1 2 -1 0 0 0 0 0	1/ 1
9/2	5/2	3	+ -1 0 1 -1 -1 0 0 0		- 4 0 -2 0 0 0 0 0	1/ 1
					- 0 2 -2 0 0 0 0 0	1/ 1
9/2	5/2	4	+ -1 -2 -1 0 -1 0 0 0	233	- 2 4 -1 0 0 0 0 0	1/233
					+ 0 0 -1 0 0 0 0 0	29/233
9/2	5/2	5	+ -2 0 0 0 -1 0 0 0		+ -1 0 -1 2 0 -1 0 0	1/ 1
					+ -1 4 -1 0 0 -1 0 0	1/ 1
9/2	5/2	5	+ -2 0 -1 1 -1 -1 0 0	41	+ -1 0 -1 0 2 0 0 0	1/ 41
					+ -1 0 -1 0 0 0 2 0	1/ 41
9/2	5/2	6	+ 0 0 0 0 -1 0 0 0		- 0 0 -1 2 0 -1 0 0	1/ 1
					- 4 0 -1 0 0 -1 0 0	1/ 1
9/2	7/2	1	+ -2 -2 -1 0 0 0 0 0	41	+ 2 0 -1 2 0 0 0 0	1/ 41
					+ 0 2 -1 0 0 0 0 0	1/ 41
9/2	7/2	3	+ 1 -1 -1 -1 -1 0 0 0	73	- -1 0 -1 0 0 0 0 0	1/ 73
					- -1 6 -1 0 0 0 0 0	1/ 73
9/2	7/2	4	+ 1 0 0 0 -1 -1 0 0		- -1 2 -1 0 0 0 0 0	1/ 1
					- -1 0 -1 0 0 0 [illegible] 0	1/ 1

9/2	7/2	5	+	-3	-1	1	0	-1	0	0	0		-	-1	0	-1	2	0	-1	0	0	1/ 1
													-	-1	4	-1	0	0	-1	0	0	1/ 1
9/2	7/2	5	+	-3	1	0	1	-1	-1	0	0		+	-1	2	-1	0	0	0	0	0	1/ 1
													+	-1	0	-1	0	0	0	0	0	1/ 1
9/2	7/2	6	+	-2	1	-1	0	-1	-1	0	0	61	-	0	0	-1	2	0	0	0	0	1/ 61
													+	8	0	-1	0	0	0	0	0	1/ 61
9/2	9/2	1	+	4	-2	-1	1	-1	0	0	0		+	-1	0	-1	0	0	0	0	0	1/ 1
													-	-1	2	-1	0	0	0	0	0	1/ 1
9/2	9/2	3	+	1	-1	-1	0	-1	-1	0	0		+	-1	0	-1	0	0	0	0	0	1/ 1
													-	-1	2	-1	0	0	0	0	0	1/ 1
9/2	9/2	4	+	0	0	0	0	-1	-1	1	0		+	2	2	-1	0	0	0	-1	0	1/ 1
													+	0	0	-1	2	0	0	-1	0	1/ 1
9/2	9/2	5	+	-3	-1	1	1	-1	-1	0	0		-	-1	0	-1	0	0	0	0	0	1/ 1
													+	-1	2	-1	0	0	0	0	0	1/ 1
9/2	9/2	5	+	-3	1	0	0	-1	0	0	0		-	-1	4	-1	0	0	-1	0	0	1/ 1
													+	-1	0	-1	2	0	-1	0	0	1/ 1
9/2	9/2	6	+	1	1	0	0	-1	-1	0	0		-	-1	0	-2	2	0	0	0	0	1/ 1
													+	-1	0	-2	0	0	0	0	0	1/ 1
9/2	9/2	1	+	1	-1	0	0	-1	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	9/2	3	+	2	1	-1	-1	-1	-1	0	0		+	0	-1	1	0	0	0	0	0	1/ 1
													-	-2	-1	1	2	0	0	0	0	1/ 1
9/2	9/2	4		0																		
9/2	9/2	5	+	-3	0	0	0	-1	1	0	0		+	-1	0	0	0	0	-2	2	0	1/ 1
													-	-1	0	0	2	0	-2	0	0	1/ 1
9/2	9/2	5	+	-3	2	0	1	-1	-1	0	0		-	-1	0	-1	0	0	0	0	0	1/ 1
													+	-1	2	-1	0	0	0	0	0	1/ 1
9/2	9/2	6		0																		
1/2	3/2	4	+	0	-2	-1	0	-1	0	0	0	41	+	2	2	-1	0	0	0	0	0	1/ 41
													-	0	0	-1	0	0	2	0	0	1/ 41

11/2 3/2	5	+ -1 1 0 1 -1 -1 0 0		+ -1 2 -1 0 0 0 0 0	1/ 1
				+ -1 0 -1 0 0 0 0 0	1/ 1
11/2 3/2	5	+ -1 -1 -1 0 -1 -1 0 0	757	- -1 0 -1 0 0 0 0 0	53/757
				+ -1 2 -1 0 0 0 0 0	23/757
11/2 3/2	6	+ 0 -1 2 -1 -1 0 0 0		+ 2 0 0 0 0 -1 0 0	1/ 1
				+ 0 2 0 0 0 -1 0 0	1/ 1
11/2 5/2	3	+ -1 -3 1 0 -1 0 0 0		+ -1 2 -1 0 0 0 0 0	1/ 1
				+ -1 0 -1 0 0 0 0 0	1/ 1
11/2 5/2	4	+ -1 -1 -1 0 -1 -1 0 0	197	- -1 0 -1 0 0 0 2 0	1/197
				+ -1 0 -1 0 0 0 0 0	41/197
11/2 5/2	5	+ 0 -3 0 0 -1 -1 0 2		+ -1 2 -1 0 0 0 0 0	1/ 1
				+ -1 0 -1 0 0 0 0 0	1/ 1
11/2 5/2	5	+ 0 -1 -1 -1 -1 -1 0 0	37	- -1 2 -1 0 0 0 0 0	1/ 37
				+ -1 0 -1 0 0 0 0 2	1/ 37
11/2 5/2	6	+ 1 0 2 -1 -1 -1 0 0		+ 0 2 -2 0 0 0 0 0	1/ 1
				- 4 0 -2 0 0 0 0 0	1/ 1
11/2 7/2	3	+ -2 -1 1 0 -1 -1 0 0	41	+ 10 0 -2 0 0 0 0 0	1/ 41
				- 0 0 -2 0 0 0 0 0	1/ 41
11/2 7/2	4	+ -2 -1 1 0 -1 -1 0 0	37	+ 4 0 -1 0 0 0 0 0	1/ 37
				+ 0 0 -1 0 0 2 0 0	1/ 37
11/2 7/2	5	+ -2 -1 1 0 -1 -1 1 0		- 0 0 -2 0 0 2 -1 0	1/ 1
				- 8 0 -2 0 0 0 -1 0	1/ 1
11/2 7/2	5	+ -2 -1 1 -1 -1 -1 0 0	149	- 0 6 -1 0 0 0 0 0	1/149
				- 4 0 -1 0 0 0 0 0	1/149
11/2 7/2	6	+ -1 -1 3 -1 -1 -1 0 0		+ 0 2 -2 0 0 0 0 0	1/ 1
				- 4 0 -2 0 0 0 0 0	1/ 1
11/2 9/2	1	+ 3 0 -1 0 -1 0 0 0		- -1 2 -1 0 0 0 0 0	1/ 1
				- -1 0 -1 0 0 0 0 0	1/ 1
11/2 9/2	3	+ -2 -1 -1 -1 -1 -1 0 0	641	- -1 0 -1 0 0 0 0 0	37/641
				+ -1 0 -1 0 0 0 0 0	71/641

11/2	9/2	4	+	-2	-1	1	1	-1	-1	0	0		-	-1	0	-2	2	0	0	0	0	1/ 1
													+	-1	0	-2	0	0	0	0	0	1/ 1
11/2	9/2	5	+	-4	-1	1	0	-1	-1	0	0	113	+	-1	0	-1	0	0	0	2	0	1/113
													+	-1	0	-1	0	0	0	0	0	29/113
11/2	9/2	5	+	-4	-1	2	1	-1	-1	0	0		+	-1	4	-3	0	0	0	0	0	1/ 1
													+	-1	0	-3	0	0	2	0	0	1/ 1
11/2	9/2	6	+	-4	-1	0	0	-1	-1	-1	0	37	-	-1	0	-2	0	0	0	0	0	1/ 37
													+	-1	0	-2	0	0	0	0	0	43/ 37
11/2	9/2	1	+	2	-1	-1	1	-1	0	0	0		+	0	0	-1	0	0	0	0	0	1/ 1
													+	2	0	-1	0	0	0	0	0	1/ 1
11/2	9/2	3	+	-2	0	-1	0	-1	-1	0	0	397	-	-1	0	-1	0	0	0	0	0	37/397
													+	-1	2	-1	0	0	0	2	0	1/397
11/2	9/2	4	+	-2	-2	1	0	-1	-1	0	0	53	+	-1	4	0	0	0	0	0	0	1/ 53
													+	-1	0	2	0	0	0	0	0	1/ 53
11/2	9/2	5	+	-4	0	1	1	-1	-1	0	0		-	-1	0	-1	0	0	0	0	0	1/ 1
													+	-1	2	-1	0	0	0	0	0	1/ 1
11/2	9/2	5	+	-4	0	2	0	-1	-1	0	0		-	-1	2	-1	0	0	0	0	0	1/ 1
													+	-1	0	-1	0	0	0	0	0	1/ 1
11/2	9/2	6	+	-4	0	0	-1	-1	-1	-1	0	37	-	-1	0	2	0	0	0	0	0	1/ 37
													+	-1	0	0	2	0	0	0	0	1/ 37
11/2	11/2	1	+	0	-2	1	0	-1	-1	0	0	41	-	0	0	-2	0	0	0	0	0	1/ 41
													-	10	0	-2	0	0	0	0	0	1/ 41
11/2	11/2	3	+	2	-3	1	-1	-1	-1	0	0		+	6	2	-2	0	0	0	0	0	1/ 1
													+	-2	0	-2	0	2	0	0	0	1/ 1
11/2	11/2	4		0																		
11/2	11/2	5	+	0	-1	0	0	-1	0	-1	0	41	-	-1	0	-1	0	0	-1	0	0	1/ 41
													+	-1	0	-1	0	0	-1	0	0	73/ 41
11/2	11/2	5	+	0	-1	1	1	-1	-1	-1	0		+	-1	2	-1	0	0	0	0	0	1/ 1
													+	-1	0	-1	0	0	0	0	0	1/ 1

11/2	3/2	4	+	4	-2	0	0	-1	0	0	0		+	-1	2	-1	0	0	0	0	0	1/ 1
													+	-1	0	-1	0	0	0	0	0	1/ 1
11/2	3/2	5	+	-3	1	1	1	-1	-1	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
11/2	3/2	5	+	-3	-1	0	0	-1	-1	0	0	233	+	2	0	-1	0	0	0	2	0	1/233
													+	0	2	-1	0	0	0	0	0	1/233
11/2	3/2	6	+	-1	-1	1	-1	-1	0	0	0		-	2	0	0	0	0	-1	0	0	1/ 1
													+	0	2	0	0	0	-1	0	0	1/ 1
11/2	5/2	3	+	-1	-3	3	0	-1	0	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
11/2	5/2	4	+	-1	-1	0	0	-1	-1	0	0	37	+	6	0	-1	0	0	0	0	0	1/ 37
													+	0	0	-1	0	2	0	0	0	1/ 37
11/2	5/2	5	+	-2	-3	1	0	-1	-1	0	0	281	-	0	6	-1	0	0	0	0	0	1/281
													+	2	0	-1	0	0	2	0	0	1/281
11/2	5/2	5	+	-2	-1	0	-1	-1	-1	0	0	113	+	0	4	-1	0	0	0	0	0	1/113
													-	2	0	-1	0	2	0	0	0	1/113
11/2	5/2	6	+	0	2	1	-1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
11/2	7/2	3	+	-1	-1	2	0	-1	-1	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
11/2	7/2	4	+	-1	-1	0	0	-1	-1	0	0	29	+	4	2	-1	0	0	0	0	0	1/ 29
													-	0	0	-1	0	0	0	0	0	1/ 29
11/2	7/2	5	+	0	-1	1	0	-1	-1	0	0		-	-1	2	-1	0	0	0	0	0	1/ 1
													-	-1	0	-1	0	0	0	0	0	1/ 1
11/2	7/2	5	+	0	-1	0	-1	-1	-1	0	0	233	+	-1	0	-1	0	0	0	0	0	31/233
													+	-1	0	-1	0	0	0	0	0	37/233
11/2	7/2	6	+	-1	-1	1	-1	-1	-1	0	0	89	+	-1	0	-1	0	0	0	0	0	29/ 89
													-	-1	0	-1	2	0	0	0	0	1/ 89
11/2	9/2	1	+	-1	0	0	0	-1	0	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1

11/2 9/2	4	+	-2	-1	1	1	-1	-1	0	0		-	-1	0	-2	2	0	0	0	0	1/ 1
												+	-1	0	-2	0	0	0	0	0	1/ 1
11/2 9/2	5	+	-4	-1	1	0	-1	-1	0	0	113	+	-1	0	-1	0	0	0	2	0	1/113
												+	-1	0	-1	0	0	0	0	0	29/113
11/2 9/2	5	+	-4	-1	2	1	-1	-1	0	0		+	-1	4	-3	0	0	0	0	0	1/ 1
												+	-1	0	-3	0	0	2	0	0	1/ 1
11/2 9/2	6	+	-4	-1	0	0	-1	-1	-1	0	37	-	-1	0	-2	0	0	0	0	0	1/ 37
												+	-1	0	-2	0	0	0	0	0	43/ 37
11/2 9/2	1	+	2	-1	-1	1	-1	0	0	0		+	0	0	-1	0	0	0	0	0	1/ 1
												+	2	0	-1	0	0	0	0	0	1/ 1
11/2 9/2	3	+	-2	0	-1	0	-1	-1	0	0	397	-	-1	0	-1	0	0	0	0	0	37/397
												+	-1	2	-1	0	0	0	2	0	1/397
11/2 9/2	4	+	-2	-2	1	0	-1	-1	0	0	53	+	-1	4	0	0	0	0	0	0	1/ 53
												+	-1	0	2	0	0	0	0	0	1/ 53
11/2 9/2	5	+	-4	0	1	1	-1	-1	0	0		-	-1	0	-1	0	0	0	0	0	1/ 1
												+	-1	2	-1	0	0	0	0	0	1/ 1
11/2 9/2	5	+	-4	0	2	0	-1	-1	0	0		-	-1	2	-1	0	0	0	0	0	1/ 1
												+	-1	0	-1	0	0	0	0	0	1/ 1
11/2 9/2	6	+	-4	0	0	-1	-1	-1	-1	0	37	-	-1	0	2	0	0	0	0	0	1/ 37
												+	-1	0	0	2	0	0	0	0	1/ 37
11/211/2	1	+	0	-2	1	0	-1	-1	0	0	41	-	0	0	-2	0	0	0	0	0	1/ 41
												-	10	0	-2	0	0	0	0	0	1/ 41
11/211/2	3	+	2	-3	1	-1	-1	-1	0	0		+	6	2	-2	0	0	0	0	0	1/ 1
												+	-2	0	-2	0	2	0	0	0	1/ 1
11/211/2	4		0																		
11/211/2	5	+	0	-1	0	0	-1	0	-1	0	41	-	-1	0	-1	0	0	-1	0	0	1/ 41
												+	-1	0	-1	0	0	-1	0	0	73/ 41
11/211/2	5	+	0	-1	1	1	-1	-1	-1	0		+	-1	2	-1	0	0	0	0	0	1/ 1
												+	-1	0	-1	0	0	0	0	0	1/ 1
11/211/2	6		0																		

11/2	9/2	3	+	-2	-1	0	-1	-1	-1	0	0	761	-	2	4	-1	0	0	0	0	0	1/761
													+	0	0	-1	0	0	0	0	0	59/761
11/2	9/2	4	+	-2	-1	-1	1	-1	-1	1	0		-	2	2	-1	0	0	0	-1	0	1/ 1
													-	0	0	-1	2	0	0	-1	0	1/ 1
11/2	9/2	5	+	-2	-1	0	0	-1	-1	0	0	53	-	0	2	-1	0	0	0	0	0	1/ 53
													-	8	0	-1	0	0	0	0	0	1/ 53
11/2	9/2	5	+	-2	-1	-1	1	-1	-1	0	0	97	+	0	0	-1	0	0	0	0	0	1/ 97
													+	2	0	-1	0	2	0	0	0	1/ 97
11/2	9/2	6	+	-7	-1	0	0	-1	-1	-1	0	1091	+	-1	0	-1	0	0	0	0	0	227/1091
													-	-1	0	-1	0	0	0	0	0	641/1091
11/2	9/2	1	+	-1	-1	0	1	-1	0	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
11/2	9/2	3	+	-2	2	0	0	-1	-1	0	0		-	2	0	-1	0	0	0	0	0	1/ 1
													+	0	0	-1	0	0	0	0	0	1/ 1
11/2	9/2	4	+	-2	-2	-1	0	-1	0	0	1		-	2	4	1	0	0	-1	0	-1	1/ 1
													+	0	0	1	0	0	-1	0	-1	1/ 1
11/2	9/2	5		0																		
11/2	9/2	5	+	0	0	1	0	-1	-1	0	0		-	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
11/2	9/2	6	+	-7	0	0	-1	-1	-1	-1	0	2671	-	-1	0	-1	0	0	0	0	0	937/2671
													-	-1	0	-1	2	0	0	0	0	67/2671
11/2	11/2	1	+	3	-2	1	0	-1	-1	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
11/2	11/2	3	+	1	-3	2	-1	-1	0	0	0		-	2	4	-2	0	0	-1	0	0	1/ 1
													-	0	0	-2	0	0	-1	0	0	1/ 1
11/2	11/2	4	+	0	-1	0	0	-1	-1	0	0	29	-	6	0	-1	0	0	0	0	0	1/ 29
													+	0	4	-1	0	0	0	0	0	1/ 29
11/2	11/2	5	+	-4	-1	1	0	-1	-1	-1	0	197	+	8	0	-1	0	0	0	0	0	1/197
													+	0	6	-1	0	0	0	0	0	1/197
11/2	11/2	5	+	-4	-1	0	1	-1	-1	-1	0	1481	+	2	6	-1	0	0	0	0	0	1/1481

				-	0	0 -1	0	0	0 0 0	67/1481
11/211/2	6	+ -2 0 1 1 -1 -1 0 0		+	2	2 -1	0	0	0 -1 0	1/ 1
				+	0	0 -1	2	0	0 -1 0	1/ 1
11/211/2	1	+ 2 -1 0 0 -1 -1 0 0		+	-2	-1 1	2	0	0 0 0	1/ 1
				+	0	-1 1	0	0	0 0 0	1/ 1
11/211/2	3	+ 3 -3 1 -1 -1 -1 0 0	53	+	-1	4 0	0	0	0 0 0	1/ 53
				+	-1	0 2	0	0	0 0 0	1/ 53
11/211/2	4	0								
11/211/2	5	+ -6 -1 2 0 0 -1 -1 0		+	-1	0 -1	2	-1	2 0 0	1/ 1
				-	-1	0 -1	0	-1	0 0 2	1/ 1
11/211/2	5	+ -3 -1 1 1 -1 -1 -1 0	29	+	0	4 -1	0	0	0 0 0	1/ 29
				+	6	0 -1	0	0	0 0 0	1/ 29
11/211/2	6	0								

								UD				UD	T2									
3/2	3/2	2	+	0	1	-1	0	0	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
3/2	3/2	3	+	0	1	0	-1	0	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
5/2	3/2	2	+	1	-1	-1	-1	0	0	0	0	29	+	2	0	0	0	0	0	0	0	1/ 29
													+	0	0	2	0	0	0	0	0	1/ 29
5/2	3/2	3	+	-1	-1	0	-1	0	1	0	0		+	-1	0	2	0	0	-1	0	0	1/ 1
													-	-1	0	0	0	0	-1	0	0	1/ 1
5/2	3/2	4	+	-1	-1	1	-1	0	0	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													+	-1	0	0	0	0	0	0	0	1/ 1
5/2	5/2	2	+	0	0	-1	-1	0	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
5/2	5/2	3	+	2	-1	0	-1	0	0	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
5/2	5/2	4	+	2	-2	1	-1	0	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
5/2	5/2	5	+	0	-1	1	0	-1	0	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
7/2	3/2	2	+	1	0	0	-1	0	0	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													+	-1	0	0	0	0	0	0	0	1/ 1
7/2	3/2	3	+	-2	-1	1	-1	0	0	0	0		-	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
7/2	3/2	4	+	-2	-1	0	-1	0	0	1	0		-	2	0	-1	0	0	0	-1	0	1/ 1
													+	0	4	-1	0	0	0	-1	0	1/ 1
7/2	3/2	5	+	0	-1	0	1	-1	0	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													+	0	0	-1	0	0	0	0	0	1/ 1
7/2	5/2	2	+	-1	-3	0	-1	0	0	0	0	113	+	6	0	0	0	0	0	0	0	1/113

													+	0	0	0	2	0	0	0	0	1/113
7/2	5/2	3	+	-1	-3	1	-1	0	1	0	0		-	0	0	-1	2	0	-1	0	0	1/ 1
													+	4	0	-1	0	0	-1	0	0	1/ 1
7/2	5/2	4	+	-1	-2	0	-1	-1	0	0	0	181	-	0	0	-1	0	2	0	0	0	1/181
													-	4	0	-1	2	0	0	0	0	1/181
7/2	5/2	5	+	-1	-3	0	0	-1	1	0	0		-	0	0	-1	2	0	-1	0	0	1/ 1
													+	4	0	-1	0	0	-1	0	0	1/ 1
7/2	5/2	6	+	-3	-3	1	1	-1	-1	0	0	29	+	-1	0	-1	0	2	0	0	0	1/ 29
													+	-1	0	-1	0	0	2	0	0	1/ 29
7/2	5/2	6	+	-3	-1	1	1	0	-1	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													+	-1	0	0	0	0	0	0	0	1/ 1
7/2	7/2	2	+	-1	2	-1	-1	0	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
7/2	7/2	3	+	1	0	1	-1	-1	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
7/2	7/2	4	+	1	-2	1	-1	-1	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
7/2	7/2	5	+	1	0	1	0	-1	-1	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
7/2	7/2	6	+	-4	2	1	1	-1	-1	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
7/2	7/2	6	+	-4	0	0	1	0	-1	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	3/2	3	+	-2	-1	-1	0	0	1	0	0		+	-1	0	0	0	0	-1	0	0	1/ 1
													+	-1	0	2	0	0	-1	0	0	1/ 1
9/2	3/2	4	+	-2	-1	-1	1	-1	0	1	0		-	-1	0	2	0	0	0	-1	0	1/ 1
													+	-1	2	0	0	0	0	-1	0	1/ 1
9/2	3/2	5	+	-1	-1	-1	0	-1	0	0	0	37	-	-1	0	2	0	0	0	0	0	1/ 37
													-	-1	0	0	2	0	0	0	0	1/ 37
9/2	3/2	6	+	-4	0	-1	0	-1	-1	0	0	937	-	-1	0	0	0	0	0	0	0	43/937

					- -1 0 2 0 0 0 0 0	1/937
9/2 3/2	6	+ -4 2 0 0 0 -1 0 0			+ -1 0 0 0 0 0 0 0	1/ 1
					- -1 0 0 0 0 0 0 0	1/ 1
9/2 5/2	2	+ -1 -3 1 0 0 0 0 0			+ 0 0 -1 0 0 0 0 0	1/ 1
					+ 2 0 -1 0 0 0 0 0	1/ 1
9/2 5/2	3	+ -1 -3 -1 0 -1 0 0 0	281		- 8 0 0 0 0 0 0 0	1/281
					- 0 0 2 0 0 0 0 0	1/281
9/2 5/2	4	+ -1 -2 -1 0 -1 0 0 0	101		- 2 0 2 0 0 0 0 0	1/101
					+ 0 0 0 0 0 0 0 0	1/101
9/2 5/2	5	+ -1 -3 -1 0 -1 -1 0 0	569		+ 4 0 2 0 0 0 0 0	1/569
					+ 0 0 0 0 0 2 0 0	1/569
9/2 5/2	6	+ -4 -3 -1 -1 -1 -1 0 0	166741		+ -1 0 0 0 0 0 0 0	131/166741
					+ -1 0 2 0 0 0 0 0	137/166741
9/2 5/2	6	+ 2 -1 0 -1 0 -1 0 0			+ -1 0 0 0 0 0 0 0	1/ 1
					- -1 0 0 0 0 0 0 0	1/ 1
9/2 7/2	2	+ -2 0 -1 0 -1 0 1 0			+ 4 0 0 0 0 0 -1 0	1/ 1
					+ 0 0 0 0 0 0 -1 0	1/ 1
9/2 7/2	3	+ 1 0 0 0 -1 0 0 0			+ -1 0 0 0 0 0 0 0	1/ 1
					- -1 0 0 0 0 0 0 0	1/ 1
9/2 7/2	4	+ 1 -2 0 0 -1 -1 1 0			- -1 0 2 0 0 0 -1 0	1/ 1
					- -1 2 0 0 0 0 -1 0	1/ 1
9/2 7/2	5	+ -2 0 0 0 -1 -1 0 0	29		- 0 0 2 0 0 0 0 0	1/ 29
					+ 2 0 0 0 0 0 0 0	1/ 29
9/2 7/2	6	+ -4 0 0 -1 -1 -1 0 0	109		+ -1 0 0 2 0 0 0 0	1/109
					+ -1 0 0 0 0 2 0 0	1/109
9/2 7/2	6	+ -4 0 -1 -1 0 -1 0 0	557		+ -1 0 2 0 0 0 0 0	1/557
					- -1 2 0 0 2 0 0 0	1/557
9/2 9/2	2	+ -2 0 1 0 -1 0 0 0			+ 0 0 0 0 0 0 0 0	1/ 1
					0	
9/2 9/2	3	0				

9/2	9/2	4	+ 0 -2 2 0 -1 -1 0 0		+ 0 0 0 0 0 0 0 0	1/ 1
					0	
9/2	9/2	5	+ -2 2 0 1 -1 -1 0 0		0	
					- 0 0 0 0 0 0 0 0	1/ 1
9/2	9/2	6	+ -5 0 0 1 -1 -1 0 0		+ 0 0 0 0 0 0 0 0	1/ 1
					0	
9/2	9/2	6	+ -5 0 1 1 0 -1 0 0		- 0 0 0 0 0 0 0 0	1/ 1
					0	
9/2	3/2	3	+ -2 0 1 -1 0 0 0 0		+ -1 0 -2 2 0 0 0 0	1/ 1
					- -1 0 -2 0 0 0 0 0	1/ 1
9/2	3/2	4	+ -2 0 -1 0 -1 0 0 0	29	- -1 2 0 0 0 0 0 0	1/ 29
					- -1 0 0 2 0 0 0 0	1/ 29
9/2	3/2	5	+ -1 0 -1 1 -1 0 0 0		+ -1 0 0 0 0 0 0 0	1/ 1
					- -1 0 0 0 0 0 0 0	1/ 1
9/2	3/2	6	+ -4 1 0 1 -1 0 0 0		+ -1 0 -1 2 0 -1 0 0	1/ 1
					+ -1 4 -1 0 0 -1 0 0	1/ 1
9/2	3/2	6	+ -4 1 0 1 0 -1 0 0		+ -1 0 0 0 0 0 0 0	1/ 1
					- -1 0 0 0 0 0 0 0	1/ 1
9/2	5/2	2	+ -1 -2 0 -1 0 0 0 0	29	- 0 0 2 0 0 0 0 0	1/ 29
					+ 2 0 0 0 0 0 0 0	1/ 29
9/2	5/2	3	+ 1 -2 -1 -1 -1 0 0 0	193	+ 4 0 0 0 0 0 0 0	1/193
					+ -2 0 0 0 0 0 0 0	31/193
9/2	5/2	4	+ -1 -1 -1 -1 -1 0 0 0	37	- 2 2 0 0 0 0 0 0	1/ 37
					+ 0 0 0 0 0 0 0 0	1/ 37
9/2	5/2	5	+ -1 -2 -1 1 -1 -1 0 0	137	- 4 0 0 0 0 0 0 0	1/137
					- 0 0 0 0 2 0 0 0	1/137
9/2	5/2	6	+ -4 -2 -1 0 -1 -1 0 0	619	- -1 0 0 0 0 0 0 2	1/619
					+ -1 0 0 0 0 0 0 0	67/619
9/2	5/2	6	+ 0 0 0 0 0 -1 0 0		- -1 0 0 0 0 0 0 0	1/ 1
					+ -1 0 0 0 0 0 0 0	1/ 1

9/2	7/2	2	+	-2	-1	0	-1	-1	0	0	0	61	-	4	0	-1	0	0	0	0	0	1/ 61
													+	0	0	-1	0	0	0	2	0	1/ 61
9/2	7/2	3	+	0	-1	1	-1	-1	0	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
9/2	7/2	4	+	0	-1	0	-1	-1	-1	0	0	389	-	2	0	2	0	0	0	0	0	1/389
													-	0	0	0	0	0	0	2	0	1/389
9/2	7/2	5	+	-2	-1	0	1	-1	-1	0	0	29	+	0	0	2	0	0	0	0	0	1/ 29
													+	2	0	0	0	0	0	0	0	1/ 29
9/2	7/2	6	+	-4	-1	0	0	-1	-1	0	0	457	+	-1	0	0	0	0	0	2	0	1/457
													-	-1	0	4	0	0	0	0	0	1/457
9/2	7/2	6	+	-4	1	-1	0	0	-1	0	0		+	-1	0	0	0	0	0	0	0	1/ 1
													-	-1	0	0	0	0	0	0	0	1/ 1
9/2	9/2	2	+	-2	-1	-1	1	-1	0	1	0		-	0	0	0	0	0	0	-1	0	1/ 1
													-	4	0	0	0	0	0	-1	0	1/ 1
9/2	9/2	3	+	0	-1	0	0	-1	-1	1	0		+	4	0	0	0	0	0	-1	0	1/ 1
													+	0	0	0	0	0	0	-1	0	1/ 1
9/2	9/2	4	+	1	-1	0	1	-1	-1	0	0		+	-1	0	0	0	0	0	0	0	1/ 1
													-	-1	0	0	0	0	0	0	0	1/ 1
9/2	9/2	5	+	-2	-1	0	0	-1	-1	0	0	113	+	6	0	0	0	0	0	0	0	1/113
													-	0	0	0	2	0	0	0	0	1/113
9/2	9/2	6	+	-5	-1	0	0	-1	-1	0	0	173	+	0	0	0	0	0	2	0	0	1/173
													-	2	0	0	0	0	0	0	0	1/173
9/2	9/2	6	+	-5	1	-1	0	0	-1	0	0	29	+	0	0	2	0	0	0	0	0	1/ 29
													-	2	0	0	0	0	0	0	0	1/ 29
9/2	9/2	2	+	-2	0	-1	0	-1	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	9/2	3	+	2	0	2	-1	-1	-1	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
9/2	9/2	4	+	0	2	0	0	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								

9/2	9/2	5	+	-2	0	0	1	-1	-1	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
9/2	9/2	6	+	-5	0	0	3	-1	-1	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	9/2	6	+	-5	2	-1	1	0	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
11/2	3/2	4	+	0	-1	0	1	-1	0	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													+	0	0	-1	0	0	0	0	0	1/ 1
11/2	3/2	5	+	0	-1	2	0	-1	-1	0	0		-	2	0	-3	0	0	0	0	0	1/ 1
													-	0	0	-3	0	2	0	0	0	1/ 1
11/2	3/2	6	+	-2	-1	1	-1	-1	-1	0	0	173	-	-1	0	-1	0	0	0	0	2	1/173
													-	-1	0	-1	0	0	0	0	0	37/173
11/2	3/2	6	+	-2	1	1	-1	0	-1	0	0		+	-1	0	0	0	0	0	0	0	1/ 1
													-	-1	0	0	0	0	0	0	0	1/ 1
11/2	5/2	3	+	-1	-1	1	0	-1	0	0	0		+	-1	2	-1	0	0	0	0	0	1/ 1
													-	-1	0	-1	0	0	0	0	0	1/ 1
11/2	5/2	4	+	-1	-2	0	1	-1	-1	0	0	29	+	-1	0	-1	0	0	2	0	0	1/ 29
													-	-1	0	-1	0	2	0	0	0	1/ 29
11/2	5/2	5	+	2	-1	-1	-1	-1	0	0	0		+	-1	2	0	0	0	-1	0	0	1/ 1
													-	1	0	0	0	0	1	0	0	1/ 1
11/2	5/2	6	+	-1	0	1	-1	-1	0	0	0		+	-1	2	-1	0	0	-1	0	0	1/ 1
													-	-1	0	-1	0	2	-1	0	0	1/ 1
11/2	5/2	6	+	-1	-2	1	-1	0	-1	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													+	-1	0	0	0	0	0	0	0	1/ 1
11/2	7/2	2	+	1	-3	-1	1	-1	1	0	0		+	-1	0	0	0	0	-1	0	0	1/ 1
													-	-1	0	2	0	0	-1	0	0	1/ 1
11/2	7/2	3	+	2	-3	1	0	-1	-1	0	0		+	-2	0	2	0	0	0	0	0	1/ 1
													+	-4	0	0	0	0	2	0	0	1/ 1
11/2	7/2	4	+	-2	-2	2	1	-1	-1	0	0		-	2	0	-1	0	0	0	0	0	1/ 1
													+	0	0	-1	0	0	0	0	0	1/ 1

11/2	7/2	5	+	-3	-3	3	-1	-1	-1	0	0	37	-	-1	0	2	0	0	0	0	0	1/ 37
													+	-1	0	0	2	0	0	0	0	1/ 37
11/2	7/2	6	+	-4	-3	1	-1	-1	-1	0	0	239	-	6	0	0	0	0	0	0	0	1/239
													-	0	0	0	0	0	0	0	0	29/239
11/2	7/2	6	+	-4	-1	0	-1	0	-1	0	0	389	+	0	0	0	0	0	0	2	0	1/389
													-	2	0	2	0	0	0	0	0	1/389
11/2	9/2	2	+	3	-3	-1	0	-1	-1	0	0	241	+	-1	0	0	0	2	0	0	0	1/241
													-	-1	0	0	0	0	0	0	2	1/241
11/2	9/2	3	+	-2	-3	0	-1	-1	-1	0	0	5641	+	-1	0	0	0	0	0	0	0	79/5641
													-	-1	0	0	0	0	0	0	0	71/5641
11/2	9/2	4	+	-2	-2	0	0	-1	0	0	0		+	-1	0	2	0	0	-1	0	0	1/ 1
													-	-1	0	0	0	0	-1	0	0	1/ 1
11/2	9/2	5	+	-2	-3	0	1	-1	-1	0	0	37	-	-1	0	2	0	0	0	0	0	1/ 37
													+	-1	0	0	2	0	0	0	0	1/ 37
11/2	9/2	6	+	-5	-3	0	1	-1	-1	-1	0	12889	-	-1	0	0	1	2	0	0	0	1/12889
													+	-1	0	0	-1	0	0	0	0	523/12889
11/2	9/2	6	+	-5	-1	-1	0	0	-1	-1	0	1327	-	-1	0	2	0	0	0	0	0	1/1327
													+	-1	0	0	0	0	0	0	0	67/1327
11/2	9/2	2	+	2	-2	0	1	-1	-1	0	0		-	0	0	-1	0	0	0	0	0	1/ 1
													+	2	0	-1	0	0	0	0	0	1/ 1
11/2	9/2	3	+	-2	-2	1	0	-1	-1	0	0	41	-	-1	0	-1	0	0	0	2	0	1/ 41
													-	-1	0	-1	0	2	0	0	0	1/ 41
11/2	9/2	4	+	-2	-1	0	1	-1	-1	1	0		+	-1	0	2	0	0	0	-1	0	1/ 1
													+	-1	2	0	0	0	0	-1	0	1/ 1
11/2	9/2	5	+	-2	-2	0	0	-1	-1	0	0	157	-	-1	0	2	0	0	0	0	0	1/157
													-	-1	0	0	0	0	0	2	0	1/157
11/2	9/2	6	+	-5	-2	0	0	-1	-1	-1	0	373	+	-1	0	2	-1	0	0	0	0	71/373
													+	-1	0	0	1	0	0	0	0	37/373
11/2	9/2	6	+	-5	0	-1	-1	0	-1	-1	0	20329	+	-1	0	0	0	0	0	0	0	43/20329
													-	-1	0	0	0	0	0	0	0	197/20329

11/211/2	2	+	0	4	-1	-1	-1	-1	0	0		-	0	0	0	0	0	0	0	0	1/ 1
													0								
11/211/2	3	+	0	-1	3	-1	-1	-1	0	0			0								
												+	0	0	0	0	0	0	0	0	1/ 1
11/211/2	4	+	3	0	1	-1	-1	-1	0	0		-	0	0	0	0	0	0	0	0	1/ 1
													0								
11/211/2	5	+	1	-1	1	1	-1	-1	-1	0			0								
												+	0	0	0	0	0	0	0	0	1/ 1
11/211/2	6	+	-2	0	1	1	-1	-1	-1	0		+	0	0	0	0	0	0	0	0	1/ 1
													0								
11/211/2	6	+	-2	-2	0	1	0	-1	-1	0		+	0	0	0	0	0	0	0	0	1/ 1
													0								
11/2 3/2	4		0																		
11/2 3/2	5	+	-1	-1	3	0	-1	-1	0	0		-	2	0	-1	0	0	0	0	0	1/ 1
												+	0	0	-1	0	0	0	0	0	1/ 1
11/2 3/2	6	+	-4	-1	2	-1	1	-1	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
												-	0	0	-1	0	0	0	0	0	1/ 1
11/2 3/2	6	+	-4	1	0	-1	0	-1	0	0	61	+	2	2	0	0	0	0	0	0	1/ 61
												+	0	0	2	0	0	0	0	0	1/ 61
11/2 5/2	3	+	-1	-1	1	0	-1	0	0	0			0								
												-	0	0	0	0	0	0	0	0	1/ 1
11/2 5/2	4	+	-1	-2	2	1	-1	-1	0	0			0								
												+	0	0	0	0	0	0	0	0	1/ 1
11/2 5/2	5	+	0	-1	2	-1	-1	0	0	0		+	0	2	0	0	0	-1	0	0	1/ 1
												-	2	0	0	0	0	-1	0	0	1/ 1
11/2 5/2	6	+	-3	0	1	-1	-1	-1	0	0	137	-	0	0	0	0	2	0	0	0	1/137
												+	4	0	0	0	0	0	0	0	1/137
11/2 5/2	6	+	-3	-2	0	-1	0	-1	0	0	569	+	0	0	0	0	0	2	0	0	1/569
												+	4	0	2	0	0	0	0	0	1/569
11/2 7/2	2	+	2	-3	0	1	-1	0	0	0		+	-1	0	0	0	0	0	0	0	1/ 1

													+	-1	0	0	0	0	0	0	0	1/ 1
11/2	7/2	3	+	-6	-3	0	0	-1	-1	0	0	1033	+	-1	0	0	0	0	4	0	0	1/1033
													+	-1	0	0	0	0	0	0	0	127/1033
11/2	7/2	4	+	-1	-2	1	1	-1	-1	0	0		+	2	0	-1	0	0	0	0	0	1/ 1
													-	0	0	-1	0	0	0	0	0	1/ 1
11/2	7/2	5	+	-6	-3	0	-1	-1	-1	0	0	21019	+	-1	0	0	2	0	0	0	0	61/21019
													+	-1	0	0	0	0	0	0	0	139/21019
11/2	7/2	6	+	0	-3	0	-1	-1	0	0	0		-	-5	0	0	0	0	-1	0	0	149/ 1
													+	-3	0	0	0	0	-1	0	0	59/ 1
11/2	7/2	6	+	-4	-1	3	-1	0	-1	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													-	-1	0	0	0	0	0	0	0	1/ 1
11/2	9/2	2	+	-1	-3	2	0	-1	-1	1	0		+	6	0	-2	0	0	0	-1	0	1/ 1
													-	0	0	-2	0	0	0	-1	2	1/ 1
11/2	9/2	3	+	0	-3	1	-1	-1	-1	0	0	257	-	0	0	0	0	0	0	0	2	1/257
													+	-2	0	0	2	0	0	0	0	1/257
11/2	9/2	4	+	-2	-2	-1	0	-1	-1	0	0	4157	-	2	0	0	0	0	2	0	0	1/4157
													-	0	0	0	0	0	0	0	0	59/4157
11/2	9/2	5	+	-2	-3	-1	1	-1	-1	0	0	53	+	2	0	0	0	0	0	0	0	1/ 53
													-	0	0	0	2	0	0	0	0	1/ 53
11/2	9/2	6	+	-4	-3	3	0	-1	-1	-1	0	433	-	-1	0	-2	0	0	0	0	0	131/433
													+	-1	0	-2	0	0	0	0	0	67/433
11/2	9/2	6	+	-3	-1	0	0	0	-1	-1	0	41	+	0	0	2	0	0	0	0	0	1/ 41
													-	4	0	0	0	0	0	0	0	1/ 41
11/2	9/2	2	+	-1	-2	1	1	-1	0	0	0		+	6	0	-1	0	0	-1	0	0	1/ 1
													-	0	0	-1	0	0	-1	0	0	1/ 1
11/2	9/2	3	+	-2	-2	1	0	-1	-1	0	0	53	-	2	0	0	0	0	0	0	0	1/ 53
													+	0	0	0	2	0	0	0	0	1/ 53
11/2	9/2	4	+	-2	-1	-1	1	-1	-1	0	0	29	+	2	0	0	0	0	0	0	0	1/ 29
													+	0	0	2	0	0	0	0	0	1/ 29
11/2	9/2	5	+	-2	-2	-1	0	-1	-1	0	0	383	+	10	0	0	0	0	0	0	0	1/383

												+	0	0	2	0	0	0	0	0	1/383
11/2 9/2	6	+	-4	-2	1	-1	-1	-1	-1	0	3541	-	-1	0	0	0	0	0	0	0	29/3541
												+	-1	0	0	0	0	0	0	0	79/3541
11/2 9/2	6	+	-1	0	0	-1	0	-1	0	0		+	4	0	0	0	0	0	-1	0	1/ 1
												+	0	0	0	0	0	0	-1	0	1/ 1
11/211/2	2	+	3	0	0	-1	-1	0	0	0		+	0	2	0	0	0	-1	0	0	1/ 1
												-	2	0	0	0	0	-1	0	0	1/ 1
11/211/2	3	+	1	-1	1	-1	-1	-1	0	0	29	-	4	2	-1	0	0	0	0	0	1/ 29
												+	0	0	-1	0	0	0	0	0	1/ 29
11/211/2	4	+	0	-2	0	-1	-1	-1	0	1		-	8	0	0	0	0	0	0	-1	1/ 1
												+	0	0	0	0	0	2	0	-1	1/ 1
11/211/2	5	+	-2	-1	1	1	-1	-1	-1	0	41	+	2	2	-1	0	0	0	0	0	1/ 41
												-	0	0	-1	0	0	2	0	0	1/ 41
11/211/2	6	+	-5	0	1	1	-1	-1	-1	0	73	-	2	0	-1	0	0	0	0	0	1/ 73
												-	0	0	-1	0	0	0	0	2	1/ 73
11/211/2	6	+	-5	-2	1	1	0	-1	-1	0	37	+	2	2	0	0	0	0	0	0	1/ 37
												+	0	0	0	0	0	0	0	0	1/ 37
11/211/2	2	+	0	0	1	-1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
													0								
11/211/2	3	+	2	-1	1	-1	-1	-1	0	0			0								
												+	0	0	0	0	0	0	0	0	1/ 1
11/211/2	4	+	1	-2	1	-1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
													0								
11/211/2	5	+	1	-1	1	1	-1	-1	-1	0			0								
												+	0	0	0	0	0	0	0	0	1/ 1
11/211/2	6	+	-4	0	1	3	-1	-1	-1	0		+	0	0	0	0	0	0	0	0	1/ 1
													0								
11/211/2	6	+	-4	-2	0	3	0	-1	-1	0		+	0	0	0	0	0	0	0	0	1/ 1
													0								

Table 2.7: The 6j Symbol $\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$ for grey O^*

These are ordered by $j_1 \geq j_2 \geq j_3$, $j_5, j_6 \leq j_2$ and $j_4 \leq j_1$. Each coefficient is given by j_1, j_2, j_3, j_4, j_5, j_6 followed by the sign, the powers of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ and $\sqrt{29}$, with in some cases another integer to multiply the coefficient. These tables were prepared nearly two years ago at the University of New South Wales and have theoretical deficiences in the multiplicity labelling. A computer is not yet available at the author's present address for easy correction and the tables are given as originally computed. Rather than attach a multiplicity lable to a triad (j_1 j_2 j_3) a J label was attached to a single ICR. This is T_1(BT_1) when the multiplicity label is one (two) in the triad ($U'U'T_1$) and T_2(BT_2) when the multiplicity label is one (two) in the triad ($U'U'T_2$). Dashes are also denoted by D. Thus the entry

UD UD BT1 UD UD T1

is the 6j

$$\begin{Bmatrix} U' & U' & T_1 \\ U' & U' & T_1 \end{Bmatrix}_{2111}$$

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A1	A1	A1	A1	A1	A1	+	0	0	0	0	0
T1	T1	A1	A1	A1	T1	+	0	-1	0	0	0
T1	T1	A1	T1	T1	A1	+	0	-2	0	0	0
T1	T1	A1	T1	T1	T1	-	0	-2	0	0	0
T1	T1	T1	A1	T1	T1	-	0	-2	0	0	0
T1	T1	T1	T1	A1	T1	-	0	-2	0	0	0
T1	T1	T1	T1	T1	A1	-	0	-2	0	0	0
T1	T1	T1	T1	T1	T1	+	-2	-2	0	0	0
E	T1	T1	A1	T1	T1	+	0	-2	0	0	0
E	T1	T1	T1	T1	T1	+	-2	-2	0	0	0
E	T1	T1	E	T1	T1	+	0	-2	0	0	0
E	E	A1	A1	A1	E	+	-1	0	0	0	0
E	E	A1	T1	T1	T1	+	-1	-1	0	0	0
E	E	A1	E	E	A1	+	-2	0	0	0	0
E	E	A1	E	E	E	+	-2	0	0	0	0
E	E	E	A1	E	E	+	-2	0	0	0	0
E	E	E	T1	T1	T1	+	-2	-1	0	0	0
E	E	E	E	A1	E	+	-2	0	0	0	0
E	E	E	E	E	A1	+	-2	0	0	0	0
E	E	E	E	E	E		0				
T2	T1	T1	A1	T1	T1	+	0	-2	0	0	0
T2	T1	T1	T1	T1	T1	+	-2	-2	0	0	0
T2	T1	T1	E	T1	T1	-	-2	-2	0	0	0
T2	T1	T1	T2	T1	T1	+	-2	-2	0	0	0
T2	E	T1	A1	T1	E	-	-1	-1	0	0	0
T2	E	T1	T1	T1	T1	-	-2	-1	0	0	0
T2	E	T1	T1	E	T1		0				

T2	E	T1	E	T1	E	+	-2	-1	0	0	0
T2	E	T1	T2	T1	T1	-	-2	-2	0	0	0
T2	E	T1	T2	E	T1	+	0	-2	0	0	0
T2	T2	A1	A1	A1	T2	+	0	-1	0	0	0
T2	T2	A1	T1	T1	T1	+	0	-2	0	0	0
T2	T2	A1	T1	T1	E	-	0	-2	0	0	0
T2	T2	A1	T1	T1	T2	-	0	-2	0	0	0
T2	T2	A1	E	E	T1	-	-1	-1	0	0	0
T2	T2	A1	E	E	T2	+	-1	-1	0	0	0
T2	T2	A1	T2	T2	A1	+	0	-2	0	0	0
T2	T2	A1	T2	T2	T1	-	0	-2	0	0	0
T2	T2	A1	T2	T2	E	+	0	-2	0	0	0
T2	T2	A1	T2	T2	T2	+	0	-2	0	0	0
T2	T2	T1	A1	T1	T2	-	0	-2	0	0	0
T2	T2	T1	T1	A1	T2	-	0	-2	0	0	0
T2	T2	T1	T1	T1	T1	-	-2	-2	0	0	0
T2	T2	T1	T1	T1	E	+	-2	-2	0	0	0
T2	T2	T1	T1	T1	T2	-	-2	-2	0	0	0
T2	T2	T1	T1	E	T1	-	-2	-1	0	0	0
T2	T2	T1	T1	E	T2	-	-2	-2	0	0	0
T2	T2	T1	T1	T2	T1	+	-2	-2	0	0	0
T2	T2	T1	T1	T2	E	+	-2	-1	0	0	0
T2	T2	T1	T1	T2	T2	+	-2	-2	0	0	0
T2	T2	T1	E	T1	T1	-	-2	-1	0	0	0
T2	T2	T1	E	T1	T2	-	-2	-2	0	0	0
T2	T2	T1	E	T2	T1	+	-2	-2	0	0	0
T2	T2	T1	E	T2	T2	-	-2	-1	0	0	0
T2	T2	T1	T2	T1	T1	+	-2	-2	0	0	0
T2	T2	T1	T2	T1	E	+	-2	-1	0	0	0
T2	T2	T1	T2	T1	T2	+	-2	-2	0	0	0

T2	T2	T1	T2	E	T1	+	-2	-2	0	0	0
T2	T2	T1	T2	E	T2	-	-2	-1	0	0	0
T2	T2	T1	T2	T2	A1	-	0	-2	0	0	0
T2	T2	T1	T2	T2	T1	+	-2	-2	0	0	0
T2	T2	T1	T2	T2	E	+	-2	-2	0	0	0
T2	T2	T1	T2	T2	T2	+	-2	-2	0	0	0
T2	T2	E	A1	E	T2	+	-1	-1	0	0	0
T2	T2	E	T1	T1	T1	+	-2	-2	0	0	0
T2	T2	E	T1	T1	E	+	0	-2	0	0	0
T2	T2	E	T1	T1	T2	-	-2	-2	0	0	0
T2	T2	E	T1	T2	T1	+	-2	-1	0	0	0
T2	T2	E	T1	T2	E		0				
T2	T2	E	T1	T2	T2	-	-2	-1	0	0	0
T2	T2	E	E	A1	T2	+	-1	-1	0	0	0
T2	T2	E	E	E	T1	-	-2	-1	0	0	0
T2	T2	E	E	E	T2	-	-2	-1	0	0	0
T2	T2	E	T2	T1	T1	+	-2	-1	0	0	0
T2	T2	E	T2	T1	E		0				
T2	T2	E	T2	T1	T2	-	-2	-1	0	0	0
T2	T2	E	T2	T2	A1	+	0	-2	0	0	0
T2	T2	E	T2	T2	T1	+	-2	-2	0	0	0
T2	T2	E	T2	T2	E	+	0	-2	0	0	0
T2	T2	E	T2	T2	T2	-	-2	-2	0	0	0
T2	T2	T2	A1	T2	T2	+	0	-2	0	0	0
T2	T2	T2	T1	T1	T1	+	-2	-2	0	0	0
T2	T2	T2	T1	T1	E	+	-2	-2	0	0	0
T2	T2	T2	T1	T1	T2	+	-2	-2	0	0	0
T2	T2	T2	T1	E	T1	+	-2	-2	0	0	0
T2	T2	T2	T1	E	T2	-	-2	-1	0	0	0
T2	T2	T2	T1	T2	T1	+	-2	-2	0	0	0

T2	T2	T2	T1	T2	E	-	-2	-1	0	0	0
T2	T2	T2	T1	T2	T2	+	-2	-2	0	0	0
T2	T2	T2	E	T1	T1	+	-2	-2	0	0	0
T2	T2	T2	E	T1	T2	-	-2	-1	0	0	0
T2	T2	T2	E	T2	T1	-	-2	-1	0	0	0
T2	T2	T2	E	T2	T2	-	-2	-2	0	0	0
T2	T2	T2	T2	A1	T2	+	0	-2	0	0	0
T2	T2	T2	T2	T1	T1	+	-2	-2	0	0	0
T2	T2	T2	T2	T1	E	-	-2	-1	0	0	0
T2	T2	T2	T2	T1	T2	+	-2	-2	0	0	0
T2	T2	T2	T2	E	T1	-	-2	-1	0	0	0
T2	T2	T2	T2	E	T2	-	-2	-2	0	0	0
T2	T2	T2	T2	T2	A1	+	0	-2	0	0	0
T2	T2	T2	T2	T2	T1	+	-2	-2	0	0	0
T2	T2	T2	T2	T2	E	-	-2	-2	0	0	0
T2	T2	T2	T2	T2	T2	+	-2	-2	0	0	0
A2	E	E	A1	E	E	-	-2	0	0	0	0
A2	E	E	E	E	E	+	-2	0	0	0	0
A2	E	E	A2	E	E	+	-2	0	0	0	0
A2	T2	T1	A1	T1	T2	+	0	-2	0	0	0
A2	T2	T1	T1	T1	T2	+	0	-2	0	0	0
A2	T2	T1	T1	E	E	-	-1	-1	0	0	0
A2	T2	T1	T1	T2	T1	+	0	-2	0	0	0
A2	T2	T1	E	T1	T2	-	0	-2	0	0	0
A2	T2	T1	E	T2	T1	+	0	-2	0	0	0
A2	T2	T1	T2	T1	T2	+	0	-2	0	0	0
A2	T2	T1	T2	E	E	-	-1	-1	0	0	0
A2	T2	T1	T2	T2	T1	-	0	-2	0	0	0
A2	T2	T1	A2	T2	T1	+	0	-2	0	0	0
A2	A2	A1	A1	A1	A2	+	0	0	0	0	0

A2	A2	A1	T1	T1	T2	+	0	−1	0	0	0
A2	A2	A1	E	E	E	−	−1	0	0	0	0
A2	A2	A1	T2	T2	T1	+	0	−1	0	0	0
A2	A2	A1	A2	A2	A1	+	0	0	0	0	0
ED	ED	A1	A1	A1	ED	+	−1	0	0	0	0
ED	ED	A1	T1	T1	ED	−	−1	−1	0	0	0
ED	ED	A1	ED	ED	A1	−	−2	0	0	0	0
ED	ED	A1	ED	ED	T1	+	−2	0	0	0	0
ED	ED	T1	A1	T1	ED	−	−1	−1	0	0	0
ED	ED	T1	T1	A1	ED	−	−1	−1	0	0	0
ED	ED	T1	T1	T1	ED	+	0	−2	0	0	0
ED	ED	T1	ED	ED	A1	+	−2	0	0	0	0
ED	ED	T1	ED	ED	T1	+	−2	−2	0	0	0
UD	ED	T1	A1	T1	ED	+	−1	−1	0	0	0
UD	ED	T1	T1	T1	ED	+	−2	−2	0	0	0
UD	ED	T1	T1	E	ED	−	−2	−1	0	0	0
UD	ED	T1	T1	T2	ED	−	−2	−1	0	0	0
UD	ED	T1	ED	ED	T1	−	0	−2	0	0	0
UD	ED	T1	UD	ED	T1	−	−4	−2	0	0	0
UD	ED	T1	UD	ED	E	+	−4	0	0	0	0
UD	ED	T1	UD	ED	T2	+	−4	0	0	0	0
UD	ED	E	A1	E	ED	−	−2	0	0	0	0
UD	ED	E	T1	T1	ED	−	−2	−1	0	0	0
UD	ED	E	T1	T2	ED	+	−2	−1	0	0	0
UD	ED	E	UD	ED	T1	+	−4	0	0	0	0
UD	ED	E	UD	ED	E	−	−4	0	0	0	0
UD	ED	E	UD	ED	T2	+	−4	0	0	0	0
UD	ED	T2	A1	T2	ED	−	−1	−1	0	0	0
UD	ED	T2	T1	T1	ED	−	−2	−1	0	0	0
UD	ED	T2	T1	E	ED	+	−2	−1	0	0	0

UD	ED	T2	T1	T2	ED	+	-2	-2	0	0	0
UD	ED	T2	UD	ED	T1	+	-4	0	0	0	0
UD	ED	T2	UD	ED	E	+	-4	0	0	0	0
UD	ED	T2	UD	ED	T2	-	-4	-2	0	0	0
UD	UD	A1	A1	A1	UD	+	-2	0	0	0	0
UD	UD	A1	T1	T1	ED	+	-2	-1	0	0	0
UD	UD	A1	T1	T1	UD	-	-2	-1	0	0	0
UD	UD	A1	T1	BT1	UD		0				
UD	UD	A1	BT1	T1	UD		0				
UD	UD	A1	BT1	BT1	UD	-	-2	-1	0	0	0
UD	UD	A1	E	E	ED	-	-3	0	0	0	0
UD	UD	A1	E	E	UD	+	-3	0	0	0	0
UD	UD	A1	T2	T2	ED	-	-2	-1	0	0	0
UD	UD	A1	T2	T2	UD	+	-2	-1	0	0	0
UD	UD	A1	T2	BT2	UD		0				
UD	UD	A1	BT2	T2	UD		0				
UD	UD	A1	BT2	BT2	UD	-	-2	-1	0	0	0
UD	UD	A1	A2	A2	UD	-	-2	0	0	0	0
UD	UD	A1	ED	ED	T1	-	-3	0	0	0	0
UD	UD	A1	ED	ED	E	+	-3	0	0	0	0
UD	UD	A1	ED	ED	T2	+	-3	0	0	0	0
UD	UD	A1	UD	UD	A1	-	-4	0	0	0	0
UD	UD	A1	UD	UD	T1	+	-4	0	0	0	0
UD	UD	A1	UD	UD	BT1	+	-4	0	0	0	0
UD	UD	A1	UD	UD	E	-	-4	0	0	0	0
UD	UD	A1	UD	UD	T2	-	-4	0	0	0	0
UD	UD	A1	UD	UD	BT2	+	-4	0	0	0	0
UD	UD	A1	UD	UD	A2	+	-4	0	0	0	0
UD	UD	T1	A1	T1	UD	-	-2	-1	0	0	0
UD	UD	T1	A1	BT1	UD		0				

UD	UD	T1	T1	A1	UD	-	-2	-1	0	0	0
UD	UD	T1	T1	T1	ED	-	-3	-2	1	0	0
UD	UD	T1	T1	T1	UD	+	-1	-2	-1	0	0
UD	UD	T1	T1	BT1	UD		0				
UD	UD	T1	T1	E	ED	-	-3	-1	-1	0	0
UD	UD	T1	T1	E	UD	+	1	-1	-2	0	0
UD	UD	T1	T1	T2	ED	-	-3	-1	-1	0	0
UD	UD	T1	T1	T2	UD	+	1	-1	-2	0	0
UD	UD	T1	T1	BT2	UD		0				
UD	UD	T1	BT1	A1	UD		0				
UD	UD	T1	BT1	T1	UD		0				
UD	UD	T1	BT1	BT1	UD	-	-3	0	-1	0	0
UD	UD	T1	BT1	E	UD	+	-3	1	-2	0	0
UD	UD	T1	BT1	T2	UD	-	-1	-1	-2	0	0
UD	UD	T1	BT1	BT2	UD	+	-3	-1	0	0	0
UD	UD	T1	E	T1	ED	-	-3	-1	-1	0	0
UD	UD	T1	E	T1	UD	+	1	-1	-2	0	0
UD	UD	T1	E	BT1	UD	+	-3	1	-2	0	0
UD	UD	T1	E	T2	ED	+	-3	1	-1	0	0
UD	UD	T1	E	T2	UD	-	-1	-1	-1	0	0
UD	UD	T1	E	BT2	UD	-	-3	-1	-1	0	0
UD	UD	T1	T2	T1	ED	-	-3	-1	-1	0	0
UD	UD	T1	T2	T1	UD	+	1	-1	-2	0	0
UD	UD	T1	T2	BT1	UD	-	-1	-1	-2	0	0
UD	UD	T1	T2	E	ED	+	-3	1	-1	0	0
UD	UD	T1	T2	E	UD	-	-1	-1	-1	0	0
UD	UD	T1	T2	T2	ED	+	-3	0	-1	0	0
UD	UD	T1	T2	T2	UD	-	-1	-2	-1	0	0
UD	UD	T1	T2	BT2	UD	+	-1	-2	-1	0	0
UD	UD	T1	T2	A2	UD	-	-2	-1	-1	0	0

UD	UD	T1	BT2	T1	UD		0					
UD	UD	T1	BT2	BT1	UD	+	-3	-1	0	0	0	
UD	UD	T1	BT2	E	UD	-	-3	-1	-1	0	0	
UD	UD	T1	BT2	T2	UD	+	-1	-2	-1	0	0	
UD	UD	T1	BT2	BT2	UD	-	-3	-2	-1	0	0	
UD	UD	T1	BT2	A2	UD	+	0	-1	-1	0	0	
UD	UD	T1	A2	T2	UD	-	-2	-1	-1	0	0	
UD	UD	T1	A2	BT2	UD	+	0	-1	-1	0	0	
UD	UD	T1	ED	ED	T1	+	-3	-2	1	0	0	
UD	UD	T1	ED	ED	E	+	-3	0	-1	0	0	
UD	UD	T1	ED	ED	T2	+	-3	0	-1	0	0	
UD	UD	T1	ED	UD	T1	+	-2	-2	0	0	0	
UD	UD	T1	ED	UD	BT1		0					
UD	UD	T1	ED	UD	E	-	-2	0	-1	0	0	
UD	UD	T1	ED	UD	T2	-	-2	0	-1	0	0	
UD	UD	T1	ED	UD	BT2		0					
UD	UD	T1	UD	ED	T1	+	-2	-2	0	0	0	
UD	UD	T1	UD	ED	BT1		0					
UD	UD	T1	UD	ED	E	-	-2	0	-1	0	0	
UD	UD	T1	UD	ED	T2	-	-2	0	-1	0	0	
UD	UD	T1	UD	ED	BT2		0					
UD	UD	T1	UD	UD	A1	+	-4	0	0	0	0	
UD	UD	T1	UD	UD	T1	-	-4	-2	-2	0	0	11
UD	UD	T1	UD	UD	BT1	+	-4	2	-2	0	0	
UD	UD	T1	UD	UD	E	+	-4	0	-2	0	0	
UD	UD	T1	UD	UD	T2	+	-4	0	-2	0	0	
UD	UD	T1	UD	UD	BT2	+	-4	2	-2	0	0	
UD	UD	T1	UD	UD	A2	+	-4	2	-2	0	0	
UD	UD	BT1	A1	T1	UD		0					
UD	UD	BT1	A1	BT1	UD	-	-2	-1	0	0	0	

UD	UD	BT1	T1	A1	UD		0				
UD	UD	BT1	T1	T1	ED		0				
UD	UD	BT1	T1	T1	UD		0				
UD	UD	BT1	T1	BT1	ED		0				
UD	UD	BT1	T1	BT1	UD	-	-3	0	-1	0	0
UD	UD	BT1	T1	E	ED	+	-3	1	-1	0	0
UD	UD	BT1	T1	E	UD	+	-3	1	-2	0	0
UD	UD	BT1	T1	T2	ED	-	-1	-1	-1	0	0
UD	UD	BT1	T1	T2	UD	-	-1	-1	-2	0	0
UD	UD	BT1	T1	BT2	ED	+	-1	-1	-1	0	0
UD	UD	BT1	T1	BT2	UD	+	-3	-1	0	0	0
UD	UD	BT1	BT1	A1	UD	-	-2	-1	0	0	0
UD	UD	BT1	BT1	T1	ED		0				
UD	UD	BT1	BT1	T1	UD	-	-3	0	-1	0	0
UD	UD	BT1	BT1	BT1	ED		0				
UD	UD	BT1	BT1	BT1	UD	-	-1	-2	-1	0	0
UD	UD	BT1	BT1	E	ED	+	-3	1	-1	0	0
UD	UD	BT1	BT1	E	UD	-	1	-1	-2	0	0
UD	UD	BT1	BT1	T2	ED	-	-1	-1	-1	0	0
UD	UD	BT1	BT1	T2	UD	+	-3	-1	-2	0	0
UD	UD	BT1	BT1	BT2	ED	-	-1	-1	-1	0	0
UD	UD	BT1	BT1	BT2	UD		0				
UD	UD	BT1	E	T1	ED	+	-3	1	-1	0	0
UD	UD	BT1	E	T1	UD	+	-3	1	-2	0	0
UD	UD	BT1	E	BT1	ED	+	-3	1	-1	0	0
UD	UD	BT1	E	BT1	UD	-	1	-1	-2	0	0
UD	UD	BT1	E	T2	ED	+	-3	-1	-1	0	0
UD	UD	BT1	E	T2	UD	+	-3	-1	-1	0	0
UD	UD	BT1	E	BT2	ED	+	-3	-1	-1	0	0
UD	UD	BT1	E	BT2	UD	-	-1	-1	-1	0	0

UD	UD	BT1	T2	T1	ED	-	-1	-1	-1	0	0
UD	UD	BT1	T2	T1	UD	-	-1	-1	-2	0	0
UD	UD	BT1	T2	BT1	ED	-	-1	-1	-1	0	0
UD	UD	BT1	T2	BT1	UD	+	-3	-1	-2	0	0
UD	UD	BT1	T2	E	ED	+	-3	-1	-1	0	0
UD	UD	BT1	T2	E	UD	+	-3	-1	-1	0	0
UD	UD	BT1	T2	T2	ED	-	1	-2	-1	0	0
UD	UD	BT1	T2	T2	UD	-	1	-2	-1	0	0
UD	UD	BT1	T2	BT2	ED	-	1	-2	-1	0	0
UD	UD	BT1	T2	BT2	UD	-	-3	-2	-1	0	0
UD	UD	BT1	T2	A2	UD	-	0	-1	-1	0	0
UD	UD	BT1	BT2	T1	ED	+	-1	-1	-1	0	0
UD	UD	BT1	BT2	T1	UD	+	-3	-1	0	0	0
UD	UD	BT1	BT2	BT1	ED	-	-1	-1	-1	0	0
UD	UD	BT1	BT2	BT1	UD		0				
UD	UD	BT1	BT2	E	ED	+	-3	-1	-1	0	0
UD	UD	BT1	BT2	E	UD	-	-1	-1	-1	0	0
UD	UD	BT1	BT2	T2	ED	-	1	-2	-1	0	0
UD	UD	BT1	BT2	T2	UD	-	-3	-2	-1	0	0
UD	UD	BT1	BT2	BT2	ED	+	1	-2	-1	0	0
UD	UD	BT1	BT2	BT2	UD	-	-1	-2	-1	0	0
UD	UD	BT1	BT2	A2	UD	-	-2	-1	-1	0	0
UD	UD	BT1	A2	T2	UD	-	0	-1	-1	0	0
UD	UD	BT1	A2	BT2	UD	-	-2	-1	-1	0	0
UD	UD	BT1	ED	ED	T1		0				
UD	UD	BT1	ED	ED	BT1		0				
UD	UD	BT1	ED	ED	E	+	-1	0	-1	0	0
UD	UD	BT1	ED	ED	T2	-	1	-2	-1	0	0
UD	UD	BT1	ED	ED	BT2	+	1	-2	-1	0	0
UD	UD	BT1	ED	UD	T1		0				

UD	UD	BT1	ED	UD	BT1	-	-4	0	0	0	0
UD	UD	BT1	ED	UD	E	+	-4	0	-1	0	0
UD	UD	BT1	ED	UD	T2	-	-2	-2	-1	0	0
UD	UD	BT1	ED	UD	BT2	+	-4	-2	1	0	0
UD	UD	BT1	UD	ED	T1		0				
UD	UD	BT1	UD	ED	BT1	-	-4	0	0	0	0
UD	UD	BT1	UD	ED	E	+	-4	0	-1	0	0
UD	UD	BT1	UD	ED	T2	-	-2	-2	-1	0	0
UD	UD	BT1	UD	ED	BT2	+	-4	-2	1	0	0
UD	UD	BT1	UD	UD	A1	+	-4	0	0	0	0
UD	UD	BT1	UD	UD	T1	+	-4	2	-2	0	0
UD	UD	BT1	UD	UD	BT1	-	-2	-2	-2	0	0
UD	UD	BT1	UD	UD	E	+	0	0	-2	0	0
UD	UD	BT1	UD	UD	T2	-	-4	0	-2	0	0
UD	UD	BT1	UD	UD	BT2	+	-2	0	-2	0	0
UD	UD	BT1	UD	UD	A2	-	-4	2	-2	0	0
UD	UD	E	A1	E	UD	+	-3	0	0	0	0
UD	UD	E	T1	T1	ED	+	-3	-1	0	0	0
UD	UD	E	T1	T1	UD	+	1	-1	-2	0	0
UD	UD	E	T1	BT1	UD	+	-3	1	-2	0	0
UD	UD	E	T1	T2	ED	+	-3	-1	0	0	0
UD	UD	E	T1	T2	UD	-	-1	-1	-1	0	0
UD	UD	E	T1	BT2	UD	-	-3	-1	-1	0	0
UD	UD	E	BT1	T1	UD	+	-3	1	-2	0	0
UD	UD	E	BT1	BT1	UD	-	1	-1	-2	0	0
UD	UD	E	BT1	T2	UD	+	-3	-1	-1	0	0
UD	UD	E	BT1	BT2	UD	-	-1	-1	-1	0	0
UD	UD	E	E	A1	UD	+	-3	0	0	0	0
UD	UD	E	E	E	ED	-	-3	0	0	0	0
UD	UD	E	E	E	UD		0				

UD	UD	E	E	A2	UD	+	-3	0	0	0	0
UD	UD	E	T2	T1	ED	+	-3	-1	0	0	0
UD	UD	E	T2	T1	UD	-	-1	-1	-1	0	0
UD	UD	E	T2	BT1	UD	+	-3	-1	-1	0	0
UD	UD	E	T2	T2	ED	-	-3	-1	0	0	0
UD	UD	E	T2	T2	UD		0				
UD	UD	E	T2	BT2	UD	+	-3	-1	0	0	0
UD	UD	E	BT2	T1	UD	-	-3	-1	-1	0	0
UD	UD	E	BT2	BT1	UD	-	-1	-1	-1	0	0
UD	UD	E	BT2	T2	UD	+	-3	-1	0	0	0
UD	UD	E	BT2	BT2	UD		0				
UD	UD	E	A2	E	UD	+	-3	0	0	0	0
UD	UD	E	ED	UD	T1	-	-2	0	-1	0	0
UD	UD	E	ED	UD	BT1	+	-4	0	-1	0	0
UD	UD	E	ED	UD	E	-	-4	0	0	0	0
UD	UD	E	ED	UD	T2		0				
UD	UD	E	ED	UD	BT2	+	-4	0	0	0	0
UD	UD	E	UD	ED	T1	-	-2	0	-1	0	0
UD	UD	E	UD	ED	BT1	+	-4	0	-1	0	0
UD	UD	E	UD	ED	E	-	-4	0	0	0	0
UD	UD	E	UD	ED	T2		0				
UD	UD	E	UD	ED	BT2	+	-4	0	0	0	0
UD	UD	E	UD	UD	A1	-	-4	0	0	0	0
UD	UD	E	UD	UD	T1	+	-4	0	-2	0	0
UD	UD	E	UD	UD	BT1	+	0	0	-2	0	0
UD	UD	E	UD	UD	E		0				
UD	UD	E	UD	UD	T2	+	-4	0	0	0	0
UD	UD	E	UD	UD	BT2		0				
UD	UD	E	UD	UD	A2	-	-4	0	0	0	0
UD	UD	T2	A1	T2	UD	+	-2	-1	0	0	0

UD	UD	T2	A1	BT2	UD		0				
UD	UD	T2	T1	T1	ED	+	-3	-1	0	0	0
UD	UD	T2	T1	T1	UD	+	1	-1	-2	0	0
UD	UD	T2	T1	BT1	UD	-	-1	-1	-2	0	0
UD	UD	T2	T1	E	ED	+	-3	-1	0	0	0
UD	UD	T2	T1	E	UD	-	-1	-1	-1	0	0
UD	UD	T2	T1	T2	ED	+	-3	-2	0	0	0
UD	UD	T2	T1	T2	UD	-	-1	-2	-1	0	0
UD	UD	T2	T1	BT2	UD	+	-1	-2	-1	0	0
UD	UD	T2	T1	A2	UD	-	-2	-1	-1	0	0
UD	UD	T2	BT1	T1	UD	-	-1	-1	-2	0	0
UD	UD	T2	BT1	BT1	UD	+	-3	-1	-2	0	0
UD	UD	T2	BT1	E	UD	+	-3	-1	-1	0	0
UD	UD	T2	BT1	T2	UD	-	1	-2	-1	0	0
UD	UD	T2	BT1	BT2	UD	-	-3	-2	-1	0	0
UD	UD	T2	BT1	A2	UD	-	0	-1	-1	0	0
UD	UD	T2	E	T1	ED	+	-3	-1	0	0	0
UD	UD	T2	E	T1	UD	-	-1	-1	-1	0	0
UD	UD	T2	E	BT1	UD	+	-3	-1	-1	0	0
UD	UD	T2	E	T2	ED	-	-3	-1	0	0	0
UD	UD	T2	E	T2	UD		0				
UD	UD	T2	E	BT2	UD	+	-3	-1	0	0	0
UD	UD	T2	T2	A1	UD	+	-2	-1	0	0	0
UD	UD	T2	T2	T1	ED	+	-3	-2	0	0	0
UD	UD	T2	T2	T1	UD	-	-1	-2	-1	0	0
UD	UD	T2	T2	BT1	UD	-	1	-2	-1	0	0
UD	UD	T2	T2	E	ED	-	-3	-1	0	0	0
UD	UD	T2	T2	E	UD		0				
UD	UD	T2	T2	T2	ED	-	-3	-1	0	0	0
UD	UD	T2	T2	T2	UD		0				

UD	UD	T2	T2	BT2	UD		0				
UD	UD	T2	BT2	A1	UD		0				
UD	UD	T2	BT2	T1	UD	+	-1	-2	-1	0	0
UD	UD	T2	BT2	BT1	UD	-	-3	-2	-1	0	0
UD	UD	T2	BT2	E	UD	+	-3	-1	0	0	0
UD	UD	T2	BT2	T2	UD		0				
UD	UD	T2	BT2	BT2	UD	+	-3	-1	0	0	0
UD	UD	T2	A2	T1	UD	-	-2	-1	-1	0	0
UD	UD	T2	A2	BT1	UD	-	0	-1	-1	0	0
UD	UD	T2	ED	UD	T1	-	-2	0	-1	0	0
UD	UD	T2	ED	UD	BT1	-	-2	-2	-1	0	0
UD	UD	T2	ED	UD	E		0				
UD	UD	T2	ED	UD	T2	-	-2	-2	0	0	0
UD	UD	T2	ED	UD	BT2	-	-2	-2	0	0	0
UD	UD	T2	UD	ED	T1	-	-2	0	-1	0	0
UD	UD	T2	UD	ED	BT1	-	-2	-2	-1	0	0
UD	UD	T2	UD	ED	E		0				
UD	UD	T2	UD	ED	T2	-	-2	-2	0	0	0
UD	UD	T2	UD	ED	BT2	-	-2	-2	0	0	0
UD	UD	T2	UD	UD	A1	-	-4	0	0	0	0
UD	UD	T2	UD	UD	T1	+	-4	0	-2	0	0
UD	UD	T2	UD	UD	BT1	-	-4	0	-2	0	0
UD	UD	T2	UD	UD	E	+	-4	0	0	0	0
UD	UD	T2	UD	UD	T2	+	-4	-2	0	0	0
UD	UD	T2	UD	UD	BT2	+	-4	-2	0	0	0
UD	UD	T2	UD	UD	A2	+	-4	0	0	0	0
UD	UD	BT2	A1	T2	UD		0				
UD	UD	BT2	A1	BT2	UD	-	-2	-1	0	0	0
UD	UD	BT2	T1	T1	ED		0				
UD	UD	BT2	T1	T1	UD		0				

UD	UD	BT2	T1	BT1	ED		0				
UD	UD	BT2	T1	BT1	UD	+	-3	-1	0	0	0
UD	UD	BT2	T1	E	ED	-	-3	-1	0	0	0
UD	UD	BT2	T1	E	UD	-	-3	-1	-1	0	0
UD	UD	BT2	T1	T2	ED	+	-1	-2	0	0	0
UD	UD	BT2	T1	T2	UD	+	-1	-2	-1	0	0
UD	UD	BT2	T1	BT2	ED	+	-1	-2	0	0	0
UD	UD	BT2	T1	BT2	UD	-	-3	-2	-1	0	0
UD	UD	BT2	T1	A2	UD	+	0	-1	-1	0	0
UD	UD	BT2	BT1	T1	ED		0				
UD	UD	BT2	BT1	T1	UD	+	-3	-1	0	0	0
UD	UD	BT2	BT1	BT1	ED		0				
UD	UD	BT2	BT1	BT1	UD		0				
UD	UD	BT2	BT1	E	ED	-	-3	-1	0	0	0
UD	UD	BT2	BT1	E	UD	-	-1	-1	-1	0	0
UD	UD	BT2	BT1	T2	ED	+	-1	-2	0	0	0
UD	UD	BT2	BT1	T2	UD	-	-3	-2	-1	0	0
UD	UD	BT2	BT1	BT2	ED	+	-1	-2	0	0	0
UD	UD	BT2	BT1	BT2	UD	-	-1	-2	-1	0	0
UD	UD	BT2	BT1	A2	UD	-	-2	-1	-1	0	0
UD	UD	BT2	E	T1	ED	-	-3	-1	0	0	0
UD	UD	BT2	E	T1	UD	-	-3	-1	-1	0	0
UD	UD	BT2	E	BT1	ED	-	-3	-1	0	0	0
UD	UD	BT2	E	BT1	UD	-	-1	-1	-1	0	0
UD	UD	BT2	E	T2	ED	+	-3	-1	0	0	0
UD	UD	BT2	E	T2	UD	+	-3	-1	0	0	0
UD	UD	BT2	E	BT2	ED	+	-3	-1	0	0	0
UD	UD	BT2	E	BT2	UD		0				
UD	UD	BT2	T2	A1	UD		0				
UD	UD	BT2	T2	T1	ED	+	-1	-2	0	0	0

UD	UD	BT2	T2	T1	UD	+	-1	-2	-1	0	0
UD	UD	BT2	T2	BT1	ED	+	-1	-2	0	0	0
UD	UD	BT2	T2	BT1	UD	-	-3	-2	-1	0	0
UD	UD	BT2	T2	E	ED	+	-3	-1	0	0	0
UD	UD	BT2	T2	E	UD	+	-3	-1	0	0	0
UD	UD	BT2	T2	T2	ED		0				
UD	UD	BT2	T2	T2	UD		0				
UD	UD	BT2	T2	BT2	ED		0				
UD	UD	BT2	T2	BT2	UD	+	-3	-1	0	0	0
UD	UD	BT2	BT2	A1	UD	-	-2	-1	0	0	0
UD	UD	BT2	BT2	T1	ED	+	-1	-2	0	0	0
UD	UD	BT2	BT2	T1	UD	-	-3	-2	-1	0	0
UD	UD	BT2	BT2	BT1	ED	+	-1	-2	0	0	0
UD	UD	BT2	BT2	BT1	UD	-	-1	-2	-1	0	0
UD	UD	BT2	BT2	E	ED	+	-3	-1	0	0	0
UD	UD	BT2	BT2	E	UD		0				
UD	UD	BT2	BT2	T2	ED		0				
UD	UD	BT2	BT2	T2	UD	+	-3	-1	0	0	0
UD	UD	BT2	BT2	BT2	ED		0				
UD	UD	BT2	BT2	BT2	UD		0				
UD	UD	BT2	A2	T1	UD	+	0	-1	-1	0	0
UD	UD	BT2	A2	BT1	UD	-	-2	-1	-1	0	0
UD	UD	BT2	ED	UD	T1		0				
UD	UD	BT2	ED	UD	BT1	+	-4	-2	1	0	0
UD	UD	BT2	ED	UD	E	+	-4	0	0	0	0
UD	UD	BT2	ED	UD	T2	-	-2	-2	0	0	0
UD	UD	BT2	ED	UD	BT2	+	-4	-2	0	0	0
UD	UD	BT2	UD	ED	T1		0				
UD	UD	BT2	UD	ED	BT1	+	-4	-2	1	0	0
UD	UD	BT2	UD	ED	E	+	-4	0	0	0	0

UD	UD	BT2	UD	ED	T2	-	-2	-2	0	0	0
UD	UD	BT2	UD	ED	BT2	+	-4	-2	0	0	0
UD	UD	BT2	UD	UD	A1	+	-4	0	0	0	0
UD	UD	BT2	UD	UD	T1	+	-4	2	-2	0	0
UD	UD	BT2	UD	UD	BT1	+	-2	0	-2	0	0
UD	UD	BT2	UD	UD	E		0				
UD	UD	BT2	UD	UD	T2	+	-4	-2	0	0	0
UD	UD	BT2	UD	UD	BT2	-	-2	-2	0	0	0
UD	UD	BT2	UD	UD	A2	+	-4	0	0	0	0
UD	UD	A2	A1	A2	UD	-	-2	0	0	0	0
UD	UD	A2	T1	T2	ED	-	-2	-1	0	0	0
UD	UD	A2	T1	T2	UD	-	-2	-1	-1	0	0
UD	UD	A2	T1	BT2	UD	+	0	-1	-1	0	0
UD	UD	A2	BT1	T2	UD	-	0	-1	-1	0	0
UD	UD	A2	BT1	BT2	UD	-	-2	-1	-1	0	0
UD	UD	A2	E	E	ED	+	-3	0	0	0	0
UD	UD	A2	E	E	UD	+	-3	0	0	0	0
UD	UD	A2	T2	T1	ED	-	-2	-1	0	0	0
UD	UD	A2	T2	T1	UD	-	-2	-1	-1	0	0
UD	UD	A2	T2	BT1	UD	-	0	-1	-1	0	0
UD	UD	A2	BT2	T1	UD	+	0	-1	-1	0	0
UD	UD	A2	BT2	BT1	UD	-	-2	-1	-1	0	0
UD	UD	A2	A2	A1	UD	-	-2	0	0	0	0
UD	UD	A2	UD	UD	A1	+	-4	0	0	0	0
UD	UD	A2	UD	UD	T1	+	-4	2	-2	0	0
UD	UD	A2	UD	UD	BT1	-	-4	2	-2	0	0
UD	UD	A2	UD	UD	E	-	-4	0	0	0	0
UD	UD	A2	UD	UD	T2	+	-4	0	0	0	0
UD	UD	A2	UD	UD	BT2	+	-4	0	0	0	0
UD	UD	A2	UD	UD	A2	-	-4	0	0	0	0

BUD	UD	T1	A1	T1	UD	+	-2	-1	0	0	0
BUD	UD	T1	T1	A1	UD		0				
BUD	UD	T1	T1	T1	ED		0				
BUD	UD	T1	T1	T1	UD	+	-3	0	-1	0	0
BUD	UD	T1	T1	BT1	ED		0				
BUD	UD	T1	T1	E	ED	-	-3	1	-1	0	0
BUD	UD	T1	T1	E	UD	-	-3	1	-2	0	0
BUD	UD	T1	T1	T2	ED	+	-1	-1	-1	0	0
BUD	UD	T1	T1	T2	UD	-	-3	-1	-2	0	1
BUD	UD	T1	T1	BT2	ED	+	-1	-1	-1	0	0
BUD	UD	T1	BT1	A1	UD	+	-2	-1	0	0	0
BUD	UD	T1	BT1	T1	ED		0				
BUD	UD	T1	BT1	T1	UD	+	-1	-2	-1	0	0
BUD	UD	T1	BT1	BT1	ED		0				
BUD	UD	T1	BT1	E	ED	-	-3	1	-1	0	0
BUD	UD	T1	BT1	E	UD	+	1	-1	-2	0	0
BUD	UD	T1	BT1	T2	ED	+	-1	-1	-1	0	0
BUD	UD	T1	BT1	T2	UD	+	-1	-1	-2	0	-1
BUD	UD	T1	BT1	BT2	ED	+	-1	-1	-1	0	0
BUD	UD	T1	E	T1	ED	-	-3	1	-1	0	0
BUD	UD	T1	E	T1	UD	+	1	-1	-2	0	0
BUD	UD	T1	E	BT1	ED	-	-3	1	-1	0	0
BUD	UD	T1	E	T2	ED	-	-3	-1	-1	0	0
BUD	UD	T1	E	T2	UD	-	3	-1	-1	0	-1
BUD	UD	T1	E	BT2	ED	-	-3	-1	-1	0	0
BUD	UD	T1	T2	T1	ED	+	-1	-1	-1	0	0
BUD	UD	T1	T2	T1	UD	-	-3	-1	-2	0	0
BUD	UD	T1	T2	BT1	ED	+	-1	-1	-1	0	0
BUD	UD	T1	T2	E	ED	-	-3	-1	-1	0	0
BUD	UD	T1	T2	E	UD	-	-3	-1	-1	0	0

BUD	UD	T1	T2	T2	ED	+	1	-2	-1	0	0	
BUD	UD	T1	T2	T2	UD	-	-3	-2	-1	0	-1	13
BUD	UD	T1	T2	BT2	ED	+	1	-2	-1	0	0	
BUD	UD	T1	T2	A2	UD	+	0	-1	-1	0	0	
BUD	UD	T1	BT2	T1	ED	+	-1	-1	-1	0	0	
BUD	UD	T1	BT2	T1	UD		0					
BUD	UD	T1	BT2	BT1	ED	+	-1	-1	-1	0	0	
BUD	UD	T1	BT2	E	ED	-	-3	-1	-1	0	0	
BUD	UD	T1	BT2	E	UD	+	-1	-1	-1	0	0	
BUD	UD	T1	BT2	T2	ED	+	1	-2	-1	0	0	
BUD	UD	T1	BT2	T2	UD	+	3	-2	-1	0	-1	
BUD	UD	T1	BT2	BT2	ED	-	1	-2	-1	0	0	
BUD	UD	T1	BT2	A2	UD	+	-2	-1	-1	0	0	
BUD	UD	T1	A2	T2	UD	+	-2	-1	-1	0	-1	
BUD	UD	T1	ED	ED	T1		0					
BUD	UD	T1	ED	ED	BT1		0					
BUD	UD	T1	ED	ED	E	+	-1	0	-1	0	0	
BUD	UD	T1	ED	ED	T2	-	1	-2	-1	0	0	
BUD	UD	T1	ED	ED	BT2	+	1	-2	-1	0	0	
BUD	UD	T1	ED	UD	T1	-	-4	0	0	0	0	
BUD	UD	T1	ED	UD	E	+	-4	0	-1	0	0	
BUD	UD	T1	ED	UD	T2	+	-4	-2	-1	0	1	
BUD	UD	T1	UD	ED	T1		0					
BUD	UD	T1	UD	ED	BT1	-	-4	0	0	0	0	
BUD	UD	T1	UD	ED	E	+	-4	0	-1	0	0	
BUD	UD	T1	UD	ED	T2	-	-2	-2	-1	0	0	
BUD	UD	T1	UD	ED	BT2	+	-4	-2	1	0	0	
BUD	UD	T1	UD	UD	A1		0					
BUD	UD	T1	UD	UD	T1	-	-2	0	-2	0	0	
BUD	UD	T1	UD	UD	E	+	-2	0	-2	0	0	

BUD	UD	T1	UD	UD	T2	+	-2	-2	-2	0	1	
BUD	UD	T1	UD	UD	A2	-	0	0	-2	0	0	
BUD	UD	T1	BUD	ED	T1	-	-4	0	0	0	0	
BUD	UD	T1	BUD	ED	E	+	-4	0	-1	0	0	
BUD	UD	T1	BUD	ED	T2	-	-4	0	-1	2	-1	
BUD	UD	T1	BUD	UD	A1	+	-4	0	0	0	0	
BUD	UD	T1	BUD	UD	T1	-	-2	-2	-2	0	0	
BUD	UD	T1	BUD	UD	E	+	0	0	-2	0	0	
BUD	UD	T1	BUD	UD	T2	+	-2	0	-2	0	-2	23
BUD	UD	T1	BUD	UD	A2	-	-4	2	-2	0	0	
BUD	UD	T2	A1	T2	UD	-	-2	-1	0	0	0	
BUD	UD	T2	T1	T1	ED	+	-1	-1	0	0	-1	
BUD	UD	T2	T1	T1	UD	-	-3	-1	-2	0	1	
BUD	UD	T2	T1	BT1	ED	+	-1	-1	0	0	-1	
BUD	UD	T2	T1	E	ED	+	-3	-1	0	2	-1	
BUD	UD	T2	T1	E	UD	+	-3	-1	-1	0	-1	
BUD	UD	T2	T1	T2	ED	-	3	-2	0	0	-1	
BUD	UD	T2	T1	T2	UD	+	-3	-2	-1	0	-2	23
BUD	UD	T2	T1	BT2	ED	+	3	-2	0	0	-1	
BUD	UD	T2	T1	A2	UD	+	4	-1	-1	0	-1	
BUD	UD	T2	BT1	T1	ED	+	-1	-1	0	0	-1	
BUD	UD	T2	BT1	T1	UD	+	-1	-1	-2	0	-1	
BUD	UD	T2	BT1	BT1	ED	+	-1	-1	0	0	-1	
BUD	UD	T2	BT1	E	ED	+	-3	-1	0	2	-1	
BUD	UD	T2	BT1	E	UD	+	1	1	-1	0	-1	
BUD	UD	T2	BT1	T2	ED	-	3	-2	0	0	-1	
BUD	UD	T2	BT1	T2	UD	-	-1	4	-1	0	-2	
BUD	UD	T2	BT1	BT2	ED	+	3	-2	0	0	-1	
BUD	UD	T2	BT1	A2	UD	-	-2	3	-1	0	-1	
BUD	UD	T2	E	T1	ED	-	-3	1	0	0	-1	

BUD	UD	T2	E	T1	UD	-	3	-1	-1	0	-1	
BUD	UD	T2	E	BT1	ED	-	-3	1	0	0	-1	
BUD	UD	T2	E	T2	ED	-	-3	-1	0	2	-1	
BUD	UD	T2	E	T2	UD	+	1	-1	2	0	-2	
BUD	UD	T2	E	BT2	ED	+	-3	-1	0	2	-1	
BUD	UD	T2	T2	A1	UD	+	0	-1	0	0	-1	
BUD	UD	T2	T2	T1	ED	+	1	0	0	0	-1	
BUD	UD	T2	T2	T1	UD	-	-3	-2	-1	0	-1	13
BUD	UD	T2	T2	BT1	ED	+	1	0	0	0	-1	
BUD	UD	T2	T2	E	ED	+	-3	1	0	0	-1	
BUD	UD	T2	T2	E	UD	+	-3	-1	2	0	-1	
BUD	UD	T2	T2	T2	ED	-	-1	-1	0	0	-1	
BUD	UD	T2	T2	T2	UD	+	-3	-1	4	0	-2	
BUD	UD	T2	T2	BT2	ED	-	-1	-1	0	0	-1	
BUD	UD	T2	BT2	A1	UD	+	-2	-1	2	0	-1	
BUD	UD	T2	BT2	T1	ED	-	1	0	0	0	-1	
BUD	UD	T2	BT2	T1	UD	+	3	-2	-1	0	-1	
BUD	UD	T2	BT2	BT1	ED	-	1	0	0	0	-1	
BUD	UD	T2	BT2	E	ED	-	-3	1	0	0	-1	
BUD	UD	T2	BT2	E	UD	+	-1	-1	0	0	-1	
BUD	UD	T2	BT2	T2	ED	-	-1	-1	0	0	-1	
BUD	UD	T2	BT2	T2	UD		0					
BUD	UD	T2	BT2	BT2	ED	+	-1	-1	0	0	-1	
BUD	UD	T2	A2	T1	UD	+	-2	-1	-1	0	-1	
BUD	UD	T2	ED	UD	T1	+	-4	-2	-1	0	1	
BUD	UD	T2	ED	UD	E	+	-4	0	2	0	-1	
BUD	UD	T2	ED	UD	T2	+	-4	0	0	0	-2	19
BUD	UD	T2	UD	ED	T1	+	0	0	-1	0	-1	
BUD	UD	T2	UD	ED	BT1	-	-4	0	-1	2	-1	
BUD	UD	T2	UD	ED	E	-	-4	0	2	0	-1	

BUD	UD	T2	UD	ED	T2	+	-2	-2	0	2	-1	
BUD	UD	T2	UD	ED	BT2	-	-4	-2	0	0	-1	
BUD	UD	T2	UD	UD	A1	+	-2	0	0	0	-1	
BUD	UD	T2	UD	UD	T1	+	-2	-2	-2	0	1	
BUD	UD	T2	UD	UD	E	-	-2	0	0	0	-1	
BUD	UD	T2	UD	UD	T2	-	-2	-2	0	0	-2	23
BUD	UD	T2	UD	UD	A2	-	-2	0	0	0	-1	
BUD	UD	T2	BUD	ED	T1	-	-4	0	-1	2	-1	
BUD	UD	T2	BUD	ED	E	-	-4	0	2	0	-1	
BUD	UD	T2	BUD	ED	T2	-	-4	-2	0	0	-2	23
BUD	UD	T2	BUD	UD	A1	+	-4	2	0	2	-2	
BUD	UD	T2	BUD	UD	T1	+	-2	0	-2	0	-2	23
BUD	UD	T2	BUD	UD	E	+	0	0	0	0	-2	
BUD	UD	T2	BUD	UD	T2	-	-2	-2	0	2	-4	131
BUD	UD	T2	BUD	UD	A2	+	-4	0	0	0	0	
EDD	ED	T2	A1	T2	ED	+	-1	-1	0	0	0	
EDD	ED	T2	T1	T2	ED	+	0	-2	0	0	0	
EDD	ED	T2	T1	A2	ED	-	-1	-1	0	0	0	
EDD	ED	T2	UD	ED	T2	-	0	-2	0	0	0	
EDD	ED	T2	EDD	ED	T2	+	-2	-2	0	0	0	
EDD	ED	T2	EDD	ED	A2	+	-2	0	0	0	0	
EDD	ED	A2	A1	A2	ED	-	-1	0	0	0	0	
EDD	ED	A2	T1	T2	ED	-	-1	-1	0	0	0	
EDD	ED	A2	EDD	ED	T2	+	-2	0	0	0	0	
EDD	ED	A2	EDD	ED	A2	-	-2	0	0	0	0	
EDD	UD	T1	A1	T1	UD	+	-2	-1	0	0	0	
EDD	UD	T1	T1	T1	UD	+	-3	0	-1	0	0	
EDD	UD	T1	T1	E	UD	-	-3	1	-1	0	0	
EDD	UD	T1	T1	T2	ED	+	-2	-1	0	0	0	
EDD	UD	T1	T1	T2	UD	-	-3	-1	-1	0	0	

EDD	UD	T1	BT1	T1	UD	-	1	-2	-1	0	0
EDD	UD	T1	BT1	BT1	UD	-	1	-2	-1	0	0
EDD	UD	T1	BT1	E	UD	-	-3	-1	-1	0	0
EDD	UD	T1	BT1	T2	UD	-	-1	-1	-1	0	0
EDD	UD	T1	BT1	BT2	UD	-	-1	-1	-1	0	0
EDD	UD	T1	E	T1	UD	-	-3	-1	0	0	0
EDD	UD	T1	E	T2	ED	+	-2	-1	0	0	0
EDD	UD	T1	E	T2	UD	-	-3	-1	0	0	0
EDD	UD	T1	T2	T1	UD	+	-3	-1	0	0	0
EDD	UD	T1	T2	E	UD	-	-3	-1	0	0	0
EDD	UD	T1	T2	T2	ED	-	-2	-2	0	0	0
EDD	UD	T1	T2	T2	UD	+	-3	-2	0	0	0
EDD	UD	T1	T2	A2	ED	-	-1	-1	0	0	0
EDD	UD	T1	BT2	T1	UD		0				
EDD	UD	T1	BT2	BT1	UD		0				
EDD	UD	T1	BT2	E	UD	-	-3	-1	0	0	0
EDD	UD	T1	BT2	T2	UD	-	-1	-2	0	0	0
EDD	UD	T1	BT2	BT2	UD	+	-1	-2	0	0	0
EDD	UD	T1	A2	T2	UD	-	-2	-1	0	0	0
EDD	UD	T1	ED	ED	T2	-	0	-2	0	0	0
EDD	UD	T1	ED	UD	T1	-	-4	0	0	0	0
EDD	UD	T1	ED	UD	E	+	-4	0	0	0	0
EDD	UD	T1	ED	UD	T2	+	-4	-2	0	0	0
EDD	UD	T1	UD	ED	T2	-	-3	-2	0	0	0
EDD	UD	T1	UD	ED	BT2	-	-1	-2	0	0	0
EDD	UD	T1	UD	ED	A2	+	-3	0	0	0	0
EDD	UD	T1	UD	UD	T1	-	-2	0	-2	0	0
EDD	UD	T1	UD	UD	BT1	-	0	0	-2	0	0
EDD	UD	T1	UD	UD	E	+	-2	0	-1	0	0
EDD	UD	T1	UD	UD	T2	+	-2	-2	-1	0	0

EDD	UD	T1	UD	UD	BT2	+	0	-2	-1	0	0
EDD	UD	T1	BUD	ED	T2	+	1	0	0	0	-1
EDD	UD	T1	BUD	UD	T1	+	-4	-2	-2	0	0
EDD	UD	T1	BUD	UD	E	-	-4	0	-1	0	0
EDD	UD	T1	BUD	UD	T2	-	-4	0	-1	0	-1
EDD	UD	T1	EDD	UD	T1	-	-4	-2	0	0	0
EDD	UD	T1	EDD	UD	E	+	-4	0	0	0	0
EDD	UD	T1	EDD	UD	T2	+	-4	0	0	0	0
EDD	UD	E	A1	E	UD	-	-3	0	0	0	0
EDD	UD	E	T1	T1	UD	-	-3	1	-1	0	0
EDD	UD	E	T1	T2	ED	-	-2	-1	0	0	0
EDD	UD	E	T1	T2	UD	+	-3	-1	-1	0	0
EDD	UD	E	BT1	T1	UD	-	-3	-1	-1	0	0
EDD	UD	E	BT1	BT1	UD	-	-3	-1	-1	0	0
EDD	UD	E	BT1	T2	UD	-	-3	1	-1	0	0
EDD	UD	E	BT1	BT2	UD	+	-3	1	-1	0	0
EDD	UD	E	E	E	UD	+	-3	0	0	0	0
EDD	UD	E	E	A2	ED	+	-2	0	0	0	0
EDD	UD	E	T2	T1	UD	-	-3	-1	0	0	0
EDD	UD	E	T2	T2	ED	-	-2	-1	0	0	0
EDD	UD	E	T2	T2	UD	+	-3	-1	0	0	0
EDD	UD	E	BT2	T1	UD	-	-3	-1	0	0	0
EDD	UD	E	BT2	BT1	UD	-	-3	-1	0	0	0
EDD	UD	E	BT2	T2	UD	+	-3	-1	0	0	0
EDD	UD	E	BT2	BT2	UD	-	-3	-1	0	0	0
EDD	UD	E	A2	E	UD	+	-3	0	0	0	0
EDD	UD	E	ED	UD	T1	+	-4	0	0	0	0
EDD	UD	E	ED	UD	E	+	-4	0	0	0	0
EDD	UD	E	ED	UD	T2	+	-4	0	0	0	0
EDD	UD	E	UD	ED	T2	+	-3	0	0	0	0

EDD	UD	E	UD	ED	BT2		0				
EDD	UD	E	UD	ED	A2	+	-3	0	0	0	0
EDD	UD	E	UD	UD	T1	+	-2	0	-1	0	0
EDD	UD	E	UD	UD	BT1	-	-4	0	-1	0	0
EDD	UD	E	UD	UD	E	-	-4	0	0	0	0
EDD	UD	E	UD	UD	T2		0				
EDD	UD	E	UD	UD	BT2	+	-4	0	0	0	0
EDD	UD	E	BUD	ED	T2	-	-1	0	0	0	-1
EDD	UD	E	BUD	UD	T1	-	-4	0	-1	0	0
EDD	UD	E	BUD	UD	T2	-	-4	0	2	0	-1
EDD	UD	E	EDD	UD	T1	+	-4	0	0	0	0
EDD	UD	E	EDD	UD	E	-	-4	0	0	0	0
EDD	UD	E	EDD	UD	T2	+	-4	0	0	0	0
EDD	UD	T2	A1	T2	UD	-	-2	-1	0	0	0
EDD	UD	T2	T1	T1	UD	-	-3	-1	-1	0	0
EDD	UD	T2	T1	E	UD	+	-3	-1	-1	0	0
EDD	UD	T2	T1	T2	ED	-	-2	-2	0	0	0
EDD	UD	T2	T1	T2	UD	-	-3	-2	1	0	0
EDD	UD	T2	T1	A2	ED	-	-1	-1	0	0	0
EDD	UD	T2	BT1	T1	UD	-	-1	-1	-1	0	0
EDD	UD	T2	BT1	BT1	UD	-	-1	-1	-1	0	0
EDD	UD	T2	BT1	E	UD	-	-3	1	-1	0	0
EDD	UD	T2	BT1	T2	UD		0				
EDD	UD	T2	BT1	BT2	UD		0				
EDD	UD	T2	E	T1	UD	-	-3	-1	0	0	0
EDD	UD	T2	E	T2	ED	+	-2	-1	0	0	0
EDD	UD	T2	E	T2	UD	+	-3	-1	0	0	0
EDD	UD	T2	T2	T1	UD	+	-3	-2	0	0	0
EDD	UD	T2	T2	E	UD	+	-3	-1	0	0	0
EDD	UD	T2	T2	T2	ED	-	-2	-1	0	0	0

EDD	UD	T2	T2	T2	UD	-	-3	-1	0	0	0	
EDD	UD	T2	BT2	T1	UD	-	-1	-2	0	0	0	
EDD	UD	T2	BT2	BT1	UD	-	-1	-2	0	0	0	
EDD	UD	T2	BT2	E	UD	+	-3	-1	0	0	0	
EDD	UD	T2	BT2	T2	UD		0					
EDD	UD	T2	BT2	BT2	UD		0					
EDD	UD	T2	A2	T1	UD	-	-2	-1	0	0	0	
EDD	UD	T2	ED	UD	T1	+	-4	-2	0	0	0	
EDD	UD	T2	ED	UD	E	+	-4	0	0	0	0	
EDD	UD	T2	ED	UD	T2	-	-4	0	0	0	0	
EDD	UD	T2	UD	ED	T2	+	-3	-2	0	0	0	
EDD	UD	T2	UD	ED	BT2	-	-1	-2	0	0	0	
EDD	UD	T2	UD	ED	A2	-	-3	0	0	0	0	
EDD	UD	T2	UD	UD	T1	+	-2	-2	-1	0	0	
EDD	UD	T2	UD	UD	BT1	-	-2	0	-1	0	0	
EDD	UD	T2	UD	UD	E		0					
EDD	UD	T2	UD	UD	T2	-	-2	-2	0	0	0	
EDD	UD	T2	UD	UD	BT2	-	-2	-2	0	0	0	
EDD	UD	T2	BUD	ED	T2	+	3	-2	0	0	-1	
EDD	UD	T2	BUD	UD	T1	-	-4	0	-1	0	-1	
EDD	UD	T2	BUD	UD	E	-	-4	0	2	0	-1	
EDD	UD	T2	BUD	UD	T2	-	-4	-2	0	0	-2	23
EDD	UD	T2	EDD	ED	T2	-	0	-2	0	0	0	
EDD	UD	T2	EDD	UD	T1	+	-4	0	0	0	0	
EDD	UD	T2	EDD	UD	E	+	-4	0	0	0	0	
EDD	UD	T2	EDD	UD	T2	-	-4	-2	0	0	0	
EDD	EDD	A1	A1	A1	EDD	+	-1	0	0	0	0	
EDD	EDD	A1	T1	T1	UD	+	-1	-1	0	0	0	
EDD	EDD	A1	T1	T1	EDD	-	-1	-1	0	0	0	
EDD	EDD	A1	E	E	UD	-	-2	0	0	0	0	

EDD	EDD	A1	E	E	UD	-	-2	0	0	0	0
EDD	EDD	A1	T2	T2	ED	+	-1	-1	0	0	0
EDD	EDD	A1	T2	T2	UD	-	-1	-1	0	0	0
EDD	EDD	A1	A2	A2	ED	-	-1	0	0	0	0
EDD	EDD	A1	ED	ED	T2	-	-2	0	0	0	0
EDD	EDD	A1	ED	ED	A2	+	-2	0	0	0	0
EDD	EDD	A1	UD	UD	T1	-	-3	0	0	0	0
EDD	EDD	A1	UD	UD	E	+	-3	0	0	0	0
EDD	EDD	A1	UD	UD	T2	+	-3	0	0	0	0
EDD	EDD	A1	EDD	EDD	A1	-	-2	0	0	0	0
EDD	EDD	A1	EDD	EDD	T1	+	-2	0	0	0	0
EDD	EDD	T1	A1	T1	EDD	-	-1	-1	0	0	0
EDD	EDD	T1	T1	A1	EDD	-	-1	-1	0	0	0
EDD	EDD	T1	T1	T1	UD	-	-2	-2	0	0	0
EDD	EDD	T1	T1	T1	EDD	-	0	-2	0	0	0
EDD	EDD	T1	T1	E	UD	-	-2	-1	0	0	0
EDD	EDD	T1	T1	T2	UD	+	-2	-1	0	0	0
EDD	EDD	T1	E	T1	UD	-	-2	-1	0	0	0
EDD	EDD	T1	E	T2	UD	+	-2	-1	0	0	0
EDD	EDD	T1	T2	T1	UD	+	-2	-1	0	0	0
EDD	EDD	T1	T2	E	UD	+	-2	-1	0	0	0
EDD	EDD	T1	T2	T2	ED	-	0	-2	0	0	0
EDD	EDD	T1	T2	T2	UD	-	-2	-2	0	0	0
EDD	EDD	T1	T2	A2	ED	+	-1	-1	0	0	0
EDD	EDD	T1	A2	T2	ED	+	-1	-1	0	0	0
EDD	EDD	T1	ED	ED	T2	+	-2	-2	0	0	0
EDD	EDD	T1	ED	ED	A2	+	-2	0	0	0	0
EDD	EDD	T1	ED	UD	T2	+	0	-2	0	0	0
EDD	EDD	T1	UD	ED	T2	+	0	-2	0	0	0
EDD	EDD	T1	UD	UD	T1	+	-3	0	-1	0	0

EDD	EDD	T1	UD	UD	E	−	−3	0	−1	0	0
EDD	EDD	T1	UD	UD	T2	+	−3	−2	1	0	0
EDD	EDD	T1	UD	BUD	T1	+	1	−2	−1	0	0
EDD	EDD	T1	UD	BUD	BT1	+	1	−2	−1	0	0
EDD	EDD	T1	UD	BUD	E	+	−1	0	−1	0	0
EDD	EDD	T1	UD	BUD	T2		0				
EDD	EDD	T1	UD	BUD	BT2		0				
EDD	EDD	T1	UD	EDD	T1	−	0	−2	0	0	0
EDD	EDD	T1	BUD	UD	T1	+	1	−2	−1	0	0
EDD	EDD	T1	BUD	UD	BT1	+	1	−2	−1	0	0
EDD	EDD	T1	BUD	UD	E	+	−1	0	−1	0	0
EDD	EDD	T1	BUD	UD	T2		0				
EDD	EDD	T1	BUD	UD	BT2		0				
EDD	EDD	T1	EDD	UD	T1	−	0	−2	0	0	0
EDD	EDD	T1	EDD	EDD	A1	+	−2	0	0	0	0
EDD	EDD	T1	EDD	EDD	T1	+	−2	−2	0	0	0

THREE: THE GREY ICOSAHEDRAL GROUP K^*

This group is the grey group of the icosahedron which is orientated so that the z-axis is the five-fold axis and the two-fold axis lies in the y-z plane making an angle of $\sin^{-1}(2/\sqrt{5})$ with the z-axis.

The serial order for the ICRs is

$$A < T_1 < V < T_2 < U < E' < U' < W' < E''$$

The $\bar{V}$ symbols are from Golding [9] with some corrections and additions.

TABLE 3.1: THE BASIS VECTORS FOR K*

$|0Aa\rangle = |00\rangle$

$|1T_1 1\rangle = -|11\rangle$

$|1T_1 0\rangle = i|10\rangle$

$|1T_1 -1\rangle = -|1-1\rangle$

$|2V2\rangle = -|22\rangle$

$|2V1\rangle = i|21\rangle$

$|2V0\rangle = |20\rangle$

$|2V-1\rangle = i|2-1\rangle$

$|2V-2\rangle = -|2-2\rangle$

$|3T_2 1\rangle = -\frac{\sqrt{2}}{\sqrt{5}}|33\rangle - i\frac{\sqrt{3}}{\sqrt{5}}|3-2\rangle$

$|3T_2 0\rangle = i|30\rangle$

$|3T_2 -1\rangle = -\frac{\sqrt{2}}{\sqrt{5}}|3-3\rangle - i\frac{\sqrt{3}}{\sqrt{5}}|32\rangle$

$|3U\kappa\rangle = \frac{\sqrt{3}}{\sqrt{5}}|33\rangle - i\frac{\sqrt{2}}{\sqrt{5}}|3-2\rangle$

$|3U\lambda\rangle = |31\rangle$

$|3U\mu\rangle = |3-1\rangle$

$$|3U\nu\rangle = \frac{\sqrt{3}}{\sqrt{5}}\,|3\ -3\rangle - i\,\frac{\sqrt{2}}{\sqrt{5}}\,|32\rangle$$

$$|4V2\rangle = \frac{-1}{\sqrt{15}}\,|42\rangle + i\,\frac{\sqrt{14}}{\sqrt{15}}\,|4\ -3\rangle$$

$$|4V1\rangle = -i\,\frac{2\sqrt{2}}{\sqrt{3}\sqrt{5}}\,|41\rangle - \frac{\sqrt{7}}{\sqrt{3}\sqrt{5}}\,|4\ -4\rangle$$

$$|4V0\rangle = |40\rangle$$

$$|4V\ -1\rangle = -\,i\frac{2\sqrt{2}}{\sqrt{3}\sqrt{5}}\,|4\ -1\rangle - \frac{\sqrt{7}}{\sqrt{3}\sqrt{5}}\,|44\rangle$$

$$|4V\ -2\rangle = -\,\frac{1}{\sqrt{15}}\,|4\ -2\rangle + i\,\frac{\sqrt{14}}{\sqrt{15}}\,|43\rangle$$

$$|4U\kappa\rangle = -i\,\frac{\sqrt{14}}{\sqrt{15}}\,|4\ -2\rangle + \frac{1}{\sqrt{15}}\,|43\rangle$$

$$|4U\lambda\rangle = \frac{\sqrt{7}}{\sqrt{3}\sqrt{5}}\,|41\rangle + i\,\frac{2\sqrt{2}}{\sqrt{3}\sqrt{5}}\,|4\ -4\rangle$$

$$|4U\mu\rangle = -\,\frac{\sqrt{7}}{\sqrt{3}\sqrt{5}}\,|4\ -1\rangle - i\,\frac{2\sqrt{2}}{\sqrt{3}\sqrt{5}}\,|44\rangle$$

$$|4U\nu\rangle = i\,\frac{\sqrt{14}}{\sqrt{15}}\,|42\rangle - \frac{1}{\sqrt{15}}\,|4\ -3\rangle$$

$$|5T_1 1\rangle = i\,\frac{\sqrt{7}}{\sqrt{10}}\,|5\ -4\rangle - \frac{\sqrt{3}}{\sqrt{10}}\,|51\rangle$$

$$|5T_1 0\rangle = -\,\frac{\sqrt{7}}{5\sqrt{2}}\,|55\rangle - i\,\frac{3\sqrt{2}}{5}\,|50\rangle - \frac{\sqrt{7}}{5\sqrt{2}}\,|5\ -5\rangle$$

$$|5T_1\ -1\rangle = i\,\frac{\sqrt{7}}{\sqrt{10}}\,|54\rangle - \frac{\sqrt{3}}{\sqrt{10}}\,|5\ -1\rangle$$

$$|5T_2 1\rangle = -\,\frac{\sqrt{2}}{\sqrt{5}}\,|53\rangle + i\,\frac{\sqrt{3}}{\sqrt{5}}\,|5\ -2\rangle$$

$$|5T_2\ 0\rangle = -\,\frac{3}{5}\,|55\rangle + i\,\frac{\sqrt{7}}{5}\,|50\rangle - \frac{3}{5}\,|5\ -5\rangle$$

$$|5T_2\ -1\rangle = -\,\frac{\sqrt{2}}{\sqrt{5}}\,|5\ -3\rangle + i\,\frac{\sqrt{3}}{\sqrt{5}}\,|52\rangle$$

$$|5V2\rangle = i\,\frac{\sqrt{3}}{\sqrt{5}}\,|5\ -3\rangle - \frac{\sqrt{2}}{\sqrt{5}}\,|52\rangle$$

$$|5V1\rangle = -i\,\frac{\sqrt{7}}{\sqrt{2}\sqrt{5}}\,|51\rangle + \frac{\sqrt{3}}{\sqrt{2}\sqrt{5}}\,|5\ -4\rangle$$

$$|5V0\rangle = -\,\frac{i}{\sqrt{2}}\,|55\rangle + \frac{i}{\sqrt{2}}\,|5\ -5\rangle$$

$$|5V\ -1\rangle = i\,\frac{\sqrt{7}}{\sqrt{2}\sqrt{5}}\,|5\ -1\rangle - \frac{\sqrt{3}}{\sqrt{2}\sqrt{5}}\,|54\rangle$$

$$|5V\ -2\rangle = -i\,\frac{\sqrt{3}}{\sqrt{5}}\,|53\rangle + \frac{\sqrt{2}}{\sqrt{5}}\,|5\ -2\rangle$$

$$|6Aa\rangle = -i\,\frac{\sqrt{7}}{5}\,|65\rangle + \frac{\sqrt{11}}{5}\,|60\rangle - i\,\frac{\sqrt{7}}{5}\,|6\ -5\rangle$$

$$|6T_1 1\rangle = i\,\frac{\sqrt{3}}{5}\,|66\rangle - \frac{\sqrt{3}\sqrt{11}}{5\sqrt{2}}\,|61\rangle + i\,\frac{\sqrt{11}}{5\sqrt{2}}\,|6\ -4\rangle$$

$$|6T_1 0\rangle = -\frac{1}{\sqrt{2}}|65\rangle + \frac{1}{\sqrt{2}}|6-5\rangle$$

$$|6T_1 -1\rangle = -i\frac{\sqrt{3}}{5}|6-6\rangle + \frac{\sqrt{3}\sqrt{11}}{5\sqrt{2}}|6-1\rangle - \frac{\sqrt{11}}{5\sqrt{2}}|64\rangle$$

$$|6U\kappa\rangle = \frac{2}{\sqrt{5}}|63\rangle - \frac{i}{\sqrt{5}}|6-2\rangle$$

$$|6U\lambda\rangle = -i\frac{\sqrt{11}}{5}|66\rangle - \frac{2\sqrt{2}}{5}|61\rangle - i\frac{\sqrt{2}\sqrt{3}}{5}|6-4\rangle$$

$$|6U\mu\rangle = i\frac{\sqrt{11}}{5}|6-6\rangle + \frac{2\sqrt{2}}{5}|6-1\rangle + i\frac{\sqrt{2}\sqrt{3}}{5}|64\rangle$$

$$|6U\nu\rangle = -\frac{2}{\sqrt{5}}|6-3\rangle + \frac{i}{\sqrt{5}}|62\rangle$$

$$|6V2\rangle = -\frac{2}{\sqrt{5}}|62\rangle + \frac{i}{\sqrt{5}}|6-3\rangle$$

$$|6V1\rangle = -\frac{\sqrt{11}}{5}|66\rangle - \frac{i}{5\sqrt{2}}|61\rangle + \frac{3\sqrt{3}}{5\sqrt{2}}|6-4\rangle$$

$$|6V0\rangle = -\frac{\sqrt{11}}{5\sqrt{2}}|65\rangle - \frac{2\sqrt{7}}{5}|60\rangle - i\frac{\sqrt{11}}{5\sqrt{2}}|6-5\rangle$$

$$|6V-1\rangle = -\frac{\sqrt{11}}{5}|6-6\rangle - \frac{i}{5\sqrt{2}}|6-1\rangle + \frac{3\sqrt{3}}{5\sqrt{2}}|64\rangle$$

$$|6V-2\rangle = -\frac{2}{\sqrt{5}}|6-2\rangle + \frac{i}{\sqrt{5}}|63\rangle$$

$$|7T_1 1\rangle = i\frac{\sqrt{2}\sqrt{3}\sqrt{13}}{5\sqrt{5}}|76\rangle - \frac{2\sqrt{11}}{5\sqrt{5}}|71\rangle + i\frac{\sqrt{3}}{5\sqrt{5}}|7-4\rangle$$

$$|7T_1 0\rangle = \frac{2\sqrt{2}\sqrt{3}}{5\sqrt{5}}|75\rangle + i\frac{\sqrt{7}\sqrt{11}}{5\sqrt{5}}|70\rangle + \frac{2\sqrt{2}\sqrt{3}}{5\sqrt{5}}|7-5\rangle$$

$$|7T_1 -1\rangle = i\frac{\sqrt{3}}{5\sqrt{5}}|74\rangle - \frac{2\sqrt{11}}{5\sqrt{5}}|7-1\rangle + i\frac{\sqrt{2}\sqrt{3}\sqrt{13}}{5\sqrt{5}}|7-6\rangle$$

$$|7T_2 1\rangle = \frac{\sqrt{11}\sqrt{13}}{10\sqrt{5}}|7-7\rangle - i\frac{\sqrt{7}}{5\sqrt{2}\sqrt{5}}|7-2\rangle - \frac{7\sqrt{7}}{10\sqrt{5}}|73\rangle$$

$$|7T_2 0\rangle^{\dagger} = \frac{\sqrt{7}\sqrt{11}}{5\sqrt{2}\sqrt{5}}|75\rangle - i\frac{4\sqrt{3}}{5\sqrt{5}}|70\rangle + \frac{\sqrt{7}\sqrt{11}}{5\sqrt{2}\sqrt{5}}|7-5\rangle$$

$$|7T_2 -1\rangle = \frac{\sqrt{11}\sqrt{13}}{10\sqrt{5}}|77\rangle - i\frac{\sqrt{7}}{5\sqrt{2}\sqrt{5}}|72\rangle - \frac{7\sqrt{7}}{10\sqrt{5}}|7-3\rangle$$

$$|7U\kappa\rangle = \frac{2\sqrt{7}\sqrt{11}}{5\sqrt{3}\sqrt{5}}|7-7\rangle - i\frac{\sqrt{3}\sqrt{13}}{5\sqrt{5}}|7-2\rangle + \frac{2\sqrt{2}\sqrt{13}}{5\sqrt{3}\sqrt{5}}|73\rangle$$

$$|7U\lambda\rangle = -i\frac{\sqrt{11}}{5\sqrt{3}\sqrt{5}}|76\rangle - \frac{\sqrt{2}\sqrt{13}}{5\sqrt{5}}|71\rangle - i\frac{\sqrt{2}\sqrt{11}\sqrt{13}}{5\sqrt{3}\sqrt{5}}|7-4\rangle$$

$$|7U\mu\rangle = -i\frac{\sqrt{11}}{5\sqrt{3}\sqrt{5}}|7-6\rangle - \frac{\sqrt{2}\sqrt{13}}{5\sqrt{5}}|7-1\rangle - i\frac{\sqrt{2}\sqrt{11}\sqrt{13}}{5\sqrt{3}\sqrt{5}}|74\rangle$$

$$|7U\nu\rangle = \frac{\sqrt{2}\sqrt{7}\sqrt{11}}{5\sqrt{3}\sqrt{5}}|77\rangle - i\frac{\sqrt{3}\sqrt{13}}{5\sqrt{5}}|72\rangle + \frac{2\sqrt{2}\sqrt{13}}{5\sqrt{3}\sqrt{5}}|7-3\rangle$$

$$|7V2\rangle = i\frac{\sqrt{7}\sqrt{13}}{10\sqrt{3}}|77\rangle - \frac{\sqrt{3}\sqrt{11}}{5\sqrt{2}}|72\rangle + i\frac{\sqrt{11}}{10\sqrt{3}}|7-3\rangle$$

$$|7V1\rangle = -\frac{\sqrt{2}\sqrt{13}}{5\sqrt{3}}|76\rangle + i\frac{\sqrt{11}}{5}|71\rangle + \frac{\sqrt{4}}{5\sqrt{3}}|7-4\rangle$$

$$|7V0\rangle = \frac{i}{\sqrt{2}}\,|75\rangle - \frac{i}{\sqrt{2}}\,|7\ -5\rangle$$

$$|7V\ -1\rangle = \frac{2\ 13}{5\sqrt{3}}\,|7\,-6\rangle - i\,\frac{\sqrt{11}}{5}\,|7\ -1\rangle - \frac{4}{5\sqrt{3}}\,|74\rangle$$

$$|7V\ -2\rangle = -i\,\frac{\sqrt{7}\sqrt{13}}{10\sqrt{3}}\,|7\ -7\rangle + \frac{\sqrt{3}\sqrt{11}}{5\sqrt{2}}\,|7\ -2\rangle - i\,\frac{\sqrt{11}}{10\sqrt{3}}\,|73\rangle$$

$$|1/2\ E'\alpha\rangle = i\,|1/2\ 1/2\rangle$$

$$|1/2\ E'\beta\rangle = i\,|1/2\ -1/2\rangle$$

$$|3/2\ U'\kappa\rangle = i\,|3/2\ 3/2\rangle$$

$$|3/2\ U'\lambda\rangle = -\,|3/2\ 1/2\rangle$$

$$|3/2\ U'\mu\rangle = -\,|3/2\ -1/2\rangle$$

$$|3/2U'\nu\rangle = i\,|3/2\ -3/2\rangle$$

$$|5/2\ W'\tau\rangle = i\,|5/2\ 5/2\rangle$$

$$|5/2\ W'\nu\rangle = -\,|5/2\ 3/2\rangle$$

$$|5/2\ W'\phi\rangle = -i\,|5/2\ 1/2\rangle$$

$$|5/2\ W'\chi\rangle = -i\,|5/2\ -1/2\rangle$$

$$|5/2\ W'\psi\rangle = -\,|5/2\ -3/2\rangle$$

$$|5/2\ W'\omega\rangle = i\,|5/2\ -5/2\rangle$$

$$|7/2\ E''\alpha\rangle = i\,\frac{\sqrt{3}}{\sqrt{10}}\,|7/2\ 7/2\rangle + \frac{\sqrt{7}}{\sqrt{10}}\,|7/2\ -3/2\rangle$$

$$|7/2\ E''\beta\rangle = i\,\frac{\sqrt{3}}{\sqrt{10}}\,|7/2\ -7/2\rangle + \frac{\sqrt{7}}{\sqrt{10}}\,|7/2\ 3/2\rangle$$

$$|7/2\ W'\tau\rangle = -\,\frac{i}{5\sqrt{2}}\,|7/2\ 5/2\rangle - \frac{7}{5\sqrt{2}}\,|7/2\ -5/2\rangle$$

$$|7/2\ W'\upsilon\rangle = -\,\frac{\sqrt{3}}{\sqrt{2}\sqrt{5}}\,|7/2\ 3/2\rangle + i\,\frac{\sqrt{7}}{\sqrt{2}\sqrt{5}}\,|7/2\ -7/2\rangle$$

$$|7/2\ W'\phi\rangle = i\,|7/2\ 1/2\rangle$$

$$|7/2\ W'\chi\rangle = -i\,|7/2\ -1/2\rangle$$

$$|7/2\ W'\psi\rangle = \frac{\sqrt{3}}{\sqrt{2}\sqrt{5}}\,|7/2\ -3/2\rangle - i\,\frac{\sqrt{7}}{\sqrt{2}\sqrt{5}}\,|7/2\ +7/2\rangle$$

$$|7/2\ W'\omega\rangle = +\,\frac{i}{5\sqrt{2}}\,|7/2\ -5/2\rangle + \frac{7}{5\sqrt{2}}\,|7/2\ 7/2\rangle$$

$$|9/2\ U'\kappa\rangle = -\frac{2i}{5}|9/2\ 3/2\rangle - \frac{\sqrt{3}\sqrt{7}}{55}|9/2\ -7/2\rangle$$

$$|9/2\ U'\lambda\rangle = -\frac{3\sqrt{2}}{5}|9/2\ 1/2\rangle + i\frac{\sqrt{7}}{5}|9/2\ -9/2\rangle$$

$$|9/2\ U'\mu\rangle = \frac{3\sqrt{2}}{5}|9/2\ -1/2\rangle - i\frac{\sqrt{7}}{5}|9/2\ 9/2\rangle$$

$$|9/2\ U'\nu\rangle = \frac{2i}{5}|9/2\ -3/2\rangle + \frac{\sqrt{3}\sqrt{7}}{5}|9/2\ 7/2\rangle$$

$$|9/2\ W'\tau\rangle = -\frac{i}{\sqrt{5}}|9/2\ 5/2\rangle - \frac{2}{\sqrt{5}}|9/2\ -5/2\rangle$$

$$|9/2\ W'\nu\rangle = -\frac{\sqrt{3}\sqrt{7}}{5}|9/2\ 3/2\rangle - \frac{2i}{5}|9/2\ -7/2\rangle$$

$$|9/2\ W'\phi\rangle = i\frac{\sqrt{7}}{5}|9/2\ 1/2\rangle - \frac{3\sqrt{2}}{5}|9/2\ -9/2\rangle$$

$$|9/2\ W'\chi\rangle = i\frac{\sqrt{7}}{5}|9/2\ -1/2\rangle - \frac{3\sqrt{2}}{5}|9/2\ 9/2\rangle$$

$$|9/2\ W'\psi\rangle = -\frac{\sqrt{3}\sqrt{7}}{5}|9/2\ -3/2\rangle - \frac{2i}{5}|9/2\ 7/2\rangle$$

$$|9/2\ W'\nu\rangle = -\frac{i}{\sqrt{5}}|9/2\ -5/2\rangle - \frac{2}{\sqrt{5}}|9/2\ 5/2\rangle$$

$$|11/2\ E'\alpha\rangle = \frac{\sqrt{7}}{5\sqrt{2}\sqrt{3}}|11/2\ 11/2\rangle + i\frac{\sqrt{11}}{5}|11/2\ 1/2\rangle + \frac{\sqrt{11}\sqrt{13}}{5\sqrt{2}\sqrt{3}}|11/2\ -9/2\rangle$$

$$|11/2\ E'\beta\rangle = -\frac{\sqrt{7}}{5\sqrt{2}\sqrt{3}}|11/2\ -11/2\rangle - i\frac{\sqrt{11}}{5}|11/2\ -1/2\rangle - \frac{\sqrt{11}\sqrt{13}}{5\sqrt{2}\sqrt{3}}|11/2\ 9/2\rangle$$

$$|11/2\ U'\kappa\rangle = -i\frac{\sqrt{3}}{\sqrt{5}}|11/2\ 3/2\rangle - \frac{\sqrt{2}}{\sqrt{5}}|11/2\ -7/2\rangle$$

$$|11/2\ U'\lambda\rangle = i\frac{\sqrt{2}\sqrt{11}}{5\sqrt{3}}|11/2\ 11/2\rangle - \frac{\sqrt{7}}{5}|11/2\ 1/2\rangle - i\frac{4\sqrt{2}}{5\sqrt{3}}|11/2\ -9/2\rangle$$

$$|11/2\ U'\mu\rangle = i\frac{\sqrt{2}\sqrt{11}}{5\sqrt{3}}|11/2\ -11/2\rangle - \frac{\sqrt{7}}{5}|11/2\ -1/2\rangle - i\frac{4\sqrt{2}}{5\sqrt{3}}|11/2\ 9/2\rangle$$

$$|11/2\ U'\nu\rangle = -i\frac{\sqrt{3}}{\sqrt{5}}|11/2\ -3/2\rangle - \frac{\sqrt{2}}{\sqrt{5}}|11/2\ 7/2\rangle$$

$$|11/2\ W'\tau\rangle = -\frac{4i}{5}|11/2\ 5/2\rangle - \frac{3}{5}|11/2\ -5/2\rangle$$

$$|11/2WW'\nu\rangle = -\frac{\sqrt{2}}{\sqrt{5}}|11/2\ 3/2\rangle - \frac{\sqrt{3}}{\sqrt{5}}|11/2\ -7/2\rangle$$

$$|11/2\ W'\phi\rangle = \frac{\sqrt{3}\sqrt{11}}{5\sqrt{2}}|11/2\ 11/2\rangle - i\frac{\sqrt{7}}{5}|11/2\ 1/2\rangle + \frac{\sqrt{3}}{5\sqrt{2}}|11/2\ -9/2\rangle$$

$$|11/2\ W'\chi\rangle = -\frac{\sqrt{3}\sqrt{11}}{5\sqrt{2}}|11/2\ -11/2\rangle + i\frac{\sqrt{7}}{5}|11/2\ -1/2\rangle - \frac{\sqrt{3}}{5\sqrt{2}}|11/2\ 9/2\rangle$$

$$|11/2\ W'\psi\rangle = \frac{\sqrt{2}}{\sqrt{5}}|11/2\ -3/2\rangle + i\frac{\sqrt{3}}{\sqrt{5}}|11/2\ 7/2\rangle$$

$$|11/2\ W'\nu\rangle = -\frac{4}{5}|11/2\ -5/2\rangle - \frac{3}{5}|11/2\ 5/2\rangle$$

$$|13/2\ E'\alpha\rangle = \frac{2\sqrt{3}}{5}\,|13/2\ 11/2\rangle + i\,\frac{\sqrt{11}}{5}\,|13/2\ 1/2\rangle + \frac{\sqrt{2}}{5}\,|13/2\ -9/2\rangle$$

$$|11/2\ E'\beta\rangle = \frac{2\sqrt{3}}{5}\,|13/2\ -11/2\rangle + i\,\frac{\sqrt{11}}{5}\,|13/2\ -1/2\rangle + \frac{\sqrt{2}}{5}\,|13/2\ 9/2\rangle$$

$$|13/2\ E''\alpha\rangle = i\,\frac{\sqrt{11}}{5\sqrt{2}}\,|13/2\ -13/2\rangle + \frac{\sqrt{13}}{5\sqrt{2}}\,|13/2\ -3/2\rangle + i\,\frac{\sqrt{13}}{5}\,|13/2\ 7/2\rangle$$

$$|13/2\ E''\beta\rangle = -i\,\frac{\sqrt{11}}{5\sqrt{2}}\,|13/2\ 13/2\rangle - \frac{\sqrt{13}}{5\sqrt{2}}\,|13/2\ 3/2\rangle - i\,\frac{\sqrt{13}}{5}\,|13/2\ -7/2\rangle$$

$$|13/2\ U'\kappa\rangle = \frac{\sqrt{2}\sqrt{13}}{5\sqrt{5}}\,|13/2\ 13/2\rangle + i\,\frac{2\sqrt{2}\sqrt{11}}{5\sqrt{5}}\,|13/2\ 3/2\rangle + \frac{\sqrt{11}}{5\sqrt{5}}\,|13/2\ -7/2\rangle$$

$$|13/2\ U'\lambda\rangle = -i\,\frac{3\sqrt{6}}{5\sqrt{5}}\,|13/2\ 11/2\rangle - \frac{\sqrt{2}\sqrt{11}}{5\sqrt{5}}\,|13/2\ 1/2\rangle + i\,\frac{7}{5\sqrt{5}}\,|13/2\ -9/2\rangle$$

$$|13/2\ U'\mu\rangle = i\,\frac{3\sqrt{6}}{5\sqrt{5}}\,|13/2\ -11/2\rangle + \frac{\sqrt{2}\sqrt{11}}{5\sqrt{5}}\,|13/2\ -1/2\rangle - i\,\frac{7}{5\sqrt{5}}\,|13/2\ 9/2\rangle$$

$$|13/2\ U'\nu\rangle = -\frac{\sqrt{2}\sqrt{13}}{5\sqrt{5}}\,|13/2\ -13/2\rangle - i\,\frac{2\sqrt{2}\sqrt{11}}{5\sqrt{5}}\,|13/2\ -3/2\rangle - \frac{\sqrt{11}}{5\sqrt{5}}\,|13/2\ 7/2\rangle$$

$$|13/2\ W'\tau\rangle = i\,\frac{3}{\sqrt{10}}\,|13/2\ 5/2\rangle + \frac{1}{\sqrt{10}}\,|13/2\ -5/2\rangle$$

$$|13/2\ W'\upsilon\rangle = -i\,\frac{\sqrt{11}\sqrt{13}}{5\sqrt{2}\sqrt{5}}\,|13/2\ 13/2\rangle - \frac{3}{5\sqrt{2}\ \cancel{5}}\,|13/2\ 3/2\rangle + i\,\frac{7}{5\sqrt{5}}\,|13/2\ -7/2\rangle$$

$$|13/2\ W'\phi\rangle = -\frac{\sqrt{11}}{5\sqrt{5}}\,|13/2\ 11/2\rangle + i\,\frac{4\sqrt{3}}{5\sqrt{5}}\,|13/2\ 1/2\rangle - \frac{2\sqrt{3}\sqrt{11}}{5\sqrt{5}}\,|13/2\ -9/2\rangle$$

$$|13/2\ W'\chi\rangle = -\frac{\sqrt{11}}{5\sqrt{5}}\,|13/2\ -11/2\rangle - i\,\frac{4\sqrt{3}}{5\sqrt{5}}\,|13/2\ -1/2\rangle + \frac{\sqrt{2}\sqrt{3}\sqrt{11}}{5\sqrt{5}}\,|13/2\ 9/2\rangle$$

$$|13/2\ W'\psi\rangle = -i\,\frac{\sqrt{11}\sqrt{13}}{5\sqrt{2}\sqrt{5}}\,|13/2\ -13/2\rangle + \frac{3}{5\sqrt{2}\sqrt{5}}\,|13/2\ -3/2\rangle - i\,\frac{7}{5\sqrt{5}}\,|13/\ 7/2\rangle$$

$$|13/2\ W'\omega\rangle = i\,\frac{3}{\sqrt{10}}\,|13/2\ -5/2\rangle + \frac{1}{\sqrt{10}}\,|13/2\ 5/2\rangle$$

$$|15/2\ U'\kappa\rangle = \frac{\sqrt{7}\sqrt{13}}{5\sqrt{5}}\,|15/2\ 13/2\rangle + i\,\frac{\sqrt{3}\sqrt{11}}{5\sqrt{5}}\,|15/2\ 3/2\rangle + \frac{1}{5\sqrt{5}}\,|15/2\ -7/2\rangle$$

$$|15/2\ U'\lambda\rangle = i\,\frac{\sqrt{3}\sqrt{13}}{5\sqrt{5}}\,|15/2\ 11/2\rangle - \frac{\sqrt{7}\sqrt{11}}{5\sqrt{5}}\,|15/2\ 1/2\rangle + i\,\frac{3}{5\sqrt{5}}\,|15/2\ -9/2\rangle$$

$$|15/2\ U'\mu\rangle = i\,\frac{\sqrt{3}\sqrt{13}}{4\sqrt{5}}\,|15/2\ -11/2\rangle - \frac{\sqrt{7}\sqrt{11}}{5\sqrt{5}}\,|15/2\ -1/2\rangle + i\,\frac{3}{5\sqrt{5}}\,|15/2\ 9/2\rangle$$

$$|15/2\ U'\nu\rangle = \frac{\sqrt{7}\sqrt{13}}{5\sqrt{5}}\,|15/2\ -13/2\rangle + i\,\frac{\sqrt{3}\sqrt{11}}{5\sqrt{5}}\,|15/2\ -3/2\rangle + \frac{1}{5\sqrt{5}}\,|15/2\ 7/2\rangle$$

$$|15/2\ aW'\tau\rangle = \frac{\sqrt{11}}{50\sqrt{2}\sqrt{17}}\,|15/2\ 15/2\rangle - i\,\frac{\sqrt{3}\sqrt{7}\sqrt{13}}{25\sqrt{2}\sqrt{17}}\,|15/2\ 5/2\rangle$$
$$+ \frac{11\sqrt{3}\sqrt{7}\sqrt{13}}{25\sqrt{2}\sqrt{17}}\,|15/2\ -5/2\rangle + i\,\frac{\sqrt{2}\sqrt{11}\sqrt{17}}{25}\,|15/2\ -15/2\rangle$$

$$|15/2\ aW'\upsilon\rangle = -i\,\frac{\sqrt{3}\sqrt{11}}{10\sqrt{2}\sqrt{17}}\,|15/2\ 13/2\rangle - \frac{\sqrt{7}\sqrt{13}}{5\sqrt{2}\sqrt{17}}\,|15/2\ 3/2\rangle$$
$$- i\,\frac{\sqrt{3}\sqrt{7}\sqrt{11}\sqrt{13}}{10\sqrt{2}\sqrt{17}}\,|15/2\ -7/2\rangle$$

$$|15/2\ aW'\phi\rangle = -\frac{\sqrt{3}\sqrt{7}\sqrt{11}}{10\sqrt{17}}|15/2\ 11/2\rangle + i\frac{5\sqrt{13}}{10\sqrt{17}}|15/2\ 1/2\rangle - \frac{\sqrt{7}\sqrt{11}\sqrt{13}}{10\sqrt{17}}|15/2\ -9/2\rangle$$

$$|15/2\ aW'\chi\rangle = \frac{3\sqrt{7}\sqrt{11}}{10\sqrt{17}}|15/2\ -11/2\rangle - i\frac{6\sqrt{13}}{10\sqrt{17}}|15/2\ -1/2\rangle + \frac{\sqrt{7}\sqrt{11}\sqrt{13}}{10\sqrt{17}}|15/2\ 9/2\rangle$$

$$|15/2\ aW'\psi\rangle = i\frac{\sqrt{3}\sqrt{11}}{10\sqrt{2}\sqrt{17}}|15/2\ -13/2\rangle + \frac{\sqrt{7}\sqrt{13}}{5\sqrt{2}\sqrt{17}}|15/2\ -3/2\rangle + i\frac{\sqrt{3}\sqrt{7}\sqrt{11}\sqrt{13}}{10\sqrt{2}\sqrt{17}}|15/2\ 7/2\rangle$$

$$|15/2\ aW'\omega\rangle = -\frac{\sqrt{11}}{50\sqrt{2}\sqrt{17}}|15/2\ -15/2\rangle + i\frac{\sqrt{3}\sqrt{7}\sqrt{13}}{25\sqrt{2}\sqrt{17}}|15/2\ -5/2\rangle - \frac{11\sqrt{3}\sqrt{7}\sqrt{13}}{25\sqrt{2}\sqrt{17}}|15/2\ 5/2\rangle - i\frac{\sqrt{2}\sqrt{11}\sqrt{17}}{25}|15/2\ 15/2\rangle$$

$$|15/2\ bW'\tau\rangle = \frac{\sqrt{3}\sqrt{7}\sqrt{13}}{2\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ 15/2\rangle + i\frac{3\sqrt{11}}{\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ 5/2\rangle + \frac{\sqrt{11}}{2\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ -5/2\rangle$$

$$|15/2\ bW'\upsilon\rangle = -i\frac{7\sqrt{7}\sqrt{13}}{10\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ 13/2\rangle - \frac{9\sqrt{3}\sqrt{11}}{5\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ 3/2\rangle + i\frac{43}{10\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ -7/2\rangle$$

$$|15/2\ bW'\phi\rangle = \frac{19\sqrt{13}}{10\sqrt{5}\sqrt{17}}|15/2\ 11/2\rangle - i\frac{\sqrt{3}\sqrt{7}\sqrt{11}}{5\sqrt{5}\sqrt{17}}|15/2\ 1/2\rangle - \frac{31\sqrt{3}}{10\sqrt{5}\sqrt{17}}|15/2\ -9/2\rangle$$

$$|15/2\ bW'\chi\rangle = -\frac{19\sqrt{13}}{10\sqrt{5}\sqrt{17}}|15/2\ -11/2\rangle + i\frac{\sqrt{3}\sqrt{7}\sqrt{11}}{5\sqrt{5}\sqrt{17}}|15/2\ -1/2\rangle + \frac{31\sqrt{3}}{10\sqrt{5}\sqrt{17}}|15/2\ 9/2\rangle$$

$$|15/2\ bW'\psi\rangle = i\frac{7\sqrt{7}\sqrt{13}}{10\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ -13/2\rangle + \frac{9\sqrt{3}\sqrt{11}}{5\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ -3/2\rangle - \frac{43}{10\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ 7/2\rangle$$

$$|15/2\ bW'\omega\rangle = \frac{\sqrt{3}\sqrt{7}\sqrt{13}}{2\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ -15/2\rangle - i\frac{3\sqrt{11}}{\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ -5/2\rangle - \frac{\sqrt{11}}{2\sqrt{2}\sqrt{5}\sqrt{17}}|15/2\ 5/2\rangle$$

TABLE 3.2: THE WIGNER TENSOR $\begin{pmatrix} m & \\ & \dot{n} \end{pmatrix}$ FOR GREY K^*

j	$\bar{\chi}$m	$\bar{\chi}\dot{n}$	Value
A	a	a	1
T_1	1	-1	1
	0	0	1
	-1	1	1
V	2	-2	1
	1	-1	1
	0	0	1
	-1	1	1
	-2	2	1
T_2	1	-1	1
	0	0	1
	-1	1	1
U	κ	ν	1
	λ	μ	1
	μ	λ	1
	ν	κ	1
E'	α	β	-1
	β	α	1
U'	κ	ν	-1
	λ	μ	-1
	μ	λ	1
	ν	κ	1
W'	τ	ω	-1
	υ	ψ	-1
	φ	χ	-1
	χ	φ	1
	ψ	υ	1
	ω	τ	1
E"	α	β	-1
	β	α	1

TABLE 3.3: $\overline{V}$ COEFFICIENTS FOR K^*

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$	Transposition Parity
A	A	A	a	a	a	1	even
T_1	T_1	A	1	-1	a	$1/\sqrt{3}$	even
			0	0	a	$1/\sqrt{3}$	
T_1	T_1	T_1	1	0	-1	$- /\sqrt{6}$	odd
V	T_1	T_1	-2	1	1	$-1/\sqrt{5}$	even
			-1	0	1	$-1/\sqrt{10}$	
			0	1	-1	$1/\sqrt{30}$	
			0	0	0	$-\sqrt{2}/\sqrt{15}$	
V	V	A	2	-2	a	$1/\sqrt{5}$	even
			1	-1	a	$1/\sqrt{5}$	
			0	0	a	$1/\sqrt{5}$	
V	V	T_1	2	-1	-1	$-i/\sqrt{15}$	odd
			1	0	-1	$-i/\sqrt{10}$	
			2	-2	0	$i\sqrt{2}/\sqrt{15}$	
			1	-1	0	$i/\sqrt{30}$	
V	V	V	2	0	-2	$\sqrt{2}/\sqrt{35}$	even
			1	1	-2	$-\sqrt{3}/\sqrt{35}$	
			1	0	-1	$-1/\sqrt{70}$	
			0	0	0	$-\sqrt{2}/\sqrt{35}$	
V	V	V	-2	2	0	$1/5\sqrt{14}$	even
			-2	1	1	$2/5\sqrt{21}$	
			-1	1	0	$-2\sqrt{2}/5\sqrt{7}$	
			0	0	0	$3\sqrt{2}/5\sqrt{7}$	
			1	2	2	$-\sqrt{7}/5\sqrt{3}$	
T_2	V	T_1	-1	2	1	$-\sqrt{2}/\sqrt{15}$	even
			1	2	0	$1/\sqrt{15}$	
			1	1	1	$\sqrt{2}/\sqrt{15}$	
			0	1	-1	$-1/\sqrt{15}$	
			0	0	0	$1/\sqrt{5}$	
T_2	V	V	-1	2	1	$-i/\sqrt{15}$	odd
			1	2	0	$i/\sqrt{10}$	
			0	2	-2	$i/\sqrt{30}$	
			0	1	-1	$-i\sqrt{2}/\sqrt{15}$	
T_2	T_2	A	1	-1	a	$1/\sqrt{3}$	even

U	U	T_2	κ	μ	1	$-i/\sqrt{6}$	odd
			κ	ν	0	$i/2\sqrt{3}$	
			λ	μ	0	$-i/2\sqrt{3}$	
U	V	V	ν	2	1	$i/2\sqrt{30}$	even
			κ	2	0	$-i/2\sqrt{5}$	
			κ	1	1	$-i\sqrt{2}/\sqrt{15}$	
			λ	2	2	$-i\sqrt{2}/\sqrt{15}$	
			μ	2	-1	$i/2\sqrt{30}$	
			μ	1	0	$-i/2\sqrt{5}$	
U	U	U	μ	κ	κ	$i/2\sqrt{3}$	even
			λ	κ	λ	$-i/2\sqrt{3}$	
			ν	λ	ν	$i/2\sqrt{3}$	
			ν	ν	λ	$-i/2\sqrt{3}$	
E'	E'	A	α	β	a	$1/\sqrt{2}$	odd
E'	E'	T_1	α	α	-1	$-1/\sqrt{3}$	even
			α	β	0	$i/\sqrt{6}$	
U'	E'	T_1	ν	α	1	$1/2$	odd
			μ	β	1	$i/2\sqrt{3}$	
			λ	β	0	$1/\sqrt{6}$	
U'	E'	V	κ	α	-2	$-1/\sqrt{5}$	even
			κ	β	-1	$i/2\sqrt{5}$	
			λ	α	-1	$\sqrt{3}/2\sqrt{5}$	
			λ	β	0	$-i/\sqrt{10}$	
U'	U'	A	κ	ν	a	$1/2$	odd
			λ	μ	a	$1/2$	
U'	U'	T_1	μ	μ	1	$-\sqrt{2}/\sqrt{15}$	even
			λ	ν	1	$i/\sqrt{10}$	
			λ	μ	0	$i/2\sqrt{15}$	
			κ	ν	0	$i\sqrt{3}/2\sqrt{5}$	
U'	U'	V	κ	λ	-2	$i/\sqrt{10}$	odd
			κ	μ	-1	$1/\sqrt{10}$	
			κ	ν	0	$1/2\sqrt{5}$	
			μ	λ	0	$1/2\sqrt{5}$	
U'	U'	T_2	κ	κ	-1	$-\sqrt{2}/\sqrt{15}$	even
			μ	ν	-1	$-1/\sqrt{10}$	
			λ	μ	0	$i/2\sqrt{15}$	
			λ	μ	0	$-i\sqrt{3}/2\sqrt{5}$	

			0	0	a	$1/\sqrt{3}$	
T_2	T_2	V	1	0	2	$1/\sqrt{10}$	even
			1	1	-1	$1/\sqrt{5}$	
			1	-1	0	$1/\sqrt{30}$	
			0	0	0	$-\sqrt{2}/\sqrt{15}$	
T_2	T_2	T_2	1	0	-1	$-i/\sqrt{6}$	odd
U	V	T_1	ν	2	1	$\sqrt{3}/2\sqrt{5}$	even
			κ	2	0	$1/\sqrt{30}$	
			κ	1	1	$1/\sqrt{15}$	
			μ	2	-1	$1/2\sqrt{15}$	
			μ	1	0	$-\sqrt{2}/\sqrt{15}$	
			μ	0	1	$-1/\sqrt{10}$	
U	V	V	ν	2	1	$i\sqrt{3}/2\sqrt{10}$	odd
			κ	2	0	$i/2\sqrt{5}$	
			μ	1	0	$-i/2\sqrt{5}$	
			μ	2	-1	$i\sqrt{3}/2\sqrt{10}$	
U	T_2	T_1	ν	-1	1	$i/2\sqrt{3}$	odd
			λ	0	-1	$i/\sqrt{6}$	
			λ	1	1	$i/2\sqrt{3}$	
			ν	1	0	$i/\sqrt{6}$	
U	T_2	V	λ	-1	2	$1/\sqrt{15}$	even
			κ	0	2	$\sqrt{2}/\sqrt{15}$	
			κ	1	-1	$-1/2\sqrt{15}$	
			λ	1	1	$\sqrt{3}/2\sqrt{5}$	
			λ	0	-1	$1/\sqrt{30}$	
			κ	-1	0	$-1/\sqrt{10}$	
U	U	A	κ	ν	a	1/2	even
			λ	μ	a	1/2	
U	U	T_1	κ	λ	1	$-i/\sqrt{6}$	odd
			κ	ν	0	$i/2\ 3$	
			λ	μ	0	$i/2\sqrt{3}$	
U	U	V	κ	μ	-2	$-1/\sqrt{30}$	even
			λ	λ	-2	$-\sqrt{2}/\sqrt{15}$	
			κ	κ	-1	$-\sqrt{2}/\sqrt{15}$	
			κ	λ	1	$1/\sqrt{30}$	
			κ	ν	0	$1/2\sqrt{5}$	
			λ	μ	0	$-1/2\sqrt{5}$	

U'	U'	U	λ	μ	ν	$\sqrt{3}/2\sqrt{5}$	even
			μ	ν	ν	$-1/2\sqrt{5}$	
			κ	μ	μ	$-i/2\sqrt{5}$	
			λ	μ	μ	$-\sqrt{3}/2\sqrt{5}$	
W'	E'	V	χ	α	0	$1/\sqrt{10}$	odd
			φ	α	-1	$i/\sqrt{15}$	
			ψ	β	2	$i/\sqrt{30}$	
			υ	β	-1	$\sqrt{2}/\sqrt{15}$	
			ω	α	2	$1/\sqrt{6}$	
W'	E'	T_2	τ	α	-1	$-\sqrt{2}/\sqrt{15}$	even
			ψ	β	-1	$-1/\sqrt{6}$	
			ω	α	-1	$-i/\sqrt{30}$	
			φ	β	0	$-i/\sqrt{6}$	
W'	E'	U	τ	α	ν	$\sqrt{3}/2\sqrt{5}$	even
			ψ	β	ν	$-1/2\sqrt{3}$	
			ω	α	ν	$-i/2\sqrt{15}$	
			ν	β	μ	$-i/2\sqrt{3}$	
			φ	α	μ	$-1/\sqrt{6}$	
W'	U'	T_1	χ	μ	1	$i/2\sqrt{5}$	odd
			φ	ν	1	$1/2\sqrt{15}$	
			ψ	λ	1	$1/\sqrt{10}$	
			τ	κ	1	$1/\sqrt{6}$	
			φ	μ	0	$1/\sqrt{10}$	
			υ	ν	0	$1/\sqrt{15}$	
W'	U'	V	ψ	κ	0	$-i\sqrt{3}/\sqrt{35}$	even
			χ	κ	-1	$-i3/2\sqrt{35}$	
			χ	λ	0	$-i/\sqrt{70}$	
			φ	κ	-2	$1/\sqrt{35}$	
			φ	λ	-1	$\sqrt{5}/2\sqrt{21}$	
			υ	λ	-2	$-2\sqrt{2}/\sqrt{105}$	
			υ	μ	-1	$-i/\sqrt{210}$	
W'	U'	V	τ	μ	-2	$i/10\sqrt{7}$	even
			υ	λ	-2	$1/2\sqrt{35}$	
			φ	κ	-2	$1/\sqrt{210}$	
			ψ	υ	-2	$\sqrt{7}/2\sqrt{15}$	
			ω	μ	-2	$\sqrt{7}/10$	
			τ	ν	-1	$-i/5\sqrt{21}$	
			υ	μ	-1	$i/\sqrt{35}$	

			ϕ	λ	-1	$\sqrt{2}/\sqrt{35}$	
			χ	κ	-1	$i\sqrt{2}/\sqrt{105}$	
			ω	ν	-1	$-\sqrt{7}/5\sqrt{3}$	
			υ	ν	0	$-i/\sqrt{70}$	
			ϕ	μ	0	$i\sqrt{3}/\sqrt{35}$	
W'	U'	T_2	τ	λ	-1	$i/2\sqrt{3}$	odd
			ψ	ν	1	$i/2\sqrt{5}$	
			χ	ν	-1	$i/\sqrt{10}$	
			υ	λ	1	$i/2\sqrt{15}$	
			ω	λ	-1	$1/2\sqrt{3}$	
			υ	ν	0	$1/\sqrt{10}$	
			χ	λ	0	$1/\sqrt{15}$	
W'	U'	U	ω	μ	κ	$i\sqrt{3}/4\sqrt{2}$	odd
			υ	κ	ν	$i3/4\sqrt{10}$	
			χ	ν	ν	$i/2\sqrt{5}$	
			υ	λ	κ	$i/2\sqrt{30}$	
			ω	λ	ν	$1/2\sqrt{6}$	
			τ	ν	μ	$1/4\sqrt{2}$	
			ψ	λ	λ	$7/4\sqrt{30}$	
			χ	μ	λ	$i/4\sqrt{15}$	
			χ	κ	μ	$3/4\sqrt{5}$	
W'	U'	**U**	χ	ν	ν	$i/2\sqrt{3}$	even
			ψ	μ	ν	$i/2\sqrt{2}$	
			τ	μ	κ	$1/2\sqrt{10}$	
			ω	μ	κ	$i/4\sqrt{10}$	
			ψ	ν	κ	$i/4\sqrt{6}$	
			τ	ν	μ	$1/4\sqrt{30}$	
			ψ	λ	λ	$1/4\sqrt{2}$	
			ϕ	λ	μ	$i/4$	
			ϕ	ν	λ	$1/4\sqrt{3}$	
			ω	ν	μ	$i\sqrt{2}/\sqrt{15}$	
W'	W'	A	τ	ω	a	$1/\sqrt{6}$	odd
			υ	ψ	a	$1/\sqrt{6}$	
			ϕ	χ	a	$1/\sqrt{6}$	
W'	W'	T_1	χ	χ	1	$-\sqrt{3}/\sqrt{35}$	even
			ϕ	ψ	1	$i2\sqrt{2}/\sqrt{105}$	
			ϕ	χ	0	$i/\sqrt{210}$	
			υ	ω	1	$i/\sqrt{21}$	

			υ	ψ	0	$i\sqrt{3}/\sqrt{70}$	
			τ	ω	0	$i\sqrt{5}/\sqrt{42}$	
W'	W'	T_1	ω	ψ	-1	$\sqrt{7}/2\sqrt{15}$	even
			τ	ψ	-1	$i/2\sqrt{105}$	
			υ	χ	-1	$-i/\sqrt{42}$	
			φ	φ	-1	$-1/\sqrt{21}$	
			τ	τ	0	$-\sqrt{7}/5\sqrt{6}$	
			τ	ω	0	$-i/5\sqrt{42}$	
			υ	ψ	0	$i/\sqrt{42}$	
			φ	χ	0	$-i\sqrt{2}/\sqrt{21}$	
W'	W'	V	υ	φ	-2	$i3/2\sqrt{35}$	odd
			φ	ω	2	$1/2\sqrt{7}$	
			φ	χ	1	$1/\sqrt{35}$	
			φ	χ	0	$2/\sqrt{105}$	
			υ	ω	1	$1/\sqrt{14}$	
			ψ	υ	0	$1/2\sqrt{105}$	
			τ	ω	0	$\sqrt{5}/2\sqrt{21}$	
W'	W'	V	τ	χ	-2	$\sqrt{3}/10\sqrt{7}$	odd
			φ	υ	-2	$i/2\sqrt{105}$	
			ω	χ	-2	$i\sqrt{7}/5\sqrt{3}$	
			ψ	τ	-1	$2\sqrt{2}/5\sqrt{21}$	
			υ	χ	-1	$2/\sqrt{105}$	
			ψ	ω	-1	$i\sqrt{7}/5\sqrt{6}$	
			τ	ω	0	$1/2\sqrt{35}$	
			ψ	υ	0	$3/2\sqrt{35}$	
			φ	χ	0	$1/\sqrt{35}$	
W'	W'	V	ν	υ	2	$i/\sqrt{15}$	even
			τ	φ	2	$i\sqrt{2}/5\sqrt{3}$	
			τ	χ	-2	$1/5\sqrt{6}$	
			ψ	χ	2	$i/\sqrt{30}$	
			τ	υ	1	$i\sqrt{3}/\sqrt{10}$	
			ω	υ	1	$1/10\sqrt{3}$	
			υ	χ	-1	$1/\sqrt{30}$	
			χ	χ	1	$i/\sqrt{15}$	
			ω	ω	0	$i/\sqrt{10}$	
W'	W'	T_2	τ	φ	-1	$1/3\sqrt{3}$	even
			υ	υ	-1	$-2\sqrt{2}/3\sqrt{15}$	

			ϕ	ω	-1	$i/2\sqrt{3}$	
			χ	ψ	-1	$-1/2\sqrt{15}$	
			τ	ω	0	$i\sqrt{5}/6\sqrt{3}$	
			υ	ψ	0	$-i7/6\sqrt{15}$	
			ϕ	χ	0	$-i2/3\sqrt{15}$	
W'	W'	U	τ	ϕ	ν	$-1/2\sqrt{6}$	even
			υ	υ	ν	$1/\sqrt{15}$	
			ϕ	ω	ν	$i/2\sqrt{6}$	
			χ	ψ	ν	$-1/2\sqrt{30}$	
			τ	ψ	μ	$-i/2\sqrt{3}$	
			υ	χ	μ	$i/2\sqrt{30}$	
			ϕ	ϕ	μ	$-1/\sqrt{15}$	
W'	W'	U	ϕ	ω	ν	$i\sqrt{3}/2\sqrt{10}$	odd
			χ	ψ	ν	$1/2\sqrt{6}$	
			ϕ	τ	ν	$1/2\sqrt{30}$	
			ψ	τ	μ	$i/2\sqrt{15}$	
			υ	χ	μ	$i/2\sqrt{6}$	
			ψ	ω	μ	$1/\sqrt{15}$	
E"	E'	U	α	β	ν	1/2	odd
			β	β	μ	$i/2$	
E"	U'	V	β	λ	-2	$-1/2\sqrt{5}$	odd
			α	ν	-2	$-\sqrt{3}/2\sqrt{5}$	
			β	μ	-1	$-i/\sqrt{5}$	
			α	κ	0	$-i/\sqrt{10}$	
E"	U'	T_2	β	κ	-1	$-i/2\sqrt{3}$	even
			α	μ	-1	$i/2$	
			α	κ	0	$-1/\sqrt{6}$	
E"	W'	T_1	α	ω	-1	$-\sqrt{3}/2\sqrt{5}$	odd
			α	ϕ	1	$-i/\sqrt{6}$	
			β	ω	1	$-i/2\sqrt{15}$	
			β	ψ	0	$-i/\sqrt{6}$	
E"	W'	V	β	ϕ	-2	$i\sqrt{2}/\sqrt{15}$	even
			α	ψ	-2	$i/\sqrt{15}$	
			α	ω	-1	$i/2\sqrt{3}$	
			β	χ	-1	$-1/\sqrt{30}$	
			α	τ	-1	$-1/2\sqrt{3}$	
			β	ψ	0	$-1/\sqrt{10}$	
E"	W'	U	α	ψ	κ	$-1/\sqrt{6}$	odd

			α	χ	ν	$-1/2\sqrt{3}$	
			α	τ	μ	$-i\sqrt{2}/\sqrt{15}$	
			α	φ	λ	$-i/2\sqrt{3}$	
			β	τ	λ	$-1/\sqrt{30}$	
E"	E"	A	α	β	α	$1/\sqrt{2}$	odd
E"	E"	T_2	α	α	1	$1/\sqrt{3}$	even
			α	β	0	$i/\sqrt{6}$	

Table 3.4: The isoscalars for Grey SU(2) Grey K^*

The ordering system and layout follows exactly the same pattern as with the actahedral group. Where the direct product is not multiplicity free the extra entries following the isoscalar give the combinations in order of the $\bar{V}$ coefficients in the previous table. This applies to the triads (V V V), (U V V), (W' U' V), (W' U' U), (W' W' T_1), (W' W' T_2), (W' W' U) and (W' W' V).

A A A

0	0	0	+	0	0	0	0	0	0	0	0
6	6	0	+	0	0	0	0	0	-1	0	0
6	6	6	-	0	0	0	0	2	-1	-1	-1

T1 T1 A

1	1	0	+	0	0	0	0	0	0	0	0
5	1	6	+	0	0	0	0	0	-1	0	0
5	5	0	+	0	1	0	0	-1	0	0	0
5	5	6	-	-2	2	1	0	0	-1	-1	0
6	1	6	+	0	0	0	0	0	-1	0	0
6	5	6	+	-2	1	0	0	1	-1	-1	0
6	6	0	+	0	1	0	0	0	-1	0	0
6	6	6	-	-2	1	0	0	2	-1	-1	-1

T1 T1 T1

1	1	1	+	0	0	0	0	0	0	0	0
5	5	1	-	-2	0	1	0	-1	0	0	0
5	5	5	-	-2	4	0	0	-1	-1	0	0
6	5	1	+	-2	0	0	1	0	-1	0	0
6	5	5		0							
6	6	1	+	-2	0	0	-1	0	-1	0	0
6	6	5	-	-2	3	0	-1	1	-1	-1	0
6	6	6		0							

V T1 T1

2	1	1	+	0	0	0	0	0	0	0	0
2	5	5	+	-2	2	0	0	-1	-1	0	0
2	6	5	-	-2	1	0	0	0	-1	0	0
2	6	6	-	-2	1	0	-1	1	-1	0	0
4	5	1	+	0	1	0	0	-1	0	0	0
4	5	5	-	0	1	1	0	-1	-1	0	0
4	6	5	-	-2	1	0	0	0	-1	0	0
4	6	6	+	-1	1	1	-1	1	-1	-1	0
5	5	1	-	-2	0	0	1	-1	0	0	0
5	5	5		0							
5	6	1	+	-2	0	1	0	0	-1	0	0
5	6	5	-	-2	4	0	0	0	-1	-1	0
5	6	6		0							
6	5	1	+	-2	0	1	1	-1	-1	0	0
6	5	5	+	-2	2	0	1	-1	-1	-1	0
6	6	1	-	-2	0	1	-1	1	-1	0	0
6	6	5	-	0	1	1	-1	0	-1	-1	0
6	6	6	-	-2	1	1	1	1	-1	-1	-1

V V A

2	2	0	+	0	0	0	0	0	0	0	0
4	2	6	+	0	0	0	0	0	-1	0	0
4	4	0	+	0	-2	1	0	0	0	0	0
4	4	6	-	2	-2	0	0	0	-1	0	0
5	2	6	+	0	0	0	0	0	-1	0	0
5	4	6	+	0	0	0	0	0	-1	0	0
5	5	0	+	0	0	1	0	-1	0	0	0
5	5	6	+	-2	1	0	0	0	-1	-1	0
6	2	6	+	0	0	0	0	0	-1	0	0
6	4	6	-	1	0	1	0	0	-1	-1	0
6	5	6	+	-2	0	1	1	0	-1	-1	0
6	6	0	+	0	0	1	0	0	-1	0	0
6	6	6	+	-2	2	1	0	0	-1	-1	-1

V V T1

2	2	1	+	0	0	0	0	0	0	0	0
4	2	5	-	1	-1	0	0	-1	0	0	0
4	2	6	+	0	-1	0	1	0	-1	0	0
4	4	1	-	3	-3	0	0	0	0	0	0
4	4	5	-	4	-1	1	0	-1	-1	0	0
4	4	6		0							
5	2	5	-	-2	2	0	1	-1	-1	0	0
5	2	6	+	-2	3	0	-1	0	-1	0	0
5	4	1	+	0	-1	0	1	-1	0	0	0
5	4	5	+	0	-1	1	1	-1	-1	0	0
5	4	6	-	-2	-1	2	-1	0	-1	0	0
5	5	1	-	-2	0	0	0	-1	0	0	0
5	5	5	-	-2	2	1	0	-1	-1	0	0
5	5	6		0							
6	2	5	-	-2	1	2	0	-1	-1	0	0
6	2	6	+	-2	1	0	-1	0	-1	0	0
6	4	5	-	-2	-1	0	0	-1	-1	0	0
6	4	6	+	-1	-1	1	-1	2	-1	-1	0
6	5	1	+	-2	0	3	0	-1	-1	0	0
6	5	5	+	0	4	0	0	-1	-1	-1	0
6	5	6	-	-2	1	1	0	0	-1	-1	0
6	6	1	+	-2	0	1	-1	0	-1	0	0
6	6	5	-	-2	1	1	-1	-1	-1	-1	2
6	6	6		0							

		T2			V			T1			
3	2	1	+	0	1	0	-1	0	0	0	0
3	2	5	-	2	1	0	-1	-1	0	0	0
3	4	1	-	2	-2	1	-1	0	0	0	0
3	4	5	+	3	2	0	-1	-1	-1	0	0
3	4	6		0							
3	5	5	+	1	1	0	0	-1	-1	0	0
3	5	6	-	0	1	1	-1	0	-1	0	0
3	6	5	+	0	1	1	0	-1	-1	0	0
3	6	6		0							
5	2	5	+	0	1	0	1	-1	-1	0	0
5	2	6	+	0	2	0	-1	0	-1	0	0
5	4	1	+	0	-2	0	1	-1	0	0	0
5	4	5		0							
5	4	6	+	0	2	0	-1	0	-1	0	0
5	5	1	-	0	1	0	0	-1	0	0	0
5	5	5	+	-2	1	1	0	-1	-1	0	0
5	5	6	-	-2	1	0	1	0	-1	-1	0
5	6	1	+	0	1	1	0	-1	-1	0	0
5	6	5	-	-2	1	0	0	1	-1	-1	0
5	6	6	-	-2	2	1	0	0	-1	-1	0

		T2			V			V			
3	2	2	+	0	1	0	-1	0	0	0	0
3	4	2	+	1	-1	0	-1	0	0	0	0
3	4	4	+	0	-3	0	-1	-1	0	2	0
3	5	2	+	1	0	0	0	-1	0	0	0
3	5	4	+	6	-1	0	0	-1	-1	0	0
3	5	5	+	4	0	0	-1	-1	-1	0	0
3	6	4	-	2	-1	1	0	-1	-1	0	0
3	6	5	-	1	0	1	-1	-1	-1	0	0
3	6	6	-	3	1	1	-1	-1	-1	0	0
5	4	2	-	0	-1	0	1	-1	0	0	0
5	4	4	+	1	-3	1	1	-1	-1	0	0
5	5	2	+	1	0	0	0	-1	-1	0	0
5	5	4	-	1	-1	1	0	-1	-1	0	0
5	5	5	-	-1	0	1	1	-1	-1	0	0
5	6	2	+	1	1	2	-1	-1	-1	0	0
5	6	4	-	1	-1	0	-1	-1	1	0	0
5	6	5	-	-1	0	0	1	-1	-1	-1	0
5	6	6	-	-1	3	1	0	-1	-1	-1	0

T2 T2 A

3	3	0	+	0	1	0	-1	0	0	0	0
3	3	6	-	2	0	0	-1	0	-1	0	0
5	3	6	+	0	0	0	0	0	-1	0	0
5	5	0	+	0	1	0	0	-1	0	0	0
5	5	6	+	0	0	1	0	0	-1	-1	0

		T2			T2			V			
3	3	2	+	1	0	0	-1	0	0	0	0
3	3	4	-	0	1	1	-1	-1	0	0	0
3	3	5		0							
3	3	6	-	2	0	1	0	-1	-1	0	0
5	3	2	-	0	0	0	0	-1	0	0	0
5	3	4	+	1	1	0	0	-1	-1	0	0
5	3	5	-	1	4	0	-1	-1	-1	0	0
5	3	6	-	2	0	1	-1	-1	-1	0	0
5	5	2	-	4	0	0	0	-1	-1	0	0
5	5	4	-	0	1	1	0	-1	-1	0	0
5	5	5		0							
5	5	6	-	4	0	0	1	-1	-1	-1	0

T2 T2 T2

3	3	3	+	0	0	0	-1	0	0	0	0
5	3	3	+	1	0	0	0	-1	0	0	0
5	5	3	+	0	0	1	-1	-1	-1	0	0
5	5	5	-	1	0	0	1	-1	-1	0	0

U V T1

3	2	1	+	2	0	0	-1	0	0	0	0
3	2	5	+	0	2	0	-1	-1	0	0	0
3	4	1	+	0	-1	1	-1	0	0	0	0
3	4	5	-	-1	-1	0	-1	1	-1	0	0
3	4	6	+	-1	-1	1	0	0	-1	0	0
3	5	5	-	1	2	0	0	-1	-1	0	0
3	5	6		0							
3	6	5		0							
3	6	6	+	0	1	1	-1	0	-1	0	0
4	2	5	+	0	-1	0	1	-1	0	0	0
4	2	6	+	1	-1	0	0	0	-1	0	0
4	4	1	-	0	-3	0	1	0	0	0	0
4	4	5	+	-1	-1	1	1	-1	-1	0	0
4	4	6	-	-1	1	0	0	0	-1	0	0
4	5	1	+	3	-1	0	0	-1	0	0	0
4	5	5	-	1	-1	1	0	-1	-1	0	0
4	5	6	-	1	-1	0	0	0	-1	0	0
4	6	5	-	3	-1	0	1	-1	-1	0	0
4	6	6	+	0	-1	1	0	0	-1	-1	0
6	2	5	+	2	1	0	0	-1	-1	0	0
6	2	6	+	2	1	0	-1	0	-1	0	0
6	4	5	-	0	-1	0	0	-1	-1	0	0
6	4	6	-	-1	-1	3	-1	0	-1	-1	0
6	5	1	+	2	0	1	0	-1	-1	0	0
6	5	5	+	0	2	0	0	-1	-1	-1	0
6	5	6	+	0	1	1	0	0	-1	-1	0
6	6	1	-	2	0	1	-1	0	-1	0	0

6	6	5	+	0	3	1	-1	-1	-1	-1	0
6	6	6	-	0	1	1	1	0	-1	-1	-1

		U			T2			T1				
3	3	1	+	0	1	0	-1	0	0	0	0
3	3	5	+	-1	1	0	-1	-1	0	0	0
3	3	6	+	-1	1	0	0	0	-1	0	0
3	5	5	-	0	1	1	0	-1	-1	0	0
3	5	6	-	1	1	0	-1	0	-1	0	0
4	3	1	-	0	-2	0	0	0	0	0	0
4	3	5	-	-1	2	1	0	-1	-1	0	0
4	3	6	+	-1	2	0	-1	0	-1	0	0
4	5	1	-	2	-2	1	0	-1	0	0	0
4	5	5	-	0	2	0	0	-1	-1	0	0
4	5	6		0							
6	3	5	-	1	1	0	0	-1	-1	0	0
6	3	6	+	-1	2	0	-1	0	-1	0	0
6	5	1	+	3	1	0	0	-1	-1	0	0
6	5	5	+	3	1	1	0	-1	-1	-1	0
6	5	6		0							

		U			T2			V			
3	3	2	+	0	0	0	-1	0	0	0	0
3	3	4	+	3	-1	1	-1	-1	0	0	0
3	3	5	-	-1	0	1	0	-1	0	0	0
3	3	6	-	-1	0	1	0	-1	-1	0	0
3	5	2	+	1	0	0	0	-1	0	0	0
3	5	4	+	6	-1	0	0	-1	-1	0	0
3	5	5	-	0	0	0	-1	-1	-1	0	0
3	5	6	+	1	0	1	-1	-1	-1	0	0
4	3	2	+	0	-1	0	0	0	0	0	0
4	3	4	-	3	-3	0	0	-1	0	0	0
4	3	5	-	-1	-1	0	1	-1	-1	0	0
4	3	6	+	-1	-1	1	-1	-1	1	0	0
4	5	2	+	1	-1	0	0	-1	0	0	0
4	5	4	-	6	-3	1	0	-1	-1	0	0
4	5	5	-	0	-1	1	1	-1	-1	0	0
4	5	6	-	4	-1	0	0	-1	-1	0	0
6	3	4	+	4	-1	1	0	-1	-1	0	0
6	3	5	+	1	0	1	-1	-1	-1	0	0
6	3	6	-	-1	3	1	-1	-1	-1	0	0
6	5	2	+	3	1	0	-1	-1	-1	0	0
6	5	4	-	1	-1	0	-1	-1	-1	0	0
6	5	5	-	5	0	0	1	-1	-1	-1	0
6	5	6	+	3	1	1	0	-1	-1	-1	0

U U A

3	3	0	+	2	0	0	-1	0	0	0	0
3	3	6	+	0	1	0	-1	0	-1	0	0
4	3	6	+	0	0	0	0	0	-1	0	0
4	4	0	+	2	-2	0	0	0	0	0	0
4	4	6	+	0	-2	1	0	0	-1	0	0
6	3	6	+	0	0	0	0	0	-1	0	0
6	4	6	-	0	0	0	1	0	-1	-1	0
6	6	0	+	2	0	0	0	0	-1	0	0
6	6	6	+	2	0	2	0	0	-1	-1	-1

U U T1

3	3	1	+	0	0	0	-1	0	0	0	0
3	3	5	-	1	2	0	-1	-1	0	0	0
3	3	6		0							
4	3	1	+	0	-1	0	0	0	0	0	0
4	3	5	+	1	-1	1	0	-1	-1	0	0
4	3	6	+	5	-1	0	-1	0	-1	0	0
4	4	1	-	0	-3	1	0	0	0	0	0
4	4	5	+	1	-1	0	0	-1	-1	0	0
4	4	6		0							
6	3	5	-	1	2	0	0	-1	-1	0	0
6	3	6	-	-1	1	0	-1	0	-1	0	0
6	4	5	-	0	-1	1	1	-1	-1	0	0
6	4	6	+	-1	-1	0	0	2	-1	-1	0
6	6	1	+	3	0	0	-1	0	-1	0	0
6	6	5	-	3	1	2	-1	-1	-1	-1	0
6	6	6		0							

U U V

3	3	2	+	0	1	0	-1	0	0	0	0	
3	3	4	-	1	-2	1	-1	-1	0	0	0	
3	3	5		0								
3	3	6	+	1	1	1	0	-1	-1	0	0	
4	3	2	-	0	-2	0	0	0	0	0	0	
4	3	4	+	1	0	0	0	-1	0	0	0	
4	3	5	-	5	-2	0	1	-1	-1	0	0	
4	3	6	+	1	-2	1	-1	-1	-1	0	0	
4	4	2	-	0	-2	0	1	-1	0	0	0	
4	4	4	-	1	-4	3	1	-1	-1	0	0	
4	4	5		0								
4	4	6	-	1	0	0	1	-1	-1	0	0	
6	3	4	+	4	-2	1	0	-1	-1	0	0	
6	3	5	-	1	1	1	-1	-1	-1	0	0	
6	3	6	-	-1	2	1	-1	-1	-1	0	0	
6	4	2	+	8	-2	0	0	-1	-1	0	0	
6	4	4	+	4	-2	0	0	-1	-1	0	0	
6	4	5	+	0	-2	0	0	1	-1	0	0	
6	4	6	+	-1	-2	1	0	-1	-1	-1	0	
6	6	2	-	3	2	0	-1	-1	-1	0	0	
6	6	4	-	0	-2	1	-1	-1	-1	-1	0	41
6	6	5		0								
6	6	6	-	3	2	1	1	-1	-1	-1	-1	

U U T2

3	3	3	+	1	0	0	-1	0	0	0	0
3	3	5	-	0	0	0	0	-1	0	0	0
4	3	3	+	1	-1	0	0	-1	0	0	0
4	3	5	-	0	-1	1	1	-1	-1	0	0
4	4	3	+	1	-3	1	1	-1	0	0	0
4	4	5	-	0	-3	0	3	-1	-1	0	0
6	3	3	+	4	0	0	0	-1	-1	0	0
6	3	5	+	2	0	2	-1	-1	-1	0	0
6	4	3	-	4	-1	0	-1	-1	-1	0	0
6	4	5	-	1	-1	1	0	-1	-1	0	0
6	6	3	-	0	1	2	-1	-1	-1	0	0
6	6	5	-	2	3	0	0	-1	-1	-1	0

U U U

3	3	3		0							
4	3	3	+	5	-2	0	0	-1	0	0	0
4	4	3		0							
4	4	4	-	5	-4	0	2	-1	-1	0	0
6	3	3	-	2	1	0	0	-1	-1	0	0
6	4	3	+	2	-2	0	-1	1	-1	0	0
6	4	4	-	2	-2	1	1	-1	-1	0	0
6	6	3		0							
6	6	4	+	4	-2	2	0	-1	-1	-1	0
6	6	6	+	6	2	0	1	-1	-1	-1	-1

ED ED A

1/2 1/2	0	+	0	0	0	0	0	0	0	0
11/2 1/2	6	+	0	0	0	0	0	-1	0	0
11/211/2	0	+	-1	-1	0	0	0	0	0	0
11/211/2	6	-	-2	-1	0	0	2	-1	-1	0

ED ED T1

1/2	1/2	1	+	0	0	0	0	0	0	0	0
11/2	1/2	5	–	-1	-1	0	0	0	0	0	0
11/2	1/2	6	+	-1	-1	0	1	0	-1	0	0
11/2	11/2	1	–	-1	-2	0	0	1	-1	0	0
11/2	11/2	5	–	-1	1	0	0	1	-1	-1	0
11/2	11/2	6		0							

	UD		ED			T1				
3/2 1/2	1	+	0	0	0	0	0	0	0	0
3/211/2	5	–	-2	-1	1	0	0	-1	0	0
3/211/2	6	–	-2	-1	0	0	1	-1	0	0
9/2 1/2	5	–	0	1	0	0	-1	0	0	0
9/211/2	1	–	-1	-1	0	0	0	0	0	0
9/211/2	5	+	-3	2	0	0	0	-1	0	0
9/211/2	6	–	-3	2	0	0	1	-1	-1	0
11/2 1/2	5	–	-1	-1	0	1	-1	0	0	0
11/2 1/2	6	–	-1	-1	0	0	1	-1	0	0
11/211/2	1	+	1	-2	0	1	0	-1	0	0
11/211/2	5	+	-3	1	0	1	0	-1	-1	0
11/211/2	6	+	-3	2	0	0	1	-1	-1	0

	UD			ED			V			
3/2 1/2	2	+	0	0	0	0	0	0	0	0
3/211/2	4	+	−1	−1	0	0	0	0	0	0
3/211/2	5	+	−2	0	0	1	0	−1	0	0
3/211/2	6	+	−2	0	1	0	0	−1	0	0
9/2 1/2	4	+	1	0	−1	0	0	0	0	0
9/2 1/2	5	−	0	0	−1	1	−1	0	0	0
9/211/2	2	+	−1	1	−1	0	0	−1	0	0
9/211/2	4	+	4	−1	−1	0	0	−1	0	0
9/211/2	5	−	−3	1	−1	1	0	−1	0	0
9/211/2	6	+	−3	1	−1	0	0	−1	−1	0
11/2 1/2	5	+	−1	0	1	0	−1	0	0	0
11/2 1/2	6	−	−1	0	1	0	0	−1	0	0
11/211/2	2	−	1	−1	0	0	0	−1	0	0
11/211/2	4	+	−2	0	1	0	0	−1	0	0
11/211/2	5	+	−3	2	1	0	0	−1	−1	0
11/211/2	6	−	−3	−1	3	0	0	−1	−1	0

		UD			UD			A			
3/2	3/2	0	+	0	0	0	0	0	0	0	0
9/2	3/2	6	−	0	0	0	0	0	−1	0	0
9/2	9/2	0	+	1	0	−1	0	0	0	0	0
9/2	9/2	6	−	0	1	−1	0	0	−1	0	0
11/2	3/2	6	−	0	0	0	0	0	−1	0	0
11/2	9/2	6	+	−1	3	0	0	0	−1	−1	0
11/2	11/2	0	+	0	−1	0	0	0	0	0	0
11/2	11/2	6	−	1	−1	0	0	0	−1	−1	0

UD UD T1

3/2	3/2	1	+	0	0	0	0	0	0	0	0
9/2	3/2	5	+	−1	2	−1	0	−1	0	0	0
9/2	3/2	6	−	−1	1	−1	1	0	−1	0	0
9/2	9/2	1	−	1	3	−2	0	−1	0	0	0
9/2	9/2	5	−	3	4	−2	0	−1	−1	0	0
9/2	9/2	6		0							
11/2	3/2	5	+	3	−1	0	1	−1	−1	0	0
11/2	3/2	6	−	1	−1	1	−1	0	−1	0	0
11/2	9/2	1	+	2	−1	−1	1	−1	0	0	0
11/2	9/2	5	+	−2	2	−1	1	−1	−1	0	0
11/2	9/2	6	−	−2	2	−1	−1	2	−1	−1	0
11/2	11/2	1	−	0	−2	1	0	−1	−1	0	0
11/2	11/2	5	−	4	1	1	0	−1	−1	−1	0
11/2	11/2	6		0							

	UD			UD			V				
3/2 3/2	2	+	0	0	0	0	0	0	0	0	
9/2 3/2	4	−	3	0	−1	0	−1	0	0	0	
9/2 3/2	5	+	−1	1	−1	1	−1	0	0	0	
9/2 3/2	6	−	−1	0	1	1	−1	−1	0	0	
9/2 9/2	2	+	2	1	−2	0	−1	0	0	0	
9/2 9/2	4	−	2	2	0	0	−1	−1	0	0	
9/2 9/2	5		0								
9/2 9/2	6	+	1	1	−2	1	−1	−1	0	0	
11/2 3/2	4	−	0	−1	0	1	−1	0	0	0	
11/2 3/2	5	+	1	0	0	0	−1	−1	0	0	
11/2 3/2	6	+	3	0	1	−1	−1	−1	0	0	
11/2 9/2	2	−	2	1	−1	1	−1	−1	0	0	
11/2 9/2	4	−	3	−1	−1	1	−1	−1	0	0	
11/2 9/2	5	−	−2	1	−1	0	1	−1	0	0	
11/2 9/2	6	−	−2	1	−1	−1	−1	−1	−1	0	53
11/211/2	2	−	0	−1	0	−1	−1	−1	2	0	
11/211/2	4	−	1	0	1	−1	−1	−1	0	0	
11/211/2	5		0								
11/211/2	6	−	4	−1	1	1	−1	−1	−1	0	

UD UD T2

3/2	3/2	3	+	0	1	0	-1	0	0	0	0
9/2	3/2	3	-	3	0	-1	-1	0	0	0	0
9/2	3/2	5	+	0	0	-1	1	-1	0	0	0
9/2	9/2	3	+	2	0	-2	-1	-1	-1	2	0
9/2	9/2	5	+	2	0	-2	1	-1	-1	0	0
11/2	3/2	5	-	0	3	0	0	-1	-1	0	0
11/2	9/2	3	+	3	2	-1	0	-1	-1	0	0
11/2	9/2	5	-	-1	2	-1	0	-1	-1	0	0
11/2	11/2	3	-	0	1	1	-1	-1	-1	0	0
11/2	11/2	5	-	1	1	1	1	-1	-1	-1	0

UD UD U

3/2	3/2	3	+	2	0	0	-1	0	0	0	0
9/2	3/2	3	+	1	1	-1	-1	0	0	0	0
9/2	3/2	4	-	1	0	-1	1	-1	0	0	0
9/2	3/2	6	-	2	0	-1	1	-1	-1	0	0
9/2	9/2	3	-	6	3	-2	-1	-1	-1	0	0
9/2	9/2	4		0							
9/2	9/2	6		0							
11/2	3/2	4	+	2	-1	0	0	-1	0	0	0
11/2	3/2	6	+	2	2	1	-1	-1	-1	0	0
11/2	9/2	3	-	1	1	-1	0	-1	-1	0	0
11/2	9/2	4	-	1	-1	-1	0	1	-1	0	0
11/2	9/2	6	+	1	1	-1	-1	1	-1	-1	0
11/2	11/2	3	+	2	2	1	-1	-1	-1	0	0
11/2	11/2	4		0							
11/2	11/2	6		0							

	WD		ED				V			
5/2 1/2	2	+	0	0	0	0	0	0	0	0
5/211/2	4	−	1	−2	0	0	0	−1	0	0
5/211/2	5	−	−1	0	0	0	0	−1	0	0
5/211/2	6	−	−1	−1	1	0	0	−1	0	0
7/2 1/2	4	−	0	−2	1	0	0	0	0	0
7/211/2	2	−	−1	−1	0	0	0	0	0	0
7/211/2	4	+	−1	−1	1	0	0	−1	0	0
7/211/2	5	−	0	−1	1	0	0	−1	0	0
7/211/2	6	+	−1	−1	1	0	0	−1	0	0
9/2 1/2	4	−	0	−2	−1	1	0	0	0	0
9/2 1/2	5	−	1	2	−1	0	−1	0	0	0
9/211/2	2	+	2	−1	−1	1	0	−1	0	0
9/211/2	4	+	1	−1	−1	1	0	−1	0	0
9/211/2	5	+	−2	−1	−1	0	0	−1	0	0
9/211/2	6	−	−2	−1	−1	1	0	1	−1	0
11/2 1/2	5	−	−1	0	1	0	−1	0	0	0
11/2 1/2	6	−	−1	0	1	0	0	−1	0	0
11/211/2	2	−	−1	1	0	0	0	−1	0	0
11/211/2	4	+	−2	−2	1	0	0	−1	0	0
11/211/2	5	+	1	0	1	0	0	−1	−1	0
11/211/2	6	+	−1	−1	1	0	0	−1	−1	0

WD ED T2

5/2 1/2	3	+	0	1	0	-1	0	0	0	0
5/211/2	3	-	1	-1	0	-1	0	0	0	0
5/211/2	5	+	0	-1	1	0	0	-1	0	0
7/2 1/2	3	-	0	1	0	-1	0	0	0	0
7/211/2	3	-	-1	0	0	-1	0	-1	0	0
7/211/2	5	+	-1	0	0	0	0	-1	0	0
9/2 1/2	5	-	0	1	0	0	-1	0	0	0
9/211/2	3	-	1	0	0	0	0	-1	0	0
9/211/2	5	-	-1	0	0	0	0	-1	0	0
11/2 1/2	5	+	0	1	0	0	-1	0	0	0
11/211/2	3	-	-1	-1	0	0	0	-1	0	0
11/211/2	5	-	-2	-1	2	0	0	-1	-1	0

	WD			ED			U			
5/2 1/2	3	+	2	0	0	-1	0	0	0	0
5/211/2	3	+	-1	0	0	-1	0	0	0	0
5/211/2	4	+	-1	-2	1	1	0	-1	0	0
5/211/2	6	+	2	-1	0	0	0	-1	0	0
7/2 1/2	3	+	-2	2	0	-1	0	0	0	0
7/2 1/2	4	-	-2	-2	0	1	0	0	0	0
7/211/2	3	+	-1	-1	2	-1	0	-1	0	0
7/211/2	4	-	-1	-1	0	1	0	-1	0	0
7/211/2	6	-	-4	-1	0	0	0	-1	0	0
9/2 1/2	4	+	2	-2	0	0	0	0	0	0
9/211/2	3	+	-1	-1	0	0	0	-1	0	0
9/211/2	4	-	-1	-1	0	0	0	-1	0	0
9/211/2	6	+	1	-1	0	1	0	-1	-1	0
11/2 1/2	6	-	2	0	0	0	0	-1	0	0
11/211/2	3	+	1	0	0	0	0	-1	0	0
11/211/2	4	+	0	-2	0	1	0	-1	0	0
11/211/2	6	-	0	-1	2	0	0	-1	-1	0

	WD			UD			T1				
5/2 3/2	1	+	0	0	0	0	0	0	0	0	
5/2 9/2	5	+	1	1	-1	0	-1	-1	0	0	
5/2 9/2	6	+	0	1	-1	0	0	-1	0	0	
5/211/2	5	-	-1	2	0	0	-1	-1	0	0	
5/211/2	6	-	-1	1	1	-1	0	-1	0	0	
7/2 3/2	5	-	0	1	0	0	-1	0	0	0	
7/2 9/2	1	+	1	0	-1	0	0	0	0	0	
7/2 9/2	5	-	0	1	1	0	-1	-1	0	0	
7/2 9/2	6	+	0	1	-1	0	0	-1	0	0	
7/211/2	5	-	0	1	1	0	-1	-1	0	0	
7/211/2	6	+	-1	1	1	-1	0	-1	0	0	
9/2 3/2	5	-	-1	1	-1	1	-1	0	0	0	
9/2 3/2	6	-	-1	2	-1	0	0	-1	0	0	
9/2 9/2	1	-	3	0	-2	1	-1	0	0	0	
9/2 9/2	5	+	1	1	-2	1	-1	-1	0	0	
9/2 9/2	6	-	-1	1	0	0	0	-1	0	0	
9/211/2	1	+	0	2	-1	0	-1	0	0	0	
9/211/2	5	-	4	1	-1	0	-1	-1	0	0	
9/211/2	6	-	2	1	-1	0	0	-1	-1	0	
11/2 3/2	5	-	-2	2	0	1	-1	-1	0	0	
11/2 3/2	6	-	-2	2	1	-1	0	-1	0	0	
11/2 9/2	1	+	-1	0	-1	1	-1	0	0	0	
11/2 9/2	5	+	-3	1	-1	1	-1	-1	0	0	
11/2 9/2	6	+	-3	1	-1	-1	0	-1	-1	0	31
11/211/2	1	-	1	1	1	0	-1	-1	0	0	
11/211/2	5	+	-3	2	1	0	-1	-1	-1	0	
11/211/2	6	-	-3	1	1	1	0	-1	-1	0	

WD UD T2

5/2 3/2	3	+	0	1	0	-1	0	0	0	0
5/2 9/2	3	-	1	2	-1	-1	-1	0	0	0
5/2 9/2	5	+	1	2	-1	1	-1	-1	0	0
5/211/2	3	+	0	-1	1	0	-1	0	0	0
5/211/2	5	-	5	-1	0	-1	-1	-1	0	0
7/2 3/2	3	+	-2	0	1	-1	0	0	0	0
7/2 3/2	5	-	-2	0	0	1	-1	0	0	0
7/2 9/2	3	+	0	4	-1	-1	-1	0	0	0
7/2 9/2	5		0							
7/211/2	3	+	-2	0	1	0	-1	-1	0	0
7/211/2	5	+	-2	0	1	-1	1	-1	0	0
9/2 3/2	3	+	0	0	-1	0	0	0	0	0
9/2 3/2	5	+	3	0	-1	0	-1	0	0	0
9/2 9/2	3	+	1	4	-2	0	-1	-1	0	0
9/2 9/2	5	+	1	4	-2	0	-1	-1	0	0
9/211/2	3	-	0	0	-1	-1	-1	-1	0	2
9/211/2	5	-	2	0	-1	1	-1	-1	0	0
11/2 3/2	5	+	2	1	0	0	-1	-1	0	0
11/2 9/2	3	+	3	2	-1	0	-1	-1	0	0
11/2 9/2	5	-	3	2	-1	0	-1	-1	0	0
11/211/2	3	+	6	-1	1	-1	-1	-1	0	0
11/211/2	5	-	1	-1	1	1	-1	-1	-1	0

		WD			WD			A			
5/2	5/2	0	+	0	0	0	0	0	0	0	0
7/2	5/2	6	-	0	0	0	0	0	-1	0	0
7/2	7/2	0	+	-2	1	0	0	0	0	0	0
7/2	7/2	6	-	-2	0	0	0	0	-1	0	0
9/2	5/2	6	-	0	0	0	0	0	-1	0	0
9/2	7/2	6	+	0	0	0	0	0	-1	0	0
9/2	9/2	0	+	0	1	-1	0	0	0	0	0
9/2	9/2	6	+	1	0	-1	0	0	-1	0	0
11/2	5/2	6	-	0	0	0	0	0	-1	0	0
11/2	7/2	6	-	0	0	0	0	0	-1	0	0
11/2	9/2	6	+	-1	0	0	1	0	-1	-1	0
11/2	11/2	0	-	-1	0	0	0	0	0	0	0
11/2	11/2	6	-	-2	0	2	0	0	-1	-1	0

	EDD			ED			U			
7/2 1/2	3	+	−2	0	0	0	0	0	0	0
7/2 1/2	4	+	−2	0	0	0	0	0	0	0
7/211/2	3	−	−1	1	0	0	0	−1	0	0
7/211/2	4	+	−1	−1	0	0	0	−1	0	0
7/211/2	6	−	−4	1	0	1	0	−1	0	0

	EDD			UD			V			
7/2 3/2	2	+	-2	0	0	0	0	0	0	0
7/2 3/2	4	+	-2	0	0	0	0	0	0	0
7/2 3/2	5	+	-1	1	0	0	-1	0	0	0
7/2 9/2	2	+	2	1	-1	0	-1	0	0	0
7/2 9/2	4	+	0	2	-1	0	-1	-1	0	0
7/2 9/2	5	+	1	3	-1	0	-1	-1	0	0
7/2 9/2	6	-	1	1	-1	1	-1	-1	0	0
7/211/2	2	-	-2	1	0	0	-1	0	0	0
7/211/2	4	+	-2	-1	3	0	-1	-1	0	0
7/211/2	5	-	-1	1	1	0	-1	-1	0	0
7/211/2	6	-	0	1	1	0	-1	-1	0	0

		EDD			UD			T2			
7/2	3/2	3	+	−2	0	0	0	0	0	0	0
7/2	3/2	5	+	−2	0	1	0	−1	0	0	0
7/2	9/2	3	−	0	0	0	0	−1	0	0	0
7/2	9/2	5	−	4	0	0	0	−1	−1	0	0
7/2	11/2	3	−	−2	2	0	1	−1	−1	0	0
7/2	11/2	5	+	−2	2	0	0	−1	−1	0	0

		EDD			WD			T1			
7/2	5/2	1	+	-2	0	0	0	0	0	0	0
7/2	5/2	5	+	-3	1	1	0	-1	0	0	0
7/2	5/2	6	+	-3	1	0	0	0	-1	0	0
7/2	7/2	1	+	-2	0	0	0	0	0	0	0
7/2	7/2	5	-	1	1	0	0	-1	-1	0	0
7/2	7/2	6	+	-1	1	0	0	0	-1	0	0
7/2	9/2	1	-	-2	0	0	0	0	0	0	0
7/2	9/2	5	-	-3	1	0	0	-1	-1	0	0
7/2	9/2	6	+	-3	1	0	0	0	-1	0	0
7/2	11/2	5	-	0	1	0	1	-1	-1	0	0
7/2	11/2	6	-	-3	1	0	0	0	-1	0	0

	EDD			WD			V			
7/2 5/2	2	+	-2	0	0	0	0	0	0	0
7/2 5/2	4	-	4	-2	0	0	-1	0	0	0
7/2 5/2	5	+	-3	0	0	1	-1	0	0	0
7/2 5/2	6	+	-3	0	3	0	-1	-1	0	0
7/2 7/2	2	-	-2	0	0	0	0	0	0	0
7/2 7/2	4	-	-2	-1	1	0	-1	0	0	0
7/2 7/2	5	+	-1	0	1	1	-1	-1	0	0
7/2 7/2	6	-	1	0	1	0	-1	-1	0	0
7/2 9/2	2	-	-2	0	-1	1	-1	0	0	0
7/2 9/2	4	+	6	-1	-1	1	-1	-1	0	0
7/2 9/2	5	+	-3	0	-1	3	-1	-1	0	0
7/2 9/2	6	+	-3	0	-1	0	-1	-1	2	0
7/211/2	2	+	1	0	0	0	-1	0	0	0
7/211/2	4	+	3	-2	1	0	-1	-1	0	0
7/211/2	5	-	0	0	1	0	-1	-1	0	0
7/211/2	6	-	-3	0	1	0	-1	-1	0	0

	EDD			WD			U			
7/2 5/2	3	+	-2	0	0	0	0	0	0	0
7/2 5/2	4	-	-2	-2	1	1	-1	0	0	0
7/2 5/2	6	+	4	0	0	0	-1	-1	0	0
7/2 7/2	3	+	0	0	0	0	-1	0	0	0
7/2 7/2	4	+	0	-1	0	1	-1	0	0	0
7/2 7/2	6	+	0	0	0	0	-1	-1	0	0
7/2 9/2	3	+	-2	0	0	1	-1	0	0	0
7/2 9/2	4	-	-2	-1	0	2	-1	-1	0	0
7/2 9/2	6	-	4	0	0	0	-1	-1	0	0
7/211/2	3	+	-1	0	0	1	-1	-1	0	0
7/211/2	4	+	-1	-2	0	3	-1	-1	0	0
7/211/2	6	-	-4	0	2	0	-1	-1	0	0

		EDD			EDD			A			
7/2	7/2	0	+	−2	0	0	0	0	0	0	0
7/2	7/2	6	+	−2	1	0	0	0	−1	0	0

EDD EDD T2

7/2	7/2	3	+	-2	0	0	1	-1	0	0	0
7/2	7/2	5	-	-2	0	0	2	-1	-1	0	0

								V					V	V								
2	2	2	+	0	0	0	0	0	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
4	2	2	+	0	-2	1	0	0	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
4	4	2	+	1	0	0	0	-1	0	0	0		+	2	-2	0	0	0	0	0	0	1/ 1
													-	0	-2	1	0	0	0	0	0	1/ 1
4	4	4	+	0	-4	2	0	-1	-1	0	0	181	-	2	2	1	-2	0	0	0	0	251/181
													-	0	0	0	-2	0	0	0	0	10987/181
5	4	2	+	2	-2	0	1	-1	0	0	0		-	-1	2	0	-1	0	0	0	0	1/ 1
													-	-1	0	1	-1	0	0	0	0	1/ 1
5	4	4		0																		
5	5	2	+	-2	3	0	1	-1	-1	0	0		-	0	-2	0	-2	2	0	0	0	1/ 1
													+	6	-2	1	-2	0	0	0	0	1/ 1
5	5	4	+	0	-2	2	1	-1	-1	0	0		-	0	2	1	-2	0	0	0	0	1/ 1
													+	2	0	0	-2	0	0	0	0	1/ 1
5	5	5		0																		
6	4	2	+	0	-2	1	2	-1	-1	0	0		+	0	2	1	-2	0	0	0	0	1/ 1
													-	2	0	0	-2	0	0	0	0	1/ 1
6	4	4	+	2	-2	0	2	-1	-1	0	0		+	2	0	0	-2	0	0	0	0	1/ 1
													+	0	2	1	-2	0	0	0	0	1/ 1
6	5	2	+	-2	0	1	-1	-1	-1	0	0	83	-	0	0	1	-1	0	0	0	0	1/ 83
													+	6	2	0	-1	0	0	0	0	1/ 83
6	5	4	+	-2	-2	0	-1	1	-1	0	0	23	-	0	4	0	-1	0	0	0	0	1/ 23
													+	4	0	1	-1	0	0	0	0	1/ 23
6	5	5	+	-2	2	0	1	-1	-1	-1	0	29	-	0	-1	0	-1	0	0	2	0	1/ 29
													-	6	-1	1	-1	0	0	0	0	1/ 29
6	6	2	+	-2	3	0	-1	-1	-1	0	0	29	-	0	-1	0	-1	0	0	2	0	1/ 29

													-	6	-1	1	-1	0	0	0	0	1/ 29
6	6	4	+	-1	-2	2	-1	-1	-1	-1	0	43	-	0	2	1	-1	0	0	0	0	1/ 43
													+	8	0	0	-1	0	0	0	0	1/ 43
6	6	5		0																		
6	6	6	+	-2	0	1	1	-1	-1	-1	-1	943	-	0	0	0	-1	0	0	0	0	61/943
													+	6	2	1	-1	0	0	0	0	1/943

								U				V										
3	2	2	+	2	0	0	-1	0	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
3	4	2	+	0	-2	0	-1	0	0	0	0	29	-	-1	2	0	0	0	0	0	0	1/ 29
													+	-1	0	0	2	0	0	0	0	1/ 29
3	4	4	+	4	0	0	-1	-1	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
3	5	2	+	0	1	0	0	-1	0	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													-	-1	0	0	0	0	0	0	0	1/ 1
3	5	4	+	-1	-2	0	0	-1	-1	0	0	317	-	-1	2	0	0	0	0	0	0	1/317
													+	-1	0	4	0	0	0	0	0	1/317
3	5	5	+	2	3	0	-1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
3	6	4	+	-1	-2	1	0	-1	-1	0	0	101	-	-1	4	0	0	0	0	0	0	1/101
													+	-1	0	0	0	2	0	0	0	1/101
3	6	5	+	4	1	1	-1	-1	-1	0	0		-	-1	0	0	0	0	0	0	0	1/ 1
													-	-1	0	0	0	0	0	0	0	1/ 1
3	6	6	+	1	0	1	-1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
4	2	2	+	2	-2	0	0	0	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
4	4	2	+	0	-2	0	0	-1	0	0	0	37	+	-1	0	0	2	0	0	0	0	1/ 37
													+	-1	0	2	0	0	0	0	0	1/ 37
4	4	4	+	4	-4	1	0	-1	-1	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
4	5	2	+	0	-2	0	0	-1	0	1	0		+	-1	2	0	0	0	0	-1	0	1/ 1
													-	-1	0	2	0	0	0	-1	0	1/ 1
4	5	4	+	-1	0	1	0	-1	0	0	0		-	-1	0	2	0	0	-1	0	0	1/ 1

														-	-1	0	0	0	0	-1	0	0	1/ 1
4	5	5	+	2	-2	1	1	-1	-1	0	0				0								
													-	0	0	0	0	0	0	0	0	1/ 1
4	6	2	+	1	-2	3	0	-1	-1	0	0		-	-1	2	-1	0	0	0	0	0	1/ 1
													+	-1	0	-1	0	0	0	0	0	1/ 1
4	6	4	+	-1	-2	0	0	-1	-1	0	0	173	-	-1	0	0	0	2	0	0	0	1/173
													-	-1	2	2	0	0	0	0	0	1/173
4	6	5	+	1	-2	0	0	-1	-1	1	0		-	-1	2	0	0	0	0	-1	0	1/ 1
													+	-1	0	2	0	0	0	-1	0	1/ 1
4	6	6	+	1	-2	1	0	-1	-1	-1	2			0								
													+	0	0	0	0	0	0	0	0	1/ 1
6	4	2	+	2	-2	1	1	-1	-1	0	0		+	-1	2	-1	0	0	0	0	0	1/ 1
													-	-1	0	-1	0	0	0	0	0	1/ 1
6	4	4	+	5	-2	0	1	-1	-1	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
6	5	2	+	3	0	2	-1	-1	-1	0	0		-	4	0	-2	0	0	0	0	0	1/ 1
													+	0	2	-2	0	0	0	0	0	1/ 1
6	5	4	+	0	-2	0	-1	-1	-1	0	0	953	+	-1	2	2	0	0	0	0	0	1/953
													-	-1	0	0	0	0	0	0	0	41/953
6	5	5	+	1	1	0	1	-1	-1	-1	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
6	6	2	+	3	0	0	-1	-1	0	0	0		-	2	0	0	0	0	-1	0	0	1/ 1
													-	0	2	0	0	0	-1	0	0	1/ 1
6	6	4	+	-1	-2	2	-1	-1	-1	-1	0	1201	-	-1	4	1	2	0	0	2	0	1/1201
													-	-1	0	-1	0	0	0	0	0	5303/1201
6	6	5	+	1	0	3	0	-1	-1	-1	0		-	4	0	-2	0	0	0	0	0	1/ 1
													+	0	2	-2	0	0	0	0	0	1/ 1
6	6	6	+	1	2	1	1	-1	-1	-1	-1			0								
													+	0	0	0	0	0	0	0	0	1/ 1

			WD	UD	V	
5/2	3/2	2	+ 0 0 0 0 0 0 0 0		+ 0 0 0 0 0 0 0 0	1/ 1
					0	
5/2	3/2	4	+ 0 −2 1 0 0 0 0 0		0	
					+ 0 0 0 0 0 0 0 0	1/ 1
5/2	9/2	2	+ 1 0 −1 0 0 0 0 0		0	
					− 0 0 0 0 0 0 0 0	1/ 1
5/2	9/2	4	+ 1 −2 −1 0 −1 0 0 0	31	− 1 1 0 0 0 0 0 0	1/ 31
					+ 0 0 2 0 0 0 0 0	1/ 31
5/2	9/2	5	+ 1 0 −1 1 0 −1 0 0		− 0 3 0 −1 −1 0 0 0	1/ 1
					− 1 0 2 −1 −1 0 0 0	1/ 1
5/2	9/2	6	+ 0 0 2 0 −1 −1 0 0		− 0 3 −1 −1 0 0 0 0	1/ 1
					+ 3 0 −1 −1 0 0 0 0	1/ 1
5/2	11/2	4	+ 0 −2 0 1 −1 −1 0 0	29	+ 7 0 0 −1 0 0 0 0	1/ 29
					+ 0 1 2 −1 0 0 0 0	1/ 29
5/2	11/2	5	+ −1 0 0 −1 0 −1 0 0	53	+ 0 0 0 −1 −1 0 0 0	547/ 53
					− 3 1 0 −1 −1 0 0 0	263/ 53
5/2	11/2	6	+ −1 −1 1 −1 −1 −1 0 0	199	+ 0 0 0 −1 0 0 0 0	23/199
					+ 5 3 0 −1 0 0 0 0	1/199
7/2	3/2	2	+ −2 1 0 0 0 0 0 0		0	
					+ 0 0 0 0 0 0 0 0	1/ 1
7/2	3/2	4	+ −2 −2 0 0 1 0 0 0		+ 3 0 0 0 −1 0 0 0	1/ 1
					− 0 1 0 0 −1 0 0 0	1/ 1
7/2	3/2	5	+ −1 0 0 1 −1 0 0 0		− 1 1 0 −1 0 0 0 0	1/ 1
					− 0 0 0 −1 0 0 0 0	1/ 1
7/2	9/2	2	+ 1 0 0 0 −1 0 0 0		− 0 1 −1 0 0 0 0 0	1/ 1
					− 1 0 −1 0 0 0 0 0	1/ 1
7/2	9/2	4	+ 0 −1 −1 0 −1 −1 0 0	601	− 3 1 2 −2 0 0 0 0	373/601

												-	0	0	0	-2	0	0	0	2	193/601
7/2 9/2	5	+	0	0	-1	1	0	-1	0	0		+	0	1	2	-1	-1	0	0	0	1/ 1
												-	1	0	0	-1	-1	0	0	0	1/ 1
7/2 9/2	6	+	0	0	-1	0	-1	-1	0	0	83	-	0	1	0	-1	0	0	0	0	1/ 83
												+	1	0	0	-1	0	0	2	0	1/ 83
7/211/2	2	+	-2	-1	1	1	-1	0	0	0		-	5	0	-1	-1	0	0	0	0	1/ 1
												+	0	1	-1	-1	0	0	0	0	1/ 1
7/211/2	4	+	-2	-1	2	1	-1	-1	0	0		-	3	0	-1	-1	0	0	0	0	1/ 1
												-	0	3	-1	-1	0	0	0	0	1/ 1
7/211/2	5	+	-1	-1	1	-1	0	-1	0	0		-	1	0	0	-1	-1	0	0	0	1/ 1
												-	0	1	2	-1	-1	0	0	0	1/ 1
7/211/2	6	+	-1	-1	1	-1	-1	-1	0	0	73	+	0	0	0	-1	0	0	0	2	1/ 73
												-	1	1	2	-1	0	0	0	0	1/ 73
9/2 3/2	4	+	0	-2	-1	1	-1	0	0	0	29	+	7	0	0	-1	0	0	0	0	1/ 29
												+	0	1	2	-1	0	0	0	0	1/ 29
9/2 3/2	5	+	-1	0	-1	0	-1	0	0	0	29	-	0	1	0	-1	0	0	0	0	1/ 29
												+	3	0	2	-1	0	0	0	0	1/ 29
9/2 3/2	6	+	-1	1	2	0	-1	-1	0	0		-	0	3	-1	-1	0	0	0	0	1/ 1
												+	3	0	-1	-1	0	0	0	0	1/ 1
9/2 9/2	2	+	1	0	-2	2	-1	0	0	0		+	3	1	0	-2	0	0	0	0	1/ 1
												-	0	0	2	-2	0	0	0	0	1/ 1
9/2 9/2	4	+	1	-1	0	1	-1	-1	0	0		-	1	1	0	-1	0	0	0	0	1/ 1
												-	0	0	0	-1	0	0	0	0	1/ 1
9/2 9/2	5	+	1	0	0	0	0	-1	0	0		-	0	1	2	-1	-1	0	0	0	1/ 1
												+	1	0	0	-1	-1	0	0	0	1/ 1
9/2 9/2	6	+	-1	0	-2	0	-1	-1	0	0	941	+	0	1	0	-1	0	0	0	0	23/941
												+	3	0	4	-1	0	0	0	0	1/941
9/211/2	2	+	0	-1	-1	0	-1	-1	0	0	463	+	0	0	0	-1	0	0	0	0	29/463
												+	5	1	2	-1	0	0	0	0	1/463
9/211/2	4	+	0	-1	-1	0	-1	-1	0	0	389	-	11	0	0	-1	0	0	0	0	1/389

					+	0 3 2 -1 0 0 0 0	1/389
9/211/2	5	+	1 -1 -1 1 0 -1 0 0		+	1 0 0 -1 -1 0 0 0	1/ 1
					+	0 1 2 -1 -1 0 0 0	1/ 1
9/211/2	6	+	1 -1 -1 1 -1 -1 -1 0	107	-	5 0 0 0 0 0 0 0	1/107
					-	0 1 2 0 0 0 0 0	1/107
11/2 3/2	4	+	-1 -2 1 1 -1 0 0 0		+	0 3 -1 -1 0 0 0 0	1/ 1
					-	3 0 -1 -1 0 0 0 0	1/ 1
11/2 3/2	5	+	-2 0 2 1 -1 -1 0 0		+	0 0 -2 -2 0 0 0 2	1/ 1
					-	5 3 -2 -2 0 0 0 0	1/ 1
11/2 3/2	6	+	-2 0 1 -1 -1 -1 0 0	199	-	0 0 0 -1 0 0 0 0	23/199
					-	5 3 0 -1 0 0 0 0	1/199
11/2 9/2	2	+	-1 0 -1 1 -1 -1 0 0	53	-	0 5 0 -1 0 0 0 0	1/ 53
					-	7 0 0 -1 0 0 0 0	1/ 53
11/2 9/2	4	+	3 -2 -1 1 -1 -1 0 0		+	1 1 0 -1 0 0 0 0	1/ 1
					+	0 0 0 -1 0 0 0 0	1/ 1
11/2 9/2	5	+	-3 0 -1 1 0 -1 0 0		-	0 3 0 -2 -1 0 0 0	1/ 1
					+	9 0 0 -2 -1 0 0 0	1/ 1
11/2 9/2	6	+	-3 0 0 -1 -1 -1 -1 0	14713	-	0 1 1 -1 0 0 0 0	11171/14713
					-	3 0 -1 -1 0 0 0 0	13829/14713
11/211/2	2	+	1 1 0 -1 -1 -1 0 0	31	+	0 0 0 -1 0 0 0 0	1/ 31
					+	3 3 0 -1 0 0 0 0	1/ 31
11/211/2	4	+	-2 -2 1 -1 -1 -1 0 0	607	-	0 0 2 -1 0 2 0 0	1/607
					+	3 1 0 -1 0 0 0 0	1/607
11/211/2	5	+	-3 0 2 1 0 -1 -1 0		-	0 0 -1 -1 -1 0 2 0	1/ 1
					-	5 1 -1 -1 -1 0 0 0	1/ 1
11/211/2	6	+	-3 -1 1 1 -1 -1 -1 0	127	+	0 0 2 -1 0 0 0 0	1/127
					-	5 3 0 -1 0 0 0 0	1/127

		WD	UD	U	
5/2 3/2	3	+ 2 0 0 -1 0 0 0 0		+ 0 0 0 0 0 0 0 0	1/ 1
				0	
5/2 3/2	4	+ 2 -2 0 0 0 0 0 0		0	
				+ 0 0 0 0 0 0 0 0	1/ 1
5/2 9/2	3	+ 3 0 -1 -1 -1 0 1 0		+ -4 3 0 0 0 0 -1 0	1/ 1
				+ -4 0 1 2 0 0 -1 0	1/ 1
5/2 9/2	4	+ 3 -2 -1 0 -1 0 1 0		+ -4 1 0 2 0 0 -1 0	1/ 1
				- -4 0 3 0 0 0 -1 0	1/ 1
5/2 9/2	6	+ 7 0 -1 0 -1 -1 0 0		+ -5 3 0 0 0 0 0 0	1/ 1
				+ -5 0 1 0 0 0 0 0	1/ 1
5/211/2	3	+ 1 0 0 0 -1 0 0 0		- -3 0 1 0 0 0 0 0	1/ 1
				+ -3 1 0 0 0 0 0 0	1/ 1
5/211/2	4	+ 1 -2 0 0 -1 -1 0 0	113	- -1 2 0 0 0 0 4 0	1/113
				- -1 -1 -1 0 0 0 0 2	47/113
5/211/2	6	+ 8 -1 0 -1 -1 -1 0 0		+ -5 0 1 0 0 0 0 0	1/ 1
				+ -5 3 0 0 0 0 0 0	1/ 1
7/2 3/2	3	+ 2 0 0 -1 0 0 0 0		- -6 1 1 0 0 0 0 0	1/ 1
				+ -6 0 0 2 0 0 0 0	1/ 1
7/2 3/2	4	+ 2 -2 0 0 0 0 0 0		+ -6 0 0 2 0 0 0 0	1/ 1
				+ -6 1 1 0 0 0 0 0	1/ 1
7/2 9/2	3	+ 1 0 -1 -1 -1 0 0 1		- -5 1 0 0 2 0 0 -1	1/ 1
				- -5 0 1 2 0 0 0 -1	1/ 1
7/2 9/2	4	+ 1 -1 0 0 -1 -1 0 0	23	+ -5 1 1 2 0 0 0 0	1/ 23
				- -5 0 0 0 0 0 0 0	1/ 23
7/2 9/2	6	+ 7 0 -1 0 -1 -1 0 0		- -7 1 0 0 0 0 0 0	1/ 1
				- -7 0 3 0 0 0 0 0	1/ 1
7/211/2	3	+ 2 -1 0 0 -1 -1 0 0	23	+ -6 0 1 0 2 0 0 0	1/ 23

												+	-6	1	0	0	0	2	0	1/ 23
7/211/2	4	+	2	-1	0	0	-1	-1	1	0		-	-6	0	1	0	0	2	-1	0	1/ 1
												+	-6	5	0	0	0	0	-1	0	1/ 1
7/211/2	6	+	8	-1	0	-1	-1	-1	0	0		-	-13	0	1	0	0	0	0	0	23/ 1
												+	-13	1	0	0	0	0	0	0	43/ 1
9/2 3/2	3	+	1	0	-1	0	0	0	0	0		-	-3	1	0	0	0	0	0	0	1/ 1
												-	-3	0	1	0	0	0	0	0	1/ 1
9/2 3/2	4	+	1	-2	-1	0	-1	0	0	0	47	+	-3	0	0	0	0	0	0	0	1/ 47
												-	-3	1	3	0	0	0	0	0	1/ 47
9/2 3/2	6	+	6	1	-1	0	-1	-1	0	0		-	-5	3	0	0	0	0	0	0	1/ 1
												+	-5	0	1	0	0	0	0	0	1/ 1
9/2 9/2	3	+	3	0	-2	0	-1	-1	0	0	107	-	-4	1	0	0	0	0	0	0	23/107
												+	-4	0	3	0	0	0	0	0	1/107
9/2 9/2	4	+	3	-1	-1	0	-1	-1	0	0	31	-	-4	1	3	0	0	0	0	0	1/ 31
												+	-4	0	0	0	2	0	0	0	1/ 31
9/2 9/2	6	+	2	0	-1	0	-1	-1	1	0		+	-3	1	1	0	0	0	-1	0	1/ 1
												-	-3	0	0	0	2	0	-1	0	1/ 1
9/211/2	3	+	2	-1	-1	-1	-1	-1	0	0	571	-	-4	0	4	0	0	4	0	0	1/571
												+	-4	9	-1	0	0	0	0	0	113/571
9/211/2	4	+	2	-1	-1	1	-1	-1	0	0		-	0	0	0	0	0	0	0	0	1/ 1
													0								
9/211/2	6	+	5	-1	-1	0	-1	-1	-1	0	173	+	-5	0	4	0	0	0	0	0	109/173
												+	-5	1	-1	0	0	0	0	0	943/173
11/2 3/2	4	+	5	-2	0	0	-1	0	0	0		-	-5	3	0	0	0	0	0	0	1/ 1
												+	-5	0	1	0	0	0	0	0	1/ 1
11/2 3/2	6	+	7	0	0	-1	-1	-1	0	0		+	-5	0	1	0	0	0	0	0	1/ 1
												-	-5	3	0	0	0	0	0	0	1/ 1
11/2 9/2	3	+	3	0	-1	0	-1	-1	1	0		+	-4	3	0	0	0	0	-1	0	1/ 1
												+	-4	0	1	2	0	0	-1	0	1/ 1
11/2 9/2	4	+	3	-2	-1	0	-1	-1	0	0	137	-	-4	1	0	0	0	0	0	0	23/137

													-	-4	0	1	0	2	0	0	0	1/137
11/2	9/2	6	+	6	0	-1	-1	-1	-1	-1	0	79	+	-1	1	0	0	0	0	0	0	103/ 79
													-	1	4	-1	0	0	0	0	0	41/ 79
11/2	11/2	3	+	5	0	0	-1	-1	-1	0	0		+	-3	0	1	0	0	0	0	0	1/ 1
													-	-3	1	0	0	0	0	0	0	1/ 1
11/2	11/2	4	+	3	-2	0	0	-1	-1	0	0	23	-	-4	0	1	0	0	0	0	0	1/ 23
													-	-4	1	0	0	2	0	0	0	1/ 23
11/2	11/2	6	+	3	-1	1	1	-1	-1	-1	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								

									WD				WD	T1								
5/2	5/2	1	+	0	0	0	0	0	0	0	0			+	0	0	0	0	0	0	0	0	1/ 1
															0								
5/2	5/2	5	+	0	1	0	0	-1	0	0	0				0								
														+	0	0	0	0	0	0	0	0	1/ 1
7/2	5/2	1	+	-2	1	0	0	0	0	0	0				0								
														+	0	0	0	0	0	0	0	0	1/ 1
7/2	5/2	5	+	-3	2	0	0	-1	0	0	0			+	2	-2	0	0	0	0	0	0	1/ 1
														-	0	-2	1	0	0	0	0	0	1/ 1
7/2	5/2	6	+	-3	1	0	1	0	-1	0	0			-	2	-1	1	-1	0	0	0	0	1/ 1
														-	0	-1	0	-1	0	0	0	0	1/ 1
7/2	7/2	1	+	-1	0	0	0	0	0	0	0			-	-1	-1	1	0	0	0	0	0	1/ 1
														-	-1	-1	0	0	0	0	0	0	1/ 1
7/2	7/2	5	+	0	3	0	0	-1	-1	0	0			-	-1	-1	1	0	0	0	0	0	1/ 1
														+	-1	-1	0	0	0	0	0	0	1/ 1
7/2	7/2	6		0																			
9/2	5/2	5	+	0	3	-1	1	-1	-1	0	0			+	6	-3	0	-1	0	0	0	0	1/ 1
														+	0	-3	3	-1	0	0	0	0	1/ 1
9/2	5/2	6	+	2	1	-1	0	0	-1	0	0			-	-1	-1	0	-2	0	2	0	0	1/ 1
														+	-1	-1	3	-2	0	0	0	0	1/ 1
9/2	7/2	1	+	-2	0	-1	1	0	0	0	0			+	4	-1	0	-1	0	0	0	0	1/ 1
														-	0	-1	1	-1	0	0	0	0	1/ 1
9/2	7/2	5	+	-3	3	0	1	-1	-1	0	0			+	2	-1	1	-1	0	0	0	0	1/ 1
														+	0	-1	0	-1	0	0	0	0	1/ 1
9/2	7/2	6	+	-3	4	-1	0	0	-1	0	0			-	2	0	0	-2	0	0	0	0	1/ 1
														-	0	2	1	-2	0	0	0	0	1/ 1
9/2	9/2	1	+	0	0	-2	0	-1	0	0	0		109	-	0	-1	0	-1	0	0	2	0	1/109
														+	4	-1	3	-1	0	0	0	0	1/109

9/2	9/2	5	+ 0 5 -2 0 -1 -1 0 0		- 6 -3 0 -1 0 0 0 0	1/ 1
					- 0 -3 3 -1 0 0 0 0	1/ 1
9/2	9/2	6	0			
11/2	5/2	5	+ -1 1 0 0 -1 -1 0 0	23	+ 0 4 0 -1 0 0 0 0	1/ 23
					- 4 0 1 -1 0 0 0 0	1/ 23
11/2	5/2	6	+ -1 3 0 -1 0 -1 0 0		- 0 -1 1 -1 0 0 0 0	1/ 1
					- 4 -1 0 -1 0 0 0 0	1/ 1
11/2	7/2	5	+ 0 1 0 0 -1 -1 0 0		- 2 -1 1 -1 0 0 0 0	1/ 1
					- 0 -1 0 -1 0 0 0 0	1/ 1
11/2	7/2	6	+ -3 1 0 -1 0 -1 0 0	29	+ 2 -1 -1 -1 2 0 0 0	61/ 29
					- 0 -1 0 -1 0 0 2 0	1/ 29
11/2	9/2	1	+ 3 1 -1 0 -1 0 0 0		+ -1 2 0 -1 0 0 0 0	1/ 1
					+ -1 0 1 -1 0 0 0 0	1/ 1
11/2	9/2	5	+ -2 1 -1 0 -1 -1 0 0	89	+ 0 -1 2 -1 0 0 0 0	269/ 89
					- 2 -1 1 -1 0 0 0 0	1/ 89
11/2	9/2	6	+ -2 1 -1 0 2 -1 -1 0		- 0 -1 0 -1 0 0 0 0	1/ 1
					+ 2 -1 1 -1 0 0 0 0	1/ 1
11/2	11/2	1	+ -1 0 0 0 -1 -1 0 0	83	- 0 0 -3 -1 0 0 0 0	73/ 83
					- 6 2 0 -1 0 0 0 0	1/ 83
11/2	11/2	5	+ -1 1 1 0 -1 -1 -1 0	23	+ 0 0 -2 -1 0 0 0 0	53/ 23
					+ 4 0 1 -1 0 0 0 0	1/ 23
11/2	11/2	6	0			

								WD				WD	T2								
5/2	5/2	3	+	0	1	0	-1	0	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
5/2	5/2	5	+	0	1	0	0	-1	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
7/2	5/2	3	+	0	1	0	-1	0	0	0	0		-	-1	-3	1	0	0	0	0	0	1/ 1
													+	-1	-3	0	2	0	0	0	0	1/ 1
7/2	5/2	5	+	0	1	0	0	-1	0	0	0		+	-1	-3	0	2	0	0	0	0	1/ 1
													+	-1	-3	1	0	0	0	0	0	1/ 1
7/2	7/2	3	+	-2	0	0	-1	-1	0	0	0	83	+	-1	-2	1	0	0	0	2	0	1/ 83
													+	-1	-2	0	2	0	0	0	0	1/ 83
7/2	7/2	5	+	-2	0	0	0	-1	-1	0	0	107	-	-1	-2	-1	0	0	0	0	0	293/107
													+	-1	-2	0	0	0	0	0	0	1681/107
9/2	5/2	3	+	0	1	-1	1	-1	0	0	0		-	6	-3	0	-1	0	0	0	0	1/ 1
													-	0	-3	3	-1	0	0	0	0	1/ 1
9/2	5/2	5	+	0	1	-1	0	-1	-1	0	0	23	+	0	-3	0	0	2	0	0	0	1/ 23
													-	2	-3	3	0	0	0	0	0	1/ 23
9/2	7/2	3	+	0	0	-1	1	-1	0	0	0		+	-1	-2	0	-1	2	0	0	0	1/ 1
													-	-1	-2	1	-1	0	0	0	0	1/ 1
9/2	7/2	5	+	0	0	0	0	-1	-1	0	0	23	-	-1	-2	3	0	0	0	0	0	1/ 23
													+	-1	-2	0	0	0	0	2	0	1/ 23
9/2	9/2	3	+	0	0	-2	1	-1	-1	0	0	149	+	0	-2	0	0	0	0	0	0	29/149
													-	2	-2	3	0	0	0	0	0	1/149
9/2	9/2	5	+	0	0	-2	1	-1	-1	0	0	89	-	2	-2	0	0	0	2	0	0	1/ 89
													-	0	-2	3	0	0	0	0	0	1/ 89
11/2	5/2	3	+	1	0	0	0	-1	0	0	0		-	-1	-1	1	0	0	0	0	0	1/ 1
													+	-1	-1	0	0	0	0	0	0	1/ 1
11/2	5/2	5	+	1	0	0	-1	-1	-1	0	0	67	-	0	-1	0	0	2	0	0	0	1/ 67

														+	4	-1	1	0	0	0	0	0	1/ 67
11/2	7/2	3	+	-1	1	0	0	-1	-1	0	0	29	+	-1	-3	-1	0	0	0	0	0	131/ 29
													-	-1	-3	0	0	0	0	0	0	961/ 29
11/2	7/2	5	+	-1	3	1	-1	-1	-1	0	0		-	-1	-5	0	0	0	0	0	2	1/ 1
													+	-1	-5	3	0	0	0	0	0	1/ 1
11/2	9/2	3	+	1	1	-1	-1	-1	-1	0	0		+	-1	-3	0	2	0	0	0	0	1/ 1
													+	-1	-3	1	0	0	0	0	0	1/ 1
11/2	9/2	5	+	0	2	-1	1	-1	-1	0	0		-	0	-4	0	0	0	0	0	0	1/ 1
													-	4	-4	1	0	0	0	0	0	1/ 1
11/2	11/2	3	+	0	0	2	0	-1	-1	0	0		+	0	-1	-3	-1	0	0	2	0	1/ 1
													+	4	-1	0	-1	0	0	0	0	1/ 1
11/2	11/2	5	+	-1	0	0	1	-1	-1	-1	0	103	+	0	-1	-3	0	0	0	0	0	11171/103
													-	6	-1	0	0	0	0	0	0	1/103

								WD				WD	U									
5/2	5/2	3	+	2	0	0	-1	0	0	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
5/2	5/2	4	+	2	-2	0	0	0	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
7/2	5/2	3	+	-1	-2	0	-1	0	0	0	0	37	+	-2	1	1	0	0	0	0	0	1/ 37
													-	-2	1	0	2	0	0	0	0	1/ 37
7/2	5/2	4	+	-1	2	0	0	-1	0	0	0		-	-2	-3	0	2	0	0	0	0	1/ 1
													-	-2	-1	1	0	0	0	0	0	1/ 1
7/2	5/2	6	+	5	-2	0	1	-1	-1	0	0		-	-4	1	1	0	0	0	0	0	1/ 1
													+	-4	1	0	0	0	0	0	0	1/ 1
7/2	7/2	3	+	1	1	1	-1	-1	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
7/2	7/2	4	+	1	-2	0	0	-1	0	0	0			0								
													+	0	0	0	0	0	0	0	0	1/ 1
7/2	7/2	6	+	1	1	0	1	-1	-1	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
9/2	5/2	3	+	2	-2	-1	0	-1	0	0	0	23	+	2	1	0	0	0	0	0	0	1/ 23
													-	0	1	1	0	0	0	0	0	1/ 23
9/2	5/2	4	+	2	-1	-1	1	-1	0	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	5/2	6	+	5	-2	0	-1	-1	-1	0	0	31	+	-4	1	-1	0	0	0	0	2	1/ 31
													-	-4	1	0	0	2	0	0	0	1/ 31
9/2	7/2	3	+	1	2	-1	0	-1	0	0	0		+	-4	-1	0	0	0	0	0	0	1/ 1
													+	-4	1	1	0	0	0	0	0	1/ 1
9/2	7/2	4	+	1	-2	0	1	-1	-1	0	0	31	-	-4	4	1	0	0	0	0	0	1/ 31
													-	-4	0	0	2	0	0	0	0	1/ 31
9/2	7/2	6	+	3	2	-1	-1	-1	-1	0	0		+	-2	1	0	0	0	0	0	0	1/ 1

													-	-2	-1	1	0	0	0	0	0	1/ 1
9/2	9/2	3	+	2	1	-2	1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
9/2	9/2	4	+	2	-2	-1	3	-1	-1	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
9/2	9/2	6	+	2	1	-1	1	-1	-1	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
11/2	5/2	3	+	2	0	0	0	-1	0	0	0		+	-4	0	1	0	0	0	0	0	1/ 1
													+	-4	2	0	0	0	0	0	0	1/ 1
11/2	5/2	4	+	2	-2	0	0	-1	-1	0	0	37	-	-4	2	0	0	0	0	0	0	1/ 37
													+	-4	0	1	0	2	0	0	0	1/ 37
11/2	5/2	6	+	3	1	1	-1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
														0								
11/2	7/2	3	+	2	-2	0	0	-1	-1	0	0	47	-	-4	1	3	0	0	0	0	0	1/ 47
													+	-4	1	0	0	0	2	0	0	1/ 47
11/2	7/2	4	+	2	-1	1	0	-1	-1	0	0		+	-4	0	0	0	0	0	0	0	1/ 1
													-	-4	4	-1	0	0	0	0	0	1/ 1
11/2	7/2	6	+	-1	-2	2	-1	-1	-1	0	0	71	-	-4	1	-1	0	0	0	0	0	1849/ 71
													+	-4	3	0	2	0	0	0	0	29/ 71
11/2	9/2	3	+	2	-2	-1	-1	-1	-1	0	0	2491	-	-4	1	0	0	0	0	0	0	11881/2491
													+	-4	-1	1	2	0	0	0	0	257/2491
11/2	9/2	4	+	2	1	-1	1	-1	-1	0	0		-	-4	-2	0	0	2	0	0	0	1/ 1
													+	-4	0	1	0	0	0	0	0	1/ 1
11/2	9/2	6	+	2	-2	-1	0	-1	-1	-1	0	2047	-	2	1	0	0	2	0	0	0	1/2047
													-	0	1	1	0	0	0	0	0	449/2047
11/2	11/2	3	+	2	0	1	-1	-1	-1	0	0		+	0	0	0	0	0	0	0	0	1/ 1
													-	2	-2	1	0	0	0	0	0	1/ 1
11/2	11/2	4	+	1	-2	2	0	-1	-1	0	0			0								
													-	0	0	0	0	0	0	0	0	1/ 1
11/2	11/2	6	+	1	3	0	1	-1	-1	-1	0			0								

- 0 0 0 0 0 0 0 0 1/ 1

WD WD V

5/2	5/2	2	+	0	0	0	0	0	0	0	0	+	0	0	0	0	0	0	0	0	1/ 1
													0								
													0								
5/2	5/2	4	+	0	−2	1	0	0	0	0	0		0								
												+	0	0	0	0	0	0	0	0	1/ 1
													0								
5/2	5/2	5	+	0	0	1	0	−1	0	0	0		0								
													0								
												+	0	0	0	0	0	0	0	0	1/ 1
7/2	5/2	2	+	−2	1	0	0	0	0	0	0		0								
												−	1	−2	0	0	0	0	0	0	1/ 1
												+	0	−2	0	1	0	0	0	0	1/ 1
7/2	5/2	4	+	0	−1	0	0	−1	1	0	0	−	−1	1	0	0	0	−1	0	0	1/ 1
												+	−1	2	0	0	0	−1	0	0	1/ 1
												+	0	0	0	1	0	−1	0	0	1/ 1
7/2	5/2	5	+	−3	1	0	0	0	0	0	0	+	1	−1	0	1	−1	0	0	0	1/ 1
												+	3	−2	0	1	−1	0	0	0	1/ 1
												+	0	−2	0	0	−1	0	0	0	1/ 1
7/2	5/2	6	+	−3	2	1	1	−1	−1	0	0	−	1	−2	2	−1	0	0	0	0	1/ 1
												+	5	−3	0	−1	0	0	0	0	1/ 1
												−	0	−3	0	0	0	0	0	0	1/ 1
7/2	7/2	2	+	−1	0	0	0	0	0	0	0	+	−2	0	0	0	0	0	0	0	1/ 1
												−	−2	1	0	0	0	0	0	0	1/ 1
													0								
7/2	7/2	4	+	−1	−2	1	0	−1	0	0	1	−	−2	1	2	0	0	0	0	−1	1/ 1
												−	−2	0	0	0	0	0	0	−1	1/ 1
													0								

7/2	7/2	5	+	1	1	1	0	-1	-1	0	0			0								
														0								
													-	0	0	0	0	0	0	0	0	1/ 1
7/2	7/2	6	+	0	0	1	1	-1	-1	0	0		+	-2	0	0	-1	0	0	0	0	1/ 1
													+	-2	3	0	-1	0	0	0	0	1/ 1
														0								
9/2	5/2	2	+	0	1	-1	0	0	0	0	0			0								
													-	0	-2	0	1	0	0	0	0	1/ 1
													-	1	-2	0	0	0	0	0	0	1/ 1
9/2	5/2	4	+	0	-1	-1	0	-1	0	0	0	71	-	0	1	0	1	0	0	0	0	1/ 71
														0								
													-	1	0	2	0	0	0	0	0	1/ 71
9/2	5/2	5	+	0	2	-1	0	-1	-1	0	1		+	1	-2	0	-1	0	0	0	1	1/ 1
													-	1	-3	4	-1	0	0	0	-1	1/ 1
													-	0	-3	2	0	0	0	0	-1	1/ 1
9/2	5/2	6	+	1	1	1	0	-1	-1	0	0		-	3	-1	0	-2	0	0	0	0	1/ 1
													-	1	-2	0	-2	2	0	0	0	1/ 1
													+	0	-2	2	-1	0	0	0	0	1/ 1
9/2	7/2	2	+	-2	0	2	0	-1	0	0	0		-	3	0	-3	1	0	0	0	0	1/ 1
													-	1	1	-3	1	0	0	0	0	1/ 1
													+	0	3	-3	0	0	0	0	0	1/ 1
9/2	7/2	4	+	0	-2	-1	0	-1	-1	0	0	347	+	-1	1	2	1	0	0	0	0	1/347
													+	-1	0	0	1	0	0	0	0	1/347
													-	0	4	0	0	0	0	0	0	1/347
9/2	7/2	5	+	-3	1	-1	0	-1	0	0	0	31	-	1	3	2	-1	0	-1	0	0	1/ 31
													+	5	2	0	-1	0	-1	0	0	1/ 31
													-	0	0	0	0	0	1	0	0	1/ 31
9/2	7/2	6	+	-3	0	-1	0	-1	-1	0	0	911	-	5	0	0	-2	0	0	0	0	1303/911
													+	1	-1	0	-2	0	0	0	0	1693/911
													+	0	3	0	-1	0	2	0	0	1/911

9/2	9/2	2	+	0	0	-2	0	-1	0	0	0	67	-	0	0	0	-1	0	2	0	0	1/ 67
													+	2	1	2	-1	0	0	0	0	1/ 67
														0								
9/2	9/2	4	+	0	-2	0	0	-1	-1	0	0	307	-	2	1	0	-1	0	2	0	0	1/307
													+	0	0	0	-1	2	0	0	0	1/307
														0								
9/2	9/2	5	+	0	1	0	1	-1	-1	0	0			0								
														0								
													+	0	0	0	0	0	0	0	0	1/ 1
9/2	9/2	6	+	3	0	-2	2	-1	-1	0	0		-	0	0	0	0	0	0	0	0	1/ 1
														0								
														0								
11/2	5/2	4	+	1	-2	1	2	-1	-1	0	0		-	-1	5	-1	-2	0	0	0	0	1/ 1
													-	-1	0	-1	-2	2	0	0	0	1/ 1
													-	0	2	-1	-1	0	0	0	0	1/ 1
11/2	5/2	5	+	0	0	3	-1	-1	-1	0	0		+	0	5	-3	-1	0	0	0	0	1/ 1
													-	6	2	-3	-1	0	0	0	0	1/ 1
													-	3	0	-3	0	0	0	0	0	1/ 1
11/2	5/2	6	+	0	0	1	-1	-1	-1	0	0	31	+	0	0	0	1	0	0	0	0	1/ 31
														0								
													+	3	1	0	0	0	0	0	0	1/ 31
11/2	7/2	2	+	-1	0	0	1	-1	0	0	0		+	-1	2	0	-1	0	0	0	0	1/ 1
													-	-1	-1	0	-1	0	0	0	0	1/ 1
													-	0	-1	0	0	0	0	0	0	1/ 1
11/2	7/2	4	+	-1	-1	1	2	-1	-1	0	0		+	-1	1	0	-2	0	0	0	0	1/ 1
													+	-1	4	0	-2	0	0	0	0	1/ 1
													-	0	0	0	-1	0	0	0	0	1/ 1
11/2	7/2	5	+	1	0	1	-1	-1	-1	0	0	23	+	-2	0	0	-1	0	0	2	0	1/ 23
													-	-2	-1	0	-1	0	0	0	0	29/ 23
													+	3	-1	0	0	0	0	0	0	1/ 23

11/2	7/2	6	+	-3	0	1	-1	-1	-1	0	0	241	+	1	0	0	1	0	0	0	0	1/241
													-	3	-1	0	1	0	0	0	0	1/241
													-	0	-1	4	0	0	0	0	0	1/241
11/2	9/2	2	+	2	0	-1	0	-1	-1	0	0	43	-	1	2	0	-1	0	0	0	0	1/ 43
													+	1	-1	0	-1	0	0	0	0	1/ 43
													+	0	-1	0	0	2	0	0	0	1/ 43
11/2	9/2	4	+	1	-1	-1	0	-1	-1	0	0	199	-	-1	1	0	-1	0	0	0	0	1/199
													+	-1	2	0	-1	2	0	0	0	1/199
													-	0	0	0	0	2	0	0	0	1/199
11/2	9/2	5	+	-1	0	-1	1	-1	-1	0	0	41	-	0	0	0	-1	2	0	0	0	1/ 41
													-	2	-1	0	-1	2	0	0	0	1/ 41
													-	1	-1	0	0	0	0	0	0	1/ 41
11/2	9/2	6	+	-1	0	0	0	-1	-1	-1	1	31	-	8	0	-1	-1	0	0	0	-1	353/ 31
													+	0	-1	-1	-1	0	0	0	-1	28597/ 31
													+	1	-1	-1	0	2	0	0	-1	1/ 31
11/2	11/2	2	+	0	0	0	-1	-1	-1	0	0	199	+	4	0	2	-1	0	0	0	0	1/199
													+	0	-1	0	-1	0	0	0	2	1/199
														0								
11/2	11/2	4	+	-1	-2	1	-1	-1	-1	0	1	31	+	4	3	0	-1	0	0	0	-1	1/ 31
													+	0	0	0	-1	0	0	0	-1	31/ 31
														0								
11/2	11/2	5	+	3	0	1	1	-1	-1	-1	0			0								
														0								
														0								
													-	0	0	0	0	0	0	0	0	1/ 1
11/2	11/2	6	+	0	0	1	2	-1	-1	-1	0		+	6	0	0	-2	0	0	0	0	1/ 1
													-	0	-1	2	-2	0	0	0	0	1/ 1
														0								

Table 3.5: The 6j Symbols for Grey K*

The ordering system follow the same pattern as for the octahedral group except that the numbers given are powers of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$ only.

W COEFFICIENTS FOR THE ICOSAHEDRAL GROUP

A	A	A	A	A	A	+	0	0	0	0
T1	T1	A	A	A	T1	+	0	-1	0	0
T1	T1	A	T1	T1	A	+	0	-2	0	0
T1	T1	A	T1	T1	T1	-	0	-2	0	0
T1	T1	T1	A	T1	T1	-	0	-2	0	0
T1	T1	T1	T1	A	T1	-	0	-2	0	0
T1	T1	T1	T1	T1	A	-	0	-2	0	0
T1	T1	T1	T1	T1	T1	+	-2	-2	0	0
V	T1	T1	A	T1	T1	+	0	-2	0	0
V	T1	T1	T1	T1	T1	+	-2	-2	0	0
V	T1	T1	V	T1	T1	+	-2	-2	-2	0
V	V	A	A	A	V	+	0	0	-1	0
V	V	A	T1	T1	T1	+	0	-1	-1	0
V	V	A	T1	T1	V	-	0	-1	-1	0
V	V	A	V	V	A	+	0	0	-2	0
V	V	A	V	V	T1	-	0	0	-2	0
V	V	A	V	V	V	+	0	0	-2	0
V	V	T1	A	T1	V	-	0	-1	-1	0
V	V	T1	T1	A	V	-	0	-1	-1	0
V	V	T1	T1	T1	T1	-	-2	0	-1	0
V	V	T1	T1	T1	V	+	-2	-2	-1	0
V	V	T1	T1	V	T1	-	-2	0	-2	0
V	V	T1	T1	V	V	+	-2	-1	-2	1
V	V	T1	V	T1	T1	-	-2	0	-2	0
V	V	T1	V	T1	V	+	-2	-1	-2	1
V	V	T1	V	V	A	-	0	0	-2	0
V	V	T1	V	V	T1	+	-2	-2	0	0

V	V	T1	V	V	V	-	-2	0	-2	0
V	V	V	A	V	V	+	0	0	-2	0
V	V	V	T1	T1	T1	+	-2	-1	-2	1
V	V	V	T1	T1	V	+	-2	-1	-2	1
V	V	V	T1	V	T1	+	-2	-1	-2	1
V	V	V	T1	V	V	-	-2	0	-2	0
V	V	V	V	A	V	+	0	0	-2	0
V	V	V	V	T1	T1	+	-2	-1	-2	1
V	V	V	V	T1	V	-	-2	0	-2	0
V	V	V	V	V	A	+	0	0	-2	0
V	V	V	V	V	T1	-	-2	0	-2	0
V	V	V	V	V	V	-	-2	2	-2	-2
BV	T1	T1	A	T1	T1	+	0	-2	0	0
BV	T1	T1	T1	T1	T1	+	-2	-2	0	0
BV	T1	T1	V	T1	T1	+	-2	-2	-2	0
BV	T1	T1	BV	T1	T1	+	-2	-2	-2	0
BV	V	A	A	A	V	+	0	0	-1	0
BV	V	A	T1	T1	T1	+	0	-1	-1	0
BV	V	A	T1	T1	V	-	0	-1	-1	0
BV	V	A	V	V	A	+	0	0	-2	0
BV	V	A	V	V	T1	-	0	0	-2	0
BV	V	A	V	V	V	-	4	2	-5	-2
BV	V	A	BV	V	A	+	0	0	-2	0
BV	V	A	BV	V	T1	-	0	0	-2	0
BV	V	A	BV	V	V	+	0	6	-4	-2
BV	V	T1	A	T1	V	-	0	-1	-1	0
BV	V	T1	T1	A	V	-	0	-1	-1	0
BV	V	T1	T1	T1	T1	-	-2	0	-1	0
BV	V	T1	T1	T1	V	+	-2	-2	-1	0
BV	V	T1	T1	V	T1	-	-2	0	-2	0

BV	V	T1	T1	V	V	-	2	1	-5	-1	
BV	V	T1	V	T1	T1	-	-2	0	-2	0	
BV	V	T1	V	T1	V	+	-2	-1	-2	1	
BV	V	T1	V	V	A	-	0	0	-2	0	
BV	V	T1	V	V	T1	+	-2	-2	0	0	
BV	V	T1	V	V	V	+	2	2	-5	-2	
BV	V	T1	BV	T1	T1	-	-2	0	-2	0	
BV	V	T1	BV	T1	V	-	2	1	-5	-1	
BV	V	T1	BV	V	A	-	0	0	-2	0	
BV	V	T1	BV	V	T1	+	-2	-2	0	0	
BV	V	T1	BV	V	V	+	2	0	-6	2	
BV	V	V	A	V	V	+	0	0	-6	-2	167
BV	V	V	T1	T1	T1	-	2	1	-5	-1	
BV	V	V	T1	T1	V	-	2	1	-5	-1	
BV	V	V	T1	V	T1	-	2	1	-5	-1	
BV	V	V	T1	V	V	+	2	-2	-6	-2	131
BV	V	V	V	A	V	-	4	2	-5	-2	
BV	V	V	V	T1	T1	-	2	1	-5	-1	
BV	V	V	V	T1	V	+	2	2	-5	-2	
BV	V	V	V	V	A	-	4	2	-5	-2	
BV	V	V	V	V	T1	+	2	2	-5	-2	
BV	V	V	V	V	V	+	2	0	-6	-4	131
BV	V	V	BV	A	V	+	0	6	-4	-2	
BV	V	V	BV	T1	T1	-	2	1	-5	-1	
BV	V	V	BV	T1	V	+	2	0	-6	2	
BV	V	V	BV	V	A	+	0	6	-4	-2	
BV	V	V	BV	V	T1	+	2	0	-6	2	
BV	V	V	BV	V	V	+	-2	0	-4	-4	79
BV	BV	A	A	A	V	+	0	0	-1	0	
BV	BV	A	A	A	BV	+	0	0	-1	0	

BV	BV	A	T1	T1	T1	+	0	-1	-1	0	
BV	BV	A	T1	T1	V	-	0	-1	-1	0	
BV	BV	A	T1	T1	BV	-	0	-1	-1	0	
BV	BV	A	V	V	A	+	0	0	-2	0	
BV	BV	A	V	V	T1	-	0	0	-2	0	
BV	BV	A	V	V	V	+	0	0	-6	-2	167
BV	BV	A	V	V	BV	+	0	0	-2	0	
BV	BV	A	V	BV	A	+	0	0	-2	0	
BV	BV	A	V	BV	T1	-	0	0	-2	0	
BV	BV	A	V	BV	V	-	4	2	-5	-2	
BV	BV	A	V	BV	BV	+	0	0	-2	0	
BV	BV	A	BV	V	A	+	0	0	-2	0	
BV	BV	A	BV	V	T1	-	0	0	-2	0	
BV	BV	A	BV	V	V	-	4	2	-5	-2	
BV	BV	A	BV	V	BV	+	0	0	-2	0	
BV	BV	A	BV	BV	A	+	0	0	-2	0	
BV	BV	A	BV	BV	T1	-	0	0	-2	0	
BV	BV	A	BV	BV	V	+	0	0	-2	0	
BV	BV	A	BV	BV	BV	+	0	0	-2	0	
BV	BV	T1	A	T1	V	-	0	-1	-1	0	
BV	BV	T1	A	T1	BV	-	0	-1	-1	0	
BV	BV	T1	T1	A	V	-	0	-1	-1	0	
BV	BV	T1	T1	A	BV	-	0	-1	-1	0	
BV	BV	T1	T1	T1	T1	-	-2	0	-1	0	
BV	BV	T1	T1	T1	V	+	-2	-2	-1	0	
BV	BV	T1	T1	T1	BV	+	-2	-2	-1	0	
BV	BV	T1	T1	V	T1	-	-2	0	-2	0	
BV	BV	T1	T1	V	V	-	2	1	-5	-1	
BV	BV	T1	T1	V	BV	+	-2	-1	-2	1	
BV	BV	T1	T1	BV	T1	-	-2	0	-2	0	

BV	BV	T1	T1	BV	V	+	-2	-1	-2	1
BV	BV	T1	T1	BV	BV	+	-2	-1	-2	1
BV	BV	T1	V	T1	T1	-	-2	0	-2	0
BV	BV	T1	V	T1	V	-	2	1	-5	-1
BV	BV	T1	V	T1	BV	+	-2	-1	-2	1
BV	BV	T1	V	V	A	-	0	0	-2	0
BV	BV	T1	V	V	T1	+	-2	-2	0	0
BV	BV	T1	V	V	V	+	2	4	-4	-2
BV	BV	T1	V	V	BV	-	-2	0	-2	0
BV	BV	T1	V	BV	A	-	0	0	-2	0
BV	BV	T1	V	BV	T1	+	-2	-2	0	0
BV	BV	T1	V	BV	V	+	2	2	-5	-2
BV	BV	T1	V	BV	BV	-	-2	0	-2	0
BV	BV	T1	BV	T1	T1	-	-2	0	-2	0
BV	BV	T1	BV	T1	V	+	-2	-1	-2	1
BV	BV	T1	BV	T1	BV	+	-2	-1	-2	1
BV	BV	T1	BV	V	A	-	0	0	-2	0
BV	BV	T1	BV	V	T1	+	-2	-2	0	0
BV	BV	T1	BV	V	V	+	2	2	-5	-2
BV	BV	T1	BV	V	BV	-	-2	0	-2	0
BV	BV	T1	BV	BV	A	-	0	0	-2	0
BV	BV	T1	BV	BV	T1	+	-2	-2	0	0
BV	BV	T1	BV	BV	V	-	-2	0	-2	0
BV	BV	T1	BV	BV	BV	-	-2	0	-2	0
BV	BV	V	A	V	V	-	4	2	-5	-2
BV	BV	V	A	V	BV	+	0	0	-2	0
BV	BV	V	A	BV	V	+	0	0	-2	0
BV	BV	V	A	BV	BV	+	0	0	-2	0
BV	BV	V	T1	T1	T1	+	-2	-1	-2	1
BV	BV	V	T1	T1	V	+	-2	-1	-2	1

BV	BV	V	T1	T1	BV	+	-2	-1	-2	1	
BV	BV	V	T1	V	T1	+	-2	-1	-2	1	
BV	BV	V	T1	V	V	+	2	2	-5	-2	
BV	BV	V	T1	V	BV	-	-2	0	-2	0	
BV	BV	V	T1	BV	T1	+	-2	-1	-2	1	
BV	BV	V	T1	BV	V	-	-2	0	-2	0	
BV	BV	V	T1	BV	BV	-	-2	0	-2	0	
BV	BV	V	V	A	V	-	4	2	-5	-2	
BV	BV	V	V	A	BV	+	0	0	-2	0	
BV	BV	V	V	T1	T1	+	-2	-1	-2	1	
BV	BV	V	V	T1	V	+	2	2	-5	-2	
BV	BV	V	V	T1	BV	-	-2	0	-2	0	
BV	BV	V	V	V	A	+	0	0	-2	0	
BV	BV	V	V	V	T1	-	-2	0	-2	0	
BV	BV	V	V	V	V	+	2	0	-6	-4	127
BV	BV	V	V	V	BV	-	-2	2	-2	-2	
BV	BV	V	V	BV	A	-	4	2	-5	-2	
BV	BV	V	V	BV	T1	+	2	2	-5	-2	
BV	BV	V	V	BV	V	+	2	0	-4	-4	31
BV	BV	V	V	BV	BV	+	2	4	-5	-4	
BV	BV	V	BV	A	V	+	0	0	-2	0	
BV	BV	V	BV	A	BV	+	0	0	-2	0	
BV	BV	V	BV	T1	T1	+	-2	-1	-2	1	
BV	BV	V	BV	T1	V	-	-2	0	-2	0	
BV	BV	V	BV	T1	BV	-	-2	0	-2	0	
BV	BV	V	BV	V	A	-	4	2	-5	-2	
BV	BV	V	BV	V	T1	+	2	2	-5	-2	
BV	BV	V	BV	V	V	+	2	0	-4	-4	31
BV	BV	V	BV	V	BV	+	2	4	-5	-4	
BV	BV	V	BV	BV	A	+	0	0	-2	0	

BV	BV	V	BV	BV	T1	-	-2	0	-2	0	
BV	BV	V	BV	BV	V	-	-2	2	-2	-2	
BV	BV	V	BV	BV	BV	-	-2	2	-2	-2	
BV	BV	BV	A	V	V	-	4	2	-5	-2	
BV	BV	BV	A	V	BV	+	0	0	-2	0	
BV	BV	BV	A	BV	V	+	0	0	-2	0	
BV	BV	BV	A	BV	BV	+	0	0	-2	0	
BV	BV	BV	T1	T1	T1	+	-2	-1	-2	1	
BV	BV	BV	T1	T1	V	+	-2	-1	-2	1	
BV	BV	BV	T1	T1	BV	+	-2	-1	-2	1	
BV	BV	BV	T1	V	T1	+	-2	-1	-2	1	
BV	BV	BV	T1	V	V	+	2	2	-5	-2	
BV	BV	BV	T1	V	BV	-	-2	0	-2	0	
BV	BV	BV	T1	BV	T1	+	-2	-1	-2	1	
BV	BV	BV	T1	BV	V	-	-2	0	-2	0	
BV	BV	BV	T1	BV	BV	-	-2	0	-2	0	
BV	BV	BV	V	A	V	-	4	2	-5	-2	
BV	BV	BV	V	A	BV	+	0	0	-2	0	
BV	BV	BV	V	T1	T1	+	-2	-1	-2	1	
BV	BV	BV	V	T1	V	+	2	2	-5	-2	
BV	BV	BV	V	T1	BV	-	-2	0	-2	0	
BV	BV	BV	V	V	A	-	4	2	-5	-2	
BV	BV	BV	V	V	T1	+	2	2	-5	-2	
BV	BV	BV	V	V	V	-	0	0	-7	-2	113
BV	BV	BV	V	V	BV	+	2	4	-5	-4	
BV	BV	BV	V	BV	A	+	0	0	-2	0	
BV	BV	BV	V	BV	T1	-	-2	0	-2	0	
BV	BV	BV	V	BV	V	+	2	4	-5	-4	
BV	BV	BV	V	BV	BV	-	-2	2	-2	-2	
BV	BV	BV	BV	A	V	+	0	0	-2	0	

BV	BV	BV	BV	A	BV	+	0	0	-2	0	
BV	BV	BV	BV	T1	T1	+	-2	-1	-2	1	
BV	BV	BV	BV	T1	V	-	-2	0	-2	0	
BV	BV	BV	BV	T1	BV	-	-2	0	-2	0	
BV	BV	BV	BV	V	A	+	0	0	-2	0	
BV	BV	BV	BV	V	T1	-	-2	0	-2	0	
BV	BV	BV	BV	V	V	+	2	4	-5	-4	
BV	BV	BV	BV	V	BV	-	-2	2	-2	-2	
BV	BV	BV	BV	BV	A	+	0	0	-2	0	
BV	BV	BV	BV	BV	T1	-	-2	0	-2	0	
BV	BV	BV	BV	BV	V	-	-2	2	-2	-2	
BV	BV	BV	BV	BV	BV	-	-2	2	-2	-2	
T2	V	T1	A	T1	V	+	0	-1	-1	0	
T2	V	T1	T1	T1	V	+	0	-2	-1	0	
T2	V	T1	T1	V	T1	+	0	0	-2	0	
T2	V	T1	T1	V	V	-	1	-1	-2	0	
T2	V	T1	V	T1	V	+	0	-1	-2	-1	
T2	V	T1	V	V	T1	+	0	-2	-2	0	
T2	V	T1	V	V	V	-	1	0	-2	-1	
T2	V	T1	BV	T1	V	-	2	-1	-5	-1	23
T2	V	T1	BV	V	T1	+	0	-2	-2	0	
T2	V	T1	BV	V	V	-	1	-2	-5	-1	17
T2	V	T1	T2	V	T1	+	0	0	-2	0	
T2	V	T1	T2	V	V	+	0	-2	-2	0	
T2	V	V	A	V	V	-	0	0	-2	0	
T2	V	V	T1	T1	V	-	1	-1	-2	0	
T2	V	V	T1	V	T1	-	1	-1	-2	0	
T2	V	V	T1	V	V		0				
T2	V	V	V	T1	V	-	1	0	-2	-1	
T2	V	V	V	V	T1	-	1	0	-2	-1	

T2	V	V	V	V	V	+	4	0	-2	-2
T2	V	V	BV	T1	V	-	1	-2	-5	-1	17
T2	V	V	BV	V	T1	-	1	-2	-5	-1	17
T2	V	V	BV	V	V	-	-2	-2	-6	-2	241
T2	V	V	T2	T1	V	+	0	-2	-2	0	
T2	V	V	T2	V	T1	+	0	-2	-2	0	
T2	V	V	T2	V	V	+	-2	-2	0	0	
T2	BV	T1	A	T1	V	+	0	-1	-1	0	
T2	BV	T1	A	T1	BV	+	0	-1	-1	0	
T2	BV	T1	T1	T1	V	+	0	-2	-1	0	
T2	BV	T1	T1	T1	BV	+	0	-2	-1	0	
T2	BV	T1	T1	V	T1	+	0	0	-2	0	
T2	BV	T1	T1	V	V	-	1	-1	-2	0	
T2	BV	T1	T1	V	BV	-	1	-1	-2	0	
T2	BV	T1	T1	BV	T1	+	0	0	-2	0	
T2	BV	T1	T1	BV	V	-	1	-1	-2	0	
T2	BV	T1	T1	BV	BV	-	1	-1	-2	0	
T2	BV	T1	V	T1	V	-	2	-1	-5	-1	23
T2	BV	T1	V	T1	BV	+	0	-1	-2	-1	
T2	BV	T1	V	V	T1	+	0	-2	-2	0	
T2	BV	T1	V	V	V	-	1	-2	-5	-1	17
T2	BV	T1	V	V	BV	-	1	0	-2	-1	
T2	BV	T1	V	BV	T1	+	0	-2	-2	0	
T2	BV	T1	V	BV	V	-	1	-2	-5	-1	17
T2	BV	T1	V	BV	BV	-	1	0	-2	-1	
T2	BV	T1	BV	T1	V	+	0	-1	-2	-1	
T2	BV	T1	BV	T1	BV	+	0	-1	-2	-1	
T2	BV	T1	BV	V	T1	+	0	-2	-2	0	
T2	BV	T1	BV	V	V	-	1	0	-2	-1	
T2	BV	T1	BV	V	BV	-	1	0	-2	-1	

T2	BV	T1	BV	BV	T1	+	0	-2	-2	0	
T2	BV	T1	BV	BV	V	-	1	0	-2	-1	
T2	BV	T1	BV	BV	BV	-	1	0	-2	-1	
T2	BV	T1	T2	V	T1	+	0	0	-2	0	
T2	BV	T1	T2	V	V	+	0	-2	-2	0	
T2	BV	T1	T2	V	BV	+	0	-2	-2	0	
T2	BV	T1	T2	BV	T1	+	0	0	-2	0	
T2	BV	T1	T2	BV	V	+	0	-2	-2	0	
T2	BV	T1	T2	BV	BV	+	0	-2	-2	0	
T2	BV	V	A	V	V	-	0	0	-2	0	
T2	BV	V	A	V	BV	-	0	0	-2	0	
T2	BV	V	A	BV	V	-	0	0	-2	0	
T2	BV	V	A	BV	BV	-	0	0	-2	0	
T2	BV	V	T1	T1	V	-	1	-1	-2	0	
T2	BV	V	T1	T1	BV	-	1	-1	-2	0	
T2	BV	V	T1	V	T1	-	1	-1	-2	0	
T2	BV	V	T1	V	V		0				
T2	BV	V	T1	V	BV		0				
T2	BV	V	T1	BV	T1	-	1	-1	-2	0	
T2	BV	V	T1	BV	V		0				
T2	BV	V	T1	BV	BV		0				
T2	BV	V	V	T1	V	-	1	-2	-5	-1	17
T2	BV	V	V	T1	BV	-	1	0	-2	-1	
T2	BV	V	V	V	T1	-	1	0	-2	-1	
T2	BV	V	V	V	V	-	0	0	-5	-2	29
T2	BV	V	V	V	BV	+	4	0	-2	-2	
T2	BV	V	V	BV	T1	-	1	-2	-5	-1	17
T2	BV	V	V	BV	V	-	-2	0	-6	-2	59
T2	BV	V	V	BV	BV	-	0	0	-5	-2	29
T2	BV	V	BV	T1	V	-	1	0	-2	-1	

T2	BV	V	BV	T1	BV	-	1	0	-2	-1	
T2	BV	V	BV	V	T1	-	1	-2	-5	-1	17
T2	BV	V	BV	V	V	-	0	0	-5	-2	29
T2	BV	V	BV	V	BV	-	0	0	-5	-2	29
T2	BV	V	BV	BV	T1	-	1	0	-2	-1	
T2	BV	V	BV	BV	V	+	4	0	-2	-2	
T2	BV	V	BV	BV	BV	+	4	0	-2	-2	
T2	BV	V	T2	T1	V	+	0	-2	-2	0	
T2	BV	V	T2	T1	BV	+	0	-2	-2	0	
T2	BV	V	T2	V	T1	+	0	-2	-2	0	
T2	BV	V	T2	V	V	+	-2	-2	0	0	
T2	BV	V	T2	V	BV	+	-2	-2	0	0	
T2	BV	V	T2	BV	T1	+	0	-2	-2	0	
T2	BV	V	T2	BV	V	+	-2	-2	0	0	
T2	BV	V	T2	BV	BV	+	-2	-2	0	0	
T2	BV	BV	A	V	V	-	0	0	-2	0	
T2	BV	BV	A	V	BV	-	0	0	-2	0	
T2	BV	BV	A	BV	V	-	0	0	-2	0	
T2	BV	BV	A	BV	BV	-	0	0	-2	0	
T2	BV	BV	T1	T1	V	-	1	-1	-2	0	
T2	BV	BV	T1	T1	BV	-	1	-1	-2	0	
T2	BV	BV	T1	V	T1	-	1	-1	-2	0	
T2	BV	BV	T1	V	V		0				
T2	BV	BV	T1	V	BV		0				
T2	BV	BV	T1	BV	T1	-	1	-1	-2	0	
T2	BV	BV	T1	BV	V		0				
T2	BV	BV	T1	BV	BV		0				
T2	BV	BV	V	T1	V	-	1	-2	-5	-1	17
T2	BV	BV	V	T1	BV	-	1	0	-2	-1	
T2	BV	BV	V	V	T1	-	1	-2	-5	-1	17

T2	BV	BV	V	V	V	-	-2	2	-2	-2	
T2	BV	BV	V	V	BV	-	0	0	-5	-2	29
T2	BV	BV	V	BV	T1	-	1	0	-2	-1	
T2	BV	BV	V	BV	V	-	0	0	-5	-2	29
T2	BV	BV	V	BV	BV	+	4	0	-2	-2	
T2	BV	BV	BV	T1	V	-	1	0	-2	-1	
T2	BV	BV	BV	T1	BV	-	1	0	-2	-1	
T2	BV	BV	BV	V	T1	-	1	0	-2	-1	
T2	BV	BV	BV	V	V	+	4	0	-2	-2	
T2	BV	BV	BV	V	BV	+	4	0	-2	-2	
T2	BV	BV	BV	BV	T1	-	1	0	-2	-1	
T2	BV	BV	BV	BV	V	+	4	0	-2	-2	
T2	BV	BV	BV	BV	BV	+	4	0	-2	-2	
T2	BV	BV	T2	T1	V	+	0	-2	-2	0	
T2	BV	BV	T2	T1	BV	+	0	-2	-2	0	
T2	BV	BV	T2	V	T1	+	0	-2	-2	0	
T2	BV	BV	T2	V	V	+	-2	-2	0	0	
T2	BV	BV	T2	V	BV	+	-2	-2	0	0	
T2	BV	BV	T2	BV	T1	+	0	-2	-2	0	
T2	BV	BV	T2	BV	V	+	-2	-2	0	0	
T2	BV	BV	T2	BV	BV	+	-2	-2	0	0	
T2	T2	A	A	A	T2	+	0	-1	0	0	
T2	T2	A	T1	T1	V	+	0	-2	0	0	
T2	T2	A	T1	T1	BV	+	0	-2	0	0	
T2	T2	A	V	V	T1	+	0	-1	-1	0	
T2	T2	A	V	V	V	-	0	-1	-1	0	
T2	T2	A	V	V	BV	-	0	-1	-1	0	
T2	T2	A	V	V	T2	+	0	-1	-1	0	
T2	T2	A	V	BV	T1	+	0	-1	-1	0	
T2	T2	A	V	BV	V	-	0	-1	-1	0	

T2	T2	A	V	BV	BV	-	0	-1	-1	0
T2	T2	A	V	BV	T2	+	0	-1	-1	0
T2	T2	A	BV	V	T1	+	0	-1	-1	0
T2	T2	A	BV	V	V	-	0	-1	-1	0
T2	T2	A	BV	V	BV	-	0	-1	-1	0
T2	T2	A	BV	V	T2	+	0	-1	-1	0
T2	T2	A	BV	BV	T1	+	0	-1	-1	0
T2	T2	A	BV	BV	V	-	0	-1	-1	0
T2	T2	A	BV	BV	BV	-	0	-1	-1	0
T2	T2	A	BV	BV	T2	+	0	-1	-1	0
T2	T2	A	T2	T2	A	+	0	-2	0	0
T2	T2	A	T2	T2	V	+	0	-2	0	0
T2	T2	A	T2	T2	BV	+	0	-2	0	0
T2	T2	A	T2	T2	T2	-	0	-2	0	0
T2	T2	V	A	V	T2	+	0	-1	-1	0
T2	T2	V	A	BV	T2	+	0	-1	-1	0
T2	T2	V	T1	T1	V	+	2	-2	-2	0
T2	T2	V	T1	T1	BV	+	2	-2	-2	0
T2	T2	V	T1	V	V	+	1	-1	-2	0
T2	T2	V	T1	V	BV	+	1	-1	-2	0
T2	T2	V	T1	BV	V	+	1	-1	-2	0
T2	T2	V	T1	BV	BV	+	1	-1	-2	0
T2	T2	V	T1	T2	V	+	0	0	-2	0
T2	T2	V	T1	T2	BV	+	0	0	-2	0
T2	T2	V	V	A	T2	+	0	-1	-1	0
T2	T2	V	V	T1	V	+	1	-1	-2	0
T2	T2	V	V	T1	BV	+	1	-1	-2	0
T2	T2	V	V	V	T1	+	4	-1	-2	-1
T2	T2	V	V	V	V	-	0	-1	-2	-1
T2	T2	V	V	V	BV	-	0	-1	-2	-1

T2	T2	V	V	V	T2	-	0	-1	-2	-1	
T2	T2	V	V	BV	T1	-	0	-1	-5	-1	29
T2	T2	V	V	BV	V	-	-2	3	-5	1	
T2	T2	V	V	BV	BV	-	-2	3	-5	1	
T2	T2	V	V	BV	T2	-	-2	3	-5	1	
T2	T2	V	V	T2	V	-	-2	0	-2	0	
T2	T2	V	V	T2	BV	-	-2	0	-2	0	
T2	T2	V	V	T2	T2	+	-2	0	-1	0	
T2	T2	V	BV	A	T2	+	0	-1	-1	0	
T2	T2	V	BV	T1	V	+	1	-1	-2	0	
T2	T2	V	BV	T1	BV	+	1	-1	-2	0	
T2	T2	V	BV	V	T1	-	0	-1	-5	-1	29
T2	T2	V	BV	V	V	-	-2	3	-5	1	
T2	T2	V	BV	V	BV	-	-2	3	-5	1	
T2	T2	V	BV	V	T2	-	-2	3	-5	1	
T2	T2	V	BV	BV	T1	+	4	-1	-2	-1	
T2	T2	V	BV	BV	V	-	0	-1	-2	-1	
T2	T2	V	BV	BV	BV	-	0	-1	-2	-1	
T2	T2	V	BV	BV	T2	-	0	-1	-2	-1	
T2	T2	V	BV	T2	V	-	-2	0	-2	0	
T2	T2	V	BV	T2	BV	-	-2	0	-2	0	
T2	T2	V	BV	T2	T2	+	-2	0	-1	0	
T2	T2	V	T2	T1	V	+	0	0	-2	0	
T2	T2	V	T2	T1	BV	+	0	0	-2	0	
T2	T2	V	T2	V	V	-	-2	0	-2	0	
T2	T2	V	T2	V	BV	-	-2	0	-2	0	
T2	T2	V	T2	V	T2	+	-2	0	-1	0	
T2	T2	V	T2	BV	V	-	-2	0	-2	0	
T2	T2	V	T2	BV	BV	-	-2	0	-2	0	
T2	T2	V	T2	BV	T2	+	-2	0	-1	0	

T2	T2	V	T2	T2	A	+	0	-2	0	0	
T2	T2	V	T2	T2	V	+	-2	-2	-2	0	
T2	T2	V	T2	T2	BV	+	-2	-2	-2	0	
T2	T2	V	T2	T2	T2	+	-2	-2	0	0	
T2	T2	BV	A	V	T2	+	0	-1	-1	0	
T2	T2	BV	A	BV	T2	+	0	-1	-1	0	
T2	T2	BV	T1	T1	V	+	2	-2	-2	0	
T2	T2	BV	T1	T1	BV	+	2	-2	-2	0	
T2	T2	BV	T1	V	V	+	1	-1	-2	0	
T2	T2	BV	T1	V	BV	+	1	-1	-2	0	
T2	T2	BV	T1	BV	V	+	1	-1	-2	0	
T2	T2	BV	T1	BV	BV	+	1	-1	-2	0	
T2	T2	BV	T1	T2	V	+	0	0	-2	0	
T2	T2	BV	T1	T2	BV	+	0	0	-2	0	
T2	T2	BV	V	A	T2	+	0	-1	-1	0	
T2	T2	BV	V	T1	V	+	1	-1	-2	0	
T2	T2	BV	V	T1	BV	+	1	-1	-2	0	
T2	T2	BV	V	V	T1	-	0	-1	-5	-1	29
T2	T2	BV	V	V	V	-	-2	3	-5	1	
T2	T2	BV	V	V	BV	-	-2	3	-5	1	
T2	T2	BV	V	V	T2	-	-2	3	-5	1	
T2	T2	BV	V	BV	T1	+	4	-1	-2	-1	
T2	T2	BV	V	BV	V	-	0	-1	-2	-1	
T2	T2	BV	V	BV	BV	-	0	-1	-2	-1	
T2	T2	BV	V	BV	T2	-	0	-1	-2	-1	
T2	T2	BV	V	T2	V	-	-2	0	-2	0	
T2	T2	BV	V	T2	BV	-	-2	0	-2	0	
T2	T2	BV	V	T2	T2	+	-2	0	-1	0	
T2	T2	BV	BV	A	T2	+	0	-1	-1	0	
T2	T2	BV	BV	T1	V	+	1	-1	-2	0	

T2	T2	BV	BV	T1	BV	+	1	-1	-2	0
T2	T2	BV	BV	V	T1	+	4	-1	-2	-1
T2	T2	BV	BV	V	V	-	0	-1	-2	-1
T2	T2	BV	BV	V	BV	-	0	-1	-2	-1
T2	T2	BV	BV	V	T2	-	0	-1	-2	-1
T2	T2	BV	BV	BV	T1	+	4	-1	-2	-1
T2	T2	BV	BV	BV	V	-	0	-1	-2	-1
T2	T2	BV	BV	BV	BV	-	0	-1	-2	-1
T2	T2	BV	BV	BV	T2	-	0	-1	-2	-1
T2	T2	BV	BV	T2	V	-	-2	0	-2	0
T2	T2	BV	BV	T2	BV	-	-2	0	-2	0
T2	T2	BV	BV	T2	T2	+	-2	0	-1	0
T2	T2	BV	T2	T1	V	+	0	0	-2	0
T2	T2	BV	T2	T1	BV	+	0	0	-2	0
T2	T2	BV	T2	V	V	-	-2	0	-2	0
T2	T2	BV	T2	V	BV	-	-2	0	-2	0
T2	T2	BV	T2	V	T2	+	-2	0	-1	0
T2	T2	BV	T2	BV	V	-	-2	0	-2	0
T2	T2	BV	T2	BV	BV	-	-2	0	-2	0
T2	T2	BV	T2	BV	T2	+	-2	0	-1	0
T2	T2	BV	T2	T2	A	+	0	-2	0	0
T2	T2	BV	T2	T2	V	+	-2	-2	-2	0
T2	T2	BV	T2	T2	BV	+	-2	-2	-2	0
T2	T2	BV	T2	T2	T2	+	-2	-2	0	0
T2	T2	T2	A	T2	T2	-	0	-2	0	0
T2	T2	T2	T1	V	V	-	0	-2	-1	0
T2	T2	T2	T1	V	BV	-	0	-2	-1	0
T2	T2	T2	T1	BV	V	-	0	-2	-1	0
T2	T2	T2	T1	BV	BV	-	0	-2	-1	0
T2	T2	T2	V	T1	V	-	0	-2	-1	0

T2	T2	T2	T2	BV	BV	+	-2	0	-1	0
T2	T2	T2	T2	BV	T2	+	-2	-2	0	0
T2	T2	T2	T2	T2	A	-	0	-2	0	0
T2	T2	T2	T2	T2	V	+	-2	-2	0	0
T2	T2	T2	T2	T2	BV	+	-2	-2	0	0
T2	T2	T2	T2	T2	T2	+	-2	-2	0	0

T2	T2	T2	V	T1	BV	-	0	-2	-1	0
T2	T2	T2	V	V	T1	-	0	-2	-1	0
T2	T2	T2	V	V	V	-	-2	-2	-1	0
T2	T2	T2	V	V	BV	-	-2	-2	-1	0
T2	T2	T2	V	V	T2	+	-2	0	-1	0
T2	T2	T2	V	BV	T1	-	0	-2	-1	0
T2	T2	T2	V	BV	V	-	-2	-2	-1	0
T2	T2	T2	V	BV	BV	-	-2	-2	-1	0
T2	T2	T2	V	BV	T2	+	-2	0	-1	0
T2	T2	T2	V	T2	V	+	-2	0	-1	0
T2	T2	T2	V	T2	BV	+	-2	0	-1	0
T2	T2	T2	V	T2	T2	+	-2	-2	0	0
T2	T2	T2	BV	T1	V	-	0	-2	-1	0
T2	T2	T2	BV	T1	BV	-	0	-2	-1	0
T2	T2	T2	BV	V	T1	-	0	-2	-1	0
T2	T2	T2	BV	V	V	-	-2	-2	-1	0
T2	T2	T2	BV	V	BV	-	-2	-2	-1	0
T2	T2	T2	BV	V	T2	+	-2	0	-1	0
T2	T2	T2	BV	BV	T1	-	0	-2	-1	0
T2	T2	T2	BV	BV	V	-	-2	-2	-1	0
T2	T2	T2	BV	BV	BV	-	-2	-2	-1	0
T2	T2	T2	BV	BV	T2	+	-2	0	-1	0
T2	T2	T2	BV	T2	V	+	-2	0	-1	0
T2	T2	T2	BV	T2	BV	+	-2	0	-1	0
T2	T2	T2	BV	T2	T2	+	-2	-2	0	0
T2	T2	T2	T2	A	T2	-	0	-2	0	0
T2	T2	T2	T2	V	V	+	-2	0	-1	0
T2	T2	T2	T2	V	BV	+	-2	0	-1	0
T2	T2	T2	T2	V	T2	+	-2	-2	0	0
T2	T2	T2	T2	BV	V	+	-2	0	-1	0

W COEFFICIENTS FOR THE ICOSAHEDRAL GROUP

U	V	T1	A	T1	V	+	0	−1	−1	0	
U	V	T1	T1	T1	V	+	0	−2	−1	0	
U	V	T1	T1	V	T1	+	0	0	−2	0	
U	V	T1	T1	V	V	−	1	−1	−2	0	
U	V	T1	V	T1	V	+	0	−1	−2	−1	
U	V	T1	V	V	T1	+	0	−2	−2	0	
U	V	T1	V	V	V	−	1	0	−2	−1	
U	V	T1	BV	T1	V	+	−2	3	−5	1	
U	V	T1	BV	V	T1	+	0	−2	−2	0	
U	V	T1	BV	V	V	+	−3	0	−5	−1	29
U	V	T1	T2	V	T1	−	2	−2	−2	0	
U	V	T1	T2	V	V	−	−2	0	−2	0	
U	V	T1	U	V	T1	+	−4	−2	−2	2	
U	V	T1	U	V	V	+	−6	0	−2	0	
U	V	V	A	V	V	−	0	0	−2	0	
U	V	V	T1	T1	V	−	1	−1	−2	0	
U	V	V	T1	V	T1	−	1	−1	−2	0	
U	V	V	T1	V	V		0				
U	V	V	V	T1	V	−	1	0	−2	−1	
U	V	V	V	V	T1	−	1	0	−2	−1	
U	V	V	V	V	V	+	4	0	−2	−2	
U	V	V	BV	T1	V	+	−3	0	−5	−1	29
U	V	V	BV	V	T1	+	−3	0	−5	−1	29
U	V	V	BV	V	V	+	0	2	−6	−2	23
U	V	V	T2	T1	V	−	−2	0	−2	0	
U	V	V	T2	V	T1	−	−2	0	−2	0	
U	V	V	T2	V	V		0				

U	V	V	U	T1	V	+	-6	0	-2	0	
U	V	V	U	V	T1	+	-6	0	-2	0	
U	V	V	U	V	V	+	-6	0	0	0	
U	BV	T1	A	T1	V	+	0	-1	-1	0	
U	BV	T1	A	T1	BV	+	0	-1	-1	0	
U	BV	T1	T1	T1	V	+	0	-2	-1	0	
U	BV	T1	T1	T1	BV	+	0	-2	-1	0	
U	BV	T1	T1	V	T1	+	0	0	-2	0	
U	BV	T1	T1	V	V	-	1	-1	-2	0	
U	BV	T1	T1	V	BV	-	1	-1	-2	0	
U	BV	T1	T1	BV	T1	+	0	0	-2	0	
U	BV	T1	T1	BV	V	-	1	-1	-2	0	
U	BV	T1	T1	BV	BV	-	1	-1	-2	0	
U	BV	T1	V	T1	V	+	-2	3	-5	1	
U	BV	T1	V	T1	BV	+	0	-1	-2	-1	
U	BV	T1	V	V	T1	+	0	-2	-2	0	
U	BV	T1	V	V	V	+	-3	0	-5	-1	29
U	BV	T1	V	V	BV	-	1	0	-2	-1	
U	BV	T1	V	BV	T1	+	0	-2	-2	0	
U	BV	T1	V	BV	V	+	-3	0	-5	-1	29
U	BV	T1	V	BV	BV	-	1	0	-2	-1	
U	BV	T1	BV	T1	V	+	0	-1	-2	-1	
U	BV	T1	BV	T1	BV	+	0	-1	-2	-1	
U	BV	T1	BV	V	T1	+	0	-2	-2	0	
U	BV	T1	BV	V	V	-	1	0	-2	-1	
U	BV	T1	BV	V	BV	-	1	0	-2	-1	
U	BV	T1	BV	BV	T1	+	0	-2	-2	0	
U	BV	T1	BV	BV	V	-	1	0	-2	-1	
U	BV	T1	BV	BV	BV	-	1	0	-2	-1	
U	BV	T1	T2	V	T1	-	2	-2	-2	0	

U	BV	T1	T2	V	V	−	−2	0	−2	0	
U	BV	T1	T2	V	BV	−	−2	0	−2	0	
U	BV	T1	T2	BV	T1	−	2	−2	−2	0	
U	BV	T1	T2	BV	V	−	−2	0	−2	0	
U	BV	T1	T2	BV	BV	−	−2	0	−2	0	
U	BV	T1	U	V	T1	+	−4	−2	−2	2	
U	BV	T1	U	V	V	+	−6	0	−2	0	
U	BV	T1	U	V	BV	+	−6	0	−2	0	
U	BV	T1	U	BV	T1	+	−4	−2	−2	2	
U	BV	T1	U	BV	V	+	−6	0	−2	0	
U	BV	T1	U	BV	BV	+	−6	0	−2	0	
U	BV	V	A	V	V	−	0	0	−2	0	
U	BV	V	A	V	BV	−	0	0	−2	0	
U	BV	V	A	BV	V	−	0	0	−2	0	
U	BV	V	A	BV	BV	−	0	0	−2	0	
U	BV	V	T1	T1	V	−	1	−1	−2	0	
U	BV	V	T1	T1	BV	−	1	−1	−2	0	
U	BV	V	T1	V	T1	−	1	−1	−2	0	
U	BV	V	T1	V	V		0				
U	BV	V	T1	V	BV		0				
U	BV	V	T1	BV	T1	−	1	−1	−2	0	
U	BV	V	T1	BV	V		0				
U	BV	V	T1	BV	BV		0				
U	BV	V	V	T1	V	+	−3	0	−5	−1	29
U	BV	V	V	T1	BV	−	1	0	−2	−1	
U	BV	V	V	V	T1	−	1	0	−2	−1	
U	BV	V	V	V	V	+	−4	2	−5	−2	13
U	BV	V	V	V	BV	+	4	0	−2	−2	
U	BV	V	V	BV	T1	+	−3	0	−5	−1	29
U	BV	V	V	BV	V	+	0	0	−6	−2	53

U	BV	V	V	BV	BV	+	-4	2	-5	-2	13
U	BV	V	BV	T1	V	-	1	0	-2	-1	
U	BV	V	BV	T1	BV	-	1	0	-2	-1	
U	BV	V	BV	V	T1	+	-3	0	-5	-1	29
U	BV	V	BV	V	V	+	-4	2	-5	-2	13
U	BV	V	BV	V	BV	+	-4	2	-5	-2	13
U	BV	V	BV	BV	T1	-	1	0	-2	-1	
U	BV	V	BV	BV	V	+	4	0	-2	-2	
U	BV	V	BV	BV	BV	+	4	0	-2	-2	
U	BV	V	T2	T1	V	-	-2	0	-2	0	
U	BV	V	T2	T1	BV	-	-2	0	-2	0	
U	BV	V	T2	V	T1	-	-2	0	-2	0	
U	BV	V	T2	V	V		0				
U	BV	V	T2	V	BV		0				
U	BV	V	T2	BV	T1	-	-2	0	-2	0	
U	BV	V	T2	BV	V		0				
U	BV	V	T2	BV	BV		0				
U	BV	V	U	T1	V	+	-6	0	-2	0	
U	BV	V	U	T1	BV	+	-6	0	-2	0	
U	BV	V	U	V	T1	+	-6	0	-2	0	
U	BV	V	U	V	V	+	-6	0	0	0	
U	BV	V	U	V	BV	+	-6	0	0	0	
U	BV	V	U	BV	T1	+	-6	0	-2	0	
U	BV	V	U	BV	V	+	-6	0	0	0	
U	BV	V	U	BV	BV	+	-6	0	0	0	
U	BV	BV	A	V	V	-	0	0	-2	0	
U	BV	BV	A	V	BV	-	0	0	-2	0	
U	BV	BV	A	BV	V	-	0	0	-2	0	
U	BV	BV	A	BV	BV	-	0	0	-2	0	
U	BV	BV	T1	T1	V	-	1	-1	-2	0	

U	BV	BV	T1	T1	BV	-	1	-1	-2	0	
U	BV	BV	T1	V	T1	-	1	-1	-2	0	
U	BV	BV	T1	V	V		0				
U	BV	BV	T1	V	BV		0				
U	BV	BV	T1	BV	T1	-	1	-1	-2	0	
U	BV	BV	T1	BV	V		0				
U	BV	BV	T1	BV	BV		0				
U	BV	BV	V	T1	V	+	-3	0	-5	-1	29
U	BV	BV	V	T1	BV	-	1	0	-2	-1	
U	BV	BV	V	V	T1	+	-3	0	-5	-1	29
U	BV	BV	V	V	V	+	0	4	-4	-2	
U	BV	BV	V	V	BV	+	-4	2	-5	-2	13
U	BV	BV	V	BV	T1	-	1	0	-2	-1	
U	BV	BV	V	BV	V	+	-4	2	-5	-2	13
U	BV	BV	V	BV	BV	+	4	0	-2	-2	
U	BV	BV	BV	T1	V	-	1	0	-2	-1	
U	BV	BV	BV	T1	BV	-	1	0	-2	-1	
U	BV	BV	BV	V	T1	-	1	0	-2	-1	
U	BV	BV	BV	V	V	+	4	0	-2	-2	
U	BV	BV	BV	V	BV	+	4	0	-2	-2	
U	BV	BV	BV	BV	T1	-	1	0	-2	-1	
U	BV	BV	BV	BV	V	+	4	0	-2	-2	
U	BV	BV	BV	BV	BV	+	4	0	-2	-2	
U	BV	BV	T2	T1	V	-	-2	0	-2	0	
U	BV	BV	T2	T1	BV	-	-2	0	-2	0	
U	BV	BV	T2	V	T1	-	-2	0	-2	0	
U	BV	BV	T2	V	V		0				
U	BV	BV	T2	V	BV		0				
U	BV	BV	T2	BV	T1	-	-2	0	-2	0	
U	BV	BV	T2	BV	V		0				

U	BV	BV	T2	BV	BV		0			
U	BV	BV	U	T1	V	+	-6	0	-2	0
U	BV	BV	U	T1	BV	+	-6	0	-2	0
U	BV	BV	U	V	T1	+	-6	0	-2	0
U	BV	BV	U	V	V	+	-6	0	0	0
U	BV	BV	U	V	BV	+	-6	0	0	0
U	BV	BV	U	BV	T1	+	-6	0	-2	0
U	BV	BV	U	BV	V	+	-6	0	0	0
U	BV	BV	U	BV	BV	+	-6	0	0	0
U	T2	T1	A	T1	T2	-	0	-2	0	0
U	T2	T1	T1	T1	V	-	-1	-2	0	0
U	T2	T1	T1	T1	BV	-	-1	-2	0	0
U	T2	T1	T1	V	V	-	-2	-1	-1	0
U	T2	T1	T1	V	BV	-	-2	-1	-1	0
U	T2	T1	T1	BV	V	-	-2	-1	-1	0
U	T2	T1	T1	BV	BV	-	-2	-1	-1	0
U	T2	T1	V	T1	V	-	-2	-1	-1	0
U	T2	T1	V	T1	BV	-	-2	-1	-1	0
U	T2	T1	V	T1	T2	+	-2	-2	0	0
U	T2	T1	V	V	T1	-	1	-2	-1	0
U	T2	T1	V	V	V	+	-3	0	-1	0
U	T2	T1	V	V	BV	+	-3	0	-1	0
U	T2	T1	V	V	T2	-	-2	-1	-1	0
U	T2	T1	V	BV	T1	-	1	-2	-1	0
U	T2	T1	V	BV	V	+	-3	0	-1	0
U	T2	T1	V	BV	BV	+	-3	0	-1	0
U	T2	T1	V	BV	T2	-	-2	-1	-1	0
U	T2	T1	V	T2	T1		0			
U	T2	T1	V	T2	V	+	1	-2	-1	0
U	T2	T1	V	T2	BV	+	1	-2	-1	0

U	T2	T1	BV	T1	V	-	-2	-1	-1	0
U	T2	T1	BV	T1	BV	-	-2	-1	-1	0
U	T2	T1	BV	T1	T2	+	-2	-2	0	0
U	T2	T1	BV	V	T1	-	1	-2	-1	0
U	T2	T1	BV	V	V	+	-3	0	-1	0
U	T2	T1	BV	V	BV	+	-3	0	-1	0
U	T2	T1	BV	V	T2	-	-2	-1	-1	0
U	T2	T1	BV	BV	T1	-	1	-2	-1	0
U	T2	T1	BV	BV	V	+	-3	0	-1	0
U	T2	T1	BV	BV	BV	+	-3	0	-1	0
U	T2	T1	BV	BV	T2	-	-2	-1	-1	0
U	T2	T1	BV	T2	T1		0			
U	T2	T1	BV	T2	V	+	1	-2	-1	0
U	T2	T1	BV	T2	BV	+	1	-2	-1	0
U	T2	T1	T2	V	V	+	-2	-1	-1	0
U	T2	T1	T2	V	BV	+	-2	-1	-1	0
U	T2	T1	T2	V	T2	-	-1	-2	0	0
U	T2	T1	T2	BV	V	+	-2	-1	-1	0
U	T2	T1	T2	BV	BV	+	-2	-1	-1	0
U	T2	T1	T2	BV	T2	-	-1	-2	0	0
U	T2	T1	U	V	T1	-	-4	-2	0	0
U	T2	T1	U	V	V	+	-5	0	-1	0
U	T2	T1	U	V	BV	+	-5	0	-1	0
U	T2	T1	U	BV	T1	-	-4	-2	0	0
U	T2	T1	U	BV	V	+	-5	0	-1	0
U	T2	T1	U	BV	BV	+	-5	0	-1	0
U	T2	T1	U	T2	T1	+	-4	0	0	0
U	T2	T1	U	T2	V	-	-4	-2	0	0
U	T2	T1	U	T2	BV	-	-4	-2	0	0
U	T2	V	A	V	T2	+	0	-1	-1	0

U	T2	V	A	BV	T2	+	0	-1	-1	0	
U	T2	V	T1	T1	V	+	-1	-1	-2	0	
U	T2	V	T1	T1	BV	+	-1	-1	-2	0	
U	T2	V	T1	V	V	+	-2	0	-2	0	
U	T2	V	T1	V	BV	+	-2	0	-2	0	
U	T2	V	T1	BV	V	+	-2	0	-2	0	
U	T2	V	T1	BV	BV	+	-2	0	-2	0	
U	T2	V	T1	T2	V	-	2	-2	-2	0	
U	T2	V	T1	T2	BV	-	2	-2	-2	0	
U	T2	V	V	T1	V	+	-2	0	-2	0	
U	T2	V	V	T1	BV	+	-2	0	-2	0	
U	T2	V	V	T1	T2	-	-2	-1	-1	0	
U	T2	V	V	V	T1	+	1	0	-2	-1	
U	T2	V	V	V	V	-	-3	0	-2	-1	
U	T2	V	V	V	BV	-	-3	0	-2	-1	
U	T2	V	V	V	T2	-	-2	-1	-2	1	
U	T2	V	V	BV	T1	+	1	-2	-5	-1	17
U	T2	V	V	BV	V	+	-1	0	-5	-1	23
U	T2	V	V	BV	BV	+	-1	0	-5	-1	23
U	T2	V	V	BV	T2	+	2	1	-5	-1	
U	T2	V	V	T2	T1	+	1	-2	-1	0	
U	T2	V	V	T2	V	+	0	-2	-2	0	
U	T2	V	V	T2	BV	+	0	-2	-2	0	
U	T2	V	BV	T1	V	+	-2	0	-2	0	
U	T2	V	BV	T1	BV	+	-2	0	-2	0	
U	T2	V	BV	T1	T2	-	-2	-1	-1	0	
U	T2	V	BV	V	T1	+	1	-2	-5	-1	17
U	T2	V	BV	V	V	+	-1	0	-5	-1	23
U	T2	V	BV	V	BV	+	-1	0	-5	-1	23
U	T2	V	BV	V	T2	+	2	1	-5	-1	

U	T2	V	BV	BV	T1	+	1	0	-2	-1
U	T2	V	BV	BV	V	-	-3	0	-2	-1
U	T2	V	BV	BV	BV	-	-3	0	-2	-1
U	T2	V	BV	BV	T2	-	-2	-1	-2	1
U	T2	V	BV	T2	T1	+	1	-2	-1	0
U	T2	V	BV	T2	V	+	0	-2	-2	0
U	T2	V	BV	T2	BV	+	0	-2	-2	0
U	T2	V	T2	T1	V	-	-1	-1	-2	0
U	T2	V	T2	T1	BV	-	-1	-1	-2	0
U	T2	V	T2	T1	T2	-	-1	-2	0	0
U	T2	V	T2	V	V	-	1	-1	-2	0
U	T2	V	T2	V	BV	-	1	-1	-2	0
U	T2	V	T2	V	T2	-	0	-2	-1	0
U	T2	V	T2	BV	V	-	1	-1	-2	0
U	T2	V	T2	BV	BV	-	1	-1	-2	0
U	T2	V	T2	BV	T2	-	0	-2	-1	0
U	T2	V	T2	T2	V	+	0	0	-2	0
U	T2	V	T2	T2	BV	+	0	0	-2	0
U	T2	V	U	T1	V	+	0	0	-2	0
U	T2	V	U	T1	BV	+	0	0	-2	0
U	T2	V	U	V	T1	+	-5	0	-1	0
U	T2	V	U	V	V	+	-6	0	-2	0
U	T2	V	U	V	BV	+	-6	0	-2	0
U	T2	V	U	BV	T1	+	-5	0	-1	0
U	T2	V	U	BV	V	+	-6	0	-2	0
U	T2	V	U	BV	BV	+	-6	0	-2	0
U	T2	V	U	T2	T1	-	-4	-2	0	0
U	T2	V	U	T2	V	+	-4	-2	-2	2
U	T2	V	U	T2	BV	+	-4	-2	-2	2
U	T2	BV	A	V	T2	+	0	-1	-1	0

U	T2	BV	A	BV	T2	+	0	-1	-1	0	
U	T2	BV	T1	T1	V	+	-1	-1	-2	0	
U	T2	BV	T1	T1	BV	+	-1	-1	-2	0	
U	T2	BV	T1	V	V	+	-2	0	-2	0	
U	T2	BV	T1	V	BV	+	-2	0	-2	0	
U	T2	BV	T1	BV	V	+	-2	0	-2	0	
U	T2	BV	T1	BV	BV	+	-2	0	-2	0	
U	T2	BV	T1	T2	V	-	2	-2	-2	0	
U	T2	BV	T1	T2	BV	-	2	-2	-2	0	
U	T2	BV	V	T1	V	+	-2	0	-2	0	
U	T2	BV	V	T1	BV	+	-2	0	-2	0	
U	T2	BV	V	T1	T2	-	-2	-1	-1	0	
U	T2	BV	V	V	T1	+	1	-2	-5	-1	17
U	T2	BV	V	V	V	+	-1	0	-5	-1	23
U	T2	BV	V	V	BV	+	-1	0	-5	-1	23
U	T2	BV	V	V	T2	+	2	1	-5	-1	
U	T2	BV	V	BV	T1	+	1	0	-2	-1	
U	T2	BV	V	BV	V	-	-3	0	-2	-1	
U	T2	BV	V	BV	BV	-	-3	0	-2	-1	
U	T2	BV	V	BV	T2	-	-2	-1	-2	1	
U	T2	BV	V	T2	T1	+	1	-2	-1	0	
U	T2	BV	V	T2	V	+	0	-2	-2	0	
U	T2	BV	V	T2	BV	+	0	-2	-2	0	
U	T2	BV	BV	T1	V	+	-2	0	-2	0	
U	T2	BV	BV	T1	BV	+	-2	0	-2	0	
U	T2	BV	BV	T1	T2	-	-2	-1	-1	0	
U	T2	BV	BV	V	T1	+	1	0	-2	-1	
U	T2	BV	BV	V	V	-	-3	0	-2	-1	
U	T2	BV	BV	V	BV	-	-3	0	-2	-1	
U	T2	BV	BV	V	T2	-	-2	-1	-2	1	

U	T2	BV	BV	BV	T1	+	1	0	-2	-1
U	T2	BV	BV	BV	V	-	-3	0	-2	-1
U	T2	BV	BV	BV	BV	-	-3	0	-2	-1
U	T2	BV	BV	BV	T2	-	-2	-1	-2	1
U	T2	BV	BV	T2	T1	+	1	-2	-1	0
U	T2	BV	BV	T2	V	+	0	-2	-2	0
U	T2	BV	BV	T2	BV	+	0	-2	-2	0
U	T2	BV	T2	T1	V	-	-1	-1	-2	0
U	T2	BV	T2	T1	BV	-	-1	-1	-2	0
U	T2	BV	T2	T1	T2	-	-1	-2	0	0
U	T2	BV	T2	V	V	-	1	-1	-2	0
U	T2	BV	T2	V	BV	-	1	-1	-2	0
U	T2	BV	T2	V	T2	-	0	-2	-1	0
U	T2	BV	T2	BV	V	-	1	-1	-2	0
U	T2	BV	T2	BV	BV	-	1	-1	-2	0
U	T2	BV	T2	BV	T2	-	0	-2	-1	0
U	T2	BV	T2	T2	V	+	0	0	-2	0
U	T2	BV	T2	T2	BV	+	0	0	-2	0
U	T2	BV	U	T1	V	+	0	0	-2	0
U	T2	BV	U	T1	BV	+	0	0	-2	0
U	T2	BV	U	V	T1	+	-5	0	-1	0
U	T2	BV	U	V	V	+	-6	0	-2	0
U	T2	BV	U	V	BV	+	-6	0	-2	0
U	T2	BV	U	BV	T1	+	-5	0	-1	0
U	T2	BV	U	BV	V	+	-6	0	-2	0
U	T2	BV	U	BV	BV	+	-6	0	-2	0
U	T2	BV	U	T2	T1	-	-4	-2	0	0
U	T2	BV	U	T2	V	+	-4	-2	-2	2
U	T2	BV	U	T2	BV	+	-4	-2	-2	2
U	U	A	A	A	U	+	-2	0	0	0

U	U	A	T1	T1	V	+	-2	-1	0	0
U	U	A	T1	T1	BV	+	-2	-1	0	0
U	U	A	T1	T1	T2	-	-2	-1	0	0
U	U	A	T1	T1	U	-	-2	-1	0	0
U	U	A	V	V	T1	+	-2	0	-1	0
U	U	A	V	V	V	-	-2	0	-1	0
U	U	A	V	V	BV	-	-2	0	-1	0
U	U	A	V	V	T2	+	-2	0	-1	0
U	U	A	V	V	U	+	-2	0	-1	0
U	U	A	V	BV	T1	+	-2	0	-1	0
U	U	A	V	BV	V	-	-2	0	-1	0
U	U	A	V	BV	BV	-	-2	0	-1	0
U	U	A	V	BV	T2	+	-2	0	-1	0
U	U	A	V	BV	U	+	-2	0	-1	0
U	U	A	BV	V	T1	+	-2	0	-1	0
U	U	A	BV	V	V	-	-2	0	-1	0
U	U	A	BV	V	BV	-	-2	0	-1	0
U	U	A	BV	V	T2	+	-2	0	-1	0
U	U	A	BV	V	U	+	-2	0	-1	0
U	U	A	BV	BV	T1	+	-2	0	-1	0
U	U	A	BV	BV	V	-	-2	0	-1	0
U	U	A	BV	BV	BV	-	-2	0	-1	0
U	U	A	BV	BV	T2	+	-2	0	-1	0
U	U	A	BV	BV	U	+	-2	0	-1	0
U	U	A	T2	T2	T1	-	-2	-1	0	0
U	U	A	T2	T2	V	+	-2	-1	0	0
U	U	A	T2	T2	BV	+	-2	-1	0	0
U	U	A	T2	T2	U	-	0	-3	0	0
U	U	A	U	U	A	+	-4	0	0	0
U	U	A	U	U	T1	-	-4	0	0	0

U	U	A	U	U	V	+	-4	0	0	0
U	U	A	U	U	BV	+	-4	0	0	0
U	U	A	U	U	T2	-	-4	0	0	0
U	U	T1	A	T1	U	-	-2	-1	0	0
U	U	T1	T1	A	U	-	-2	-1	0	0
U	U	T1	T1	T1	V	-	-3	-2	0	0
U	U	T1	T1	T1	BV	-	-3	-2	0	0
U	U	T1	T1	T1	T2	+	-3	-2	0	0
U	U	T1	T1	T1	U	-	-1	-2	0	0
U	U	T1	T1	V	V	-	-4	-1	-1	0
U	U	T1	T1	V	BV	-	-4	-1	-1	0
U	U	T1	T1	V	T2	+	-3	-1	0	0
U	U	T1	T1	V	U		0			
U	U	T1	T1	BV	V	-	-4	-1	-1	0
U	U	T1	T1	BV	BV	-	-4	-1	-1	0
U	U	T1	T1	BV	T2	+	-3	-1	0	0
U	U	T1	T1	BV	U		0			
U	U	T1	V	T1	V	-	-4	-1	-1	0
U	U	T1	V	T1	BV	-	-4	-1	-1	0
U	U	T1	V	T1	T2	+	-3	-1	0	0
U	U	T1	V	T1	U		0			
U	U	T1	V	V	T1	-	-1	-2	-1	0
U	U	T1	V	V	V	+	-5	0	-1	0
U	U	T1	V	V	BV	+	-5	0	-1	0
U	U	T1	V	V	T2	+	-3	0	-1	0
U	U	T1	V	V	U	-	-1	-2	-1	0
U	U	T1	V	BV	T1	-	-1	-2	-1	0
U	U	T1	V	BV	V	+	-5	0	-1	0
U	U	T1	V	BV	BV	+	-5	0	-1	0
U	U	T1	V	BV	T2	+	-3	0	-1	0

U	U	T1	V	BV	U	-	-1	-2	-1	0
U	U	T1	V	T2	T1	-	-2	-2	0	0
U	U	T1	V	T2	V	-	-3	0	-1	0
U	U	T1	V	T2	BV	-	-3	0	-1	0
U	U	T1	V	T2	U	-	-4	-4	2	0
U	U	T1	V	U	T1	+	-2	-2	0	0
U	U	T1	V	U	V	+	-3	0	-1	0
U	U	T1	V	U	BV	+	-3	0	-1	0
U	U	T1	V	U	T2		0			
U	U	T1	BV	T1	V	-	-4	-1	-1	0
U	U	T1	BV	T1	BV	-	-4	-1	-1	0
U	U	T1	BV	T1	T2	+	-3	-1	0	0
U	U	T1	BV	T1	U		0			
U	U	T1	BV	V	T1	-	-1	-2	-1	0
U	U	T1	BV	V	V	+	-5	0	-1	0
U	U	T1	BV	V	BV	+	-5	0	-1	0
U	U	T1	BV	V	T2	+	-3	0	-1	0
U	U	T1	BV	V	U	-	-1	-2	-1	0
U	U	T1	BV	BV	T1	-	-1	-2	-1	0
U	U	T1	BV	BV	V	+	-5	0	-1	0
U	U	T1	BV	BV	BV	+	-5	0	-1	0
U	U	T1	BV	BV	T2	+	-3	0	-1	0
U	U	T1	BV	BV	U	-	-1	-2	-1	0
U	U	T1	BV	T2	T1	-	-2	-2	0	0
U	U	T1	BV	T2	V	-	-3	0	-1	0
U	U	T1	BV	T2	BV	-	-3	0	-1	0
U	U	T1	BV	T2	U	-	-4	-4	2	0
U	U	T1	BV	U	T1	+	-2	-2	0	0
U	U	T1	BV	U	V	+	-3	0	-1	0
U	U	T1	BV	U	BV	+	-3	0	-1	0

U	U	T1	BV	U	T2		0			
U	U	T1	T2	V	T1	-	-2	-2	0	0
U	U	T1	T2	V	V	-	-3	0	-1	0
U	U	T1	T2	V	BV	-	-3	0	-1	0
U	U	T1	T2	V	U	-	-4	-4	2	0
U	U	T1	T2	BV	T1	-	-2	-2	0	0
U	U	T1	T2	BV	V	-	-3	0	-1	0
U	U	T1	T2	BV	BV	-	-3	0	-1	0
U	U	T1	T2	BV	U	-	-4	-4	2	0
U	U	T1	T2	U	T1	+	-2	-2	0	0
U	U	T1	T2	U	V	-	-2	-2	0	0
U	U	T1	T2	U	BV	-	-2	-2	0	0
U	U	T1	U	V	T1	+	-2	-2	0	0
U	U	T1	U	V	V	+	-3	0	-1	0
U	U	T1	U	V	BV	+	-3	0	-1	0
U	U	T1	U	V	T2		0			
U	U	T1	U	BV	T1	+	-2	-2	0	0
U	U	T1	U	BV	V	+	-3	0	-1	0
U	U	T1	U	BV	BV	+	-3	0	-1	0
U	U	T1	U	BV	T2		0			
U	U	T1	U	T2	T1	+	-2	-2	0	0
U	U	T1	U	T2	V	-	-2	-2	0	0
U	U	T1	U	T2	BV	-	-2	-2	0	0
U	U	T1	U	U	A	-	-4	0	0	0
U	U	T1	U	U	T1	-	-4	-2	0	0
U	U	T1	U	U	V	+	-4	-2	0	0
U	U	T1	U	U	BV	+	-4	-2	0	0
U	U	T1	U	U	T2	+	-4	-4	0	2
U	U	V	A	V	U	+	-2	0	-1	0
U	U	V	A	BV	U	+	-2	0	-1	[illegible]

U	U	V	T1	T1	V	+	-3	1	-2	0	
U	U	V	T1	T1	BV	+	-3	1	-2	0	
U	U	V	T1	T1	T2	+	-3	-1	0	0	
U	U	V	T1	T1	U		0				
U	U	V	T1	V	V	+	-4	2	-2	0	
U	U	V	T1	V	BV	+	-4	2	-2	0	
U	U	V	T1	V	T2	-	-3	-2	-1	0	
U	U	V	T1	V	U	-	-1	-2	-1	0	
U	U	V	T1	BV	V	+	-4	2	-2	0	
U	U	V	T1	BV	BV	+	-4	2	-2	0	
U	U	V	T1	BV	T2	-	-3	-2	-1	0	
U	U	V	T1	BV	U	-	-1	-2	-1	0	
U	U	V	T1	T2	V	+	2	-2	-2	0	
U	U	V	T1	T2	BV	+	2	-2	-2	0	
U	U	V	T1	T2	U	-	-4	-4	2	0	
U	U	V	T1	U	V	+	-2	-2	-2	0	
U	U	V	T1	U	BV	+	-2	-2	-2	0	
U	U	V	T1	U	T2	-	-4	-4	2	0	
U	U	V	V	A	U	+	-2	0	-1	0	
U	U	V	V	T1	V	+	-4	2	-2	0	
U	U	V	V	T1	BV	+	-4	2	-2	0	
U	U	V	V	T1	T2	-	-3	-2	-1	0	
U	U	V	V	T1	U	-	-1	-2	-1	0	
U	U	V	V	V	T1	+	-1	2	-2	-1	
U	U	V	V	V	V	-	-5	2	-2	-1	
U	U	V	V	V	BV	-	-5	2	-2	-1	
U	U	V	V	V	T2	+	-3	0	-2	-1	
U	U	V	V	V	U	-	1	0	-2	-1	
U	U	V	V	BV	T1	-	-3	-2	-5	-1	53
U	U	V	V	BV	V	-	-5	0	-5	-1	17

U	U	V	V	BV	BV	−	−5	0	−5	−1	17
U	U	V	V	BV	T2	+	1	−2	−5	−1	43
U	U	V	V	BV	U	−	1	−2	−5	−1	17
U	U	V	V	T2	T1	+	−3	−2	−1	0	
U	U	V	V	T2	V	−	−4	2	−2	0	
U	U	V	V	T2	BV	−	−4	2	−2	0	
U	U	V	V	T2	U	+	−3	−4	1	0	
U	U	V	V	U	T1	+	−3	0	−1	0	
U	U	V	V	U	V	−	−2	0	−2	0	
U	U	V	V	U	BV	−	−2	0	−2	0	
U	U	V	V	U	T2	+	−5	−2	1	0	
U	U	V	BV	A	U	+	−2	0	−1	0	
U	U	V	BV	T1	V	+	−4	2	−2	0	
U	U	V	BV	T1	BV	+	−4	2	−2	0	
U	U	V	BV	T1	T2	−	−3	−2	−1	0	
U	U	V	BV	T1	U	−	−1	−2	−1	0	
U	U	V	BV	V	T1	−	−3	−2	−5	−1	53
U	U	V	BV	V	V	−	−5	0	−5	−1	17
U	U	V	BV	V	BV	−	−5	0	−5	−1	17
U	U	V	BV	V	T2	+	1	−2	−5	−1	43
U	U	V	BV	V	U	−	1	−2	−5	−1	17
U	U	V	BV	BV	T1	+	−1	2	−2	−1	
U	U	V	BV	BV	V	−	−5	2	−2	−1	
U	U	V	BV	BV	BV	−	−5	2	−2	−1	
U	U	V	BV	BV	T2	+	−3	0	−2	−1	
U	U	V	BV	BV	U	−	1	0	−2	−1	
U	U	V	BV	T2	T1	+	−3	−2	−1	0	
U	U	V	BV	T2	V	−	−4	2	−2	0	
U	U	V	BV	T2	BV	−	−4	2	−2	0	
U	U	V	BV	T2	U	+	−3	−4	1	0	

U	U	V	BV	U	T1	+	-3	0	-1	0
U	U	V	BV	U	V	-	-2	0	-2	0
U	U	V	BV	U	BV	-	-2	0	-2	0
U	U	V	BV	U	T2	+	-5	-2	1	0
U	U	V	T2	T1	V	+	2	-2	-2	0
U	U	V	T2	T1	BV	+	2	-2	-2	0
U	U	V	T2	T1	U	-	-4	-4	2	0
U	U	V	T2	V	T1	+	-3	-2	-1	0
U	U	V	T2	V	V	-	-4	2	-2	0
U	U	V	T2	V	BV	-	-4	2	-2	0
U	U	V	T2	V	U	+	-3	-4	1	0
U	U	V	T2	BV	T1	+	-3	-2	-1	0
U	U	V	T2	BV	V	-	-4	2	-2	0
U	U	V	T2	BV	BV	-	-4	2	-2	0
U	U	V	T2	BV	U	+	-3	-4	1	0
U	U	V	T2	T2	T1	-	-3	-1	0	0
U	U	V	T2	T2	V	-	-3	1	-2	0
U	U	V	T2	T2	BV	-	-3	1	-2	0
U	U	V	T2	T2	U		0			
U	U	V	T2	U	T1	-	-2	-2	0	0
U	U	V	T2	U	V	+	-2	-2	-2	0
U	U	V	T2	U	BV	+	-2	-2	-2	0
U	U	V	U	T1	V	+	-2	-2	-2	0
U	U	V	U	T1	BV	+	-2	-2	-2	0
U	U	V	U	T1	T2	-	-4	-4	2	0
U	U	V	U	V	T1	+	-3	0	-1	0
U	U	V	U	V	V	-	-2	0	-2	0
U	U	V	U	V	BV	-	-2	0	-2	0
U	U	V	U	V	T2	+	-5	-2	1	0
U	U	V	U	BV	T1	+	-3	0	-1	0

U	U	V	U	BV	V	−	−2	0	−2	0
U	U	V	U	BV	BV	−	−2	0	−2	0
U	U	V	U	BV	T2	+	−5	−2	1	0
U	U	V	U	T2	T1	−	−2	−2	0	0
U	U	V	U	T2	V	+	−2	−2	−2	0
U	U	V	U	T2	BV	+	−2	−2	−2	0
U	U	V	U	U	A	+	−4	0	0	0
U	U	V	U	U	T1	+	−4	−2	0	0
U	U	V	U	U	V	+	−4	−2	−2	2
U	U	V	U	U	BV	+	−4	−2	−2	2
U	U	V	U	U	T2	+	−4	−4	−2	2
U	U	BV	A	V	U	+	−2	0	−1	0
U	U	BV	A	BV	U	+	−2	0	−1	0
U	U	BV	T1	T1	V	+	−3	1	−2	0
U	U	BV	T1	T1	BV	+	−3	1	−2	0
U	U	BV	T1	T1	T2	+	−3	−1	0	0
U	U	BV	T1	T1	U		0			
U	U	BV	T1	V	V	+	−4	2	−2	0
U	U	BV	T1	V	BV	+	−4	2	−2	0
U	U	BV	T1	V	T2	−	−3	−2	−1	0
U	U	BV	T1	V	U	−	−1	−2	−1	0
U	U	BV	T1	BV	V	+	−4	2	−2	0
U	U	BV	T1	BV	BV	+	−4	2	−2	0
U	U	BV	T1	BV	T2	−	−3	−2	−1	0
U	U	BV	T1	BV	U	−	−1	−2	−1	0
U	U	BV	T1	T2	V	+	2	−2	−2	0
U	U	BV	T1	T2	BV	+	2	−2	−2	0
U	U	BV	T1	T2	U	−	−4	−4	2	0
U	U	BV	T1	U	V	+	−2	−2	−2	0
U	U	BV	T1	U	BV	+	−2	−2	−2	0

U	U	BV	T1	U	T2	-	-4	-4	2	0.	
U	U	BV	V	A	U	+	-2	0	-1	0	
U	U	BV	V	T1	V	+	-4	2	-2	0	
U	U	BV	V	T1	BV	+	-4	2	-2	0	
U	U	BV	V	T1	T2	-	-3	-2	-1	0	
U	U	BV	V	T1	U	-	-1	-2	-1	0	
U	U	BV	V	V	T1	-	-3	-2	-5	-1	53
U	U	BV	V	V	V	-	-5	0	-5	-1	17
U	U	BV	V	V	BV	-	-5	0	-5	-1	17
U	U	BV	V	V	T2	+	1	-2	-5	-1	43
U	U	BV	V	V	U	-	1	-2	-5	-1	17
U	U	BV	V	BV	T1	+	-1	2	-2	-1	
U	U	BV	V	BV	V	-	-5	2	-2	-1	
U	U	BV	V	BV	BV	-	-5	2	-2	-1	
U	U	BV	V	BV	T2	+	-3	0	-2	-1	
U	U	BV	V	BV	U	-	1	0	-2	-1	
U	U	BV	V	T2	T1	+	-3	-2	-1	0	
U	U	BV	V	T2	V	-	-4	2	-2	0	
U	U	BV	V	T2	BV	-	-4	2	-2	0	
U	U	BV	V	T2	U	+	-3	-4	1	0	
U	U	BV	V	U	T1	+	-3	0	-1	0	
U	U	BV	V	U	V	-	-2	0	-2	0	
U	U	BV	V	U	BV	-	-2	0	-2	0	
U	U	BV	V	U	T2	+	-5	-2	1	0	
U	U	BV	BV	A	U	+	-2	0	-1	0	
U	U	BV	BV	T1	V	+	-4	2	-2	0	
U	U	BV	BV	T1	BV	+	-4	2	-2	0	
U	U	BV	BV	T1	T2	-	-3	-2	-1	0	
U	U	BV	BV	T1	U	-	-1	-2	-1	0	
U	U	BV	BV	V	T1	+	-1	2	-2	-1	

U	U	BV	BV	V	V	-	-5	2	-2	-1
U	U	BV	BV	V	BV	-	-5	2	-2	-1
U	U	BV	BV	V	T2	+	-3	0	-2	-1
U	U	BV	BV	V	U	-	1	0	-2	-1
U	U	BV	BV	BV	T1	+	-1	2	-2	-1
U	U	BV	BV	BV	V	-	-5	2	-2	-1
U	U	BV	BV	BV	BV	-	-5	2	-2	-1
U	U	BV	BV	BV	T2	+	-3	0	-2	-1
U	U	BV	BV	BV	U	-	1	0	-2	-1
U	U	BV	BV	T2	T1	+	-3	-2	-1	0
U	U	BV	BV	T2	V	-	-4	2	-2	0
U	U	BV	BV	T2	BV	-	-4	2	-2	0
U	U	BV	BV	T2	U	+	-3	-4	1	0
U	U	BV	BV	U	T1	+	-3	0	-1	0
U	U	BV	BV	U	V	-	-2	0	-2	0
U	U	BV	BV	U	BV	-	-2	0	-2	0
U	U	BV	BV	U	T2	+	-5	-2	1	0
U	U	BV	T2	T1	V	+	2	-2	-2	0
U	U	BV	T2	T1	BV	+	2	-2	-2	0
U	U	BV	T2	T1	U	-	-4	-4	2	0
U	U	BV	T2	V	T1	+	-3	-2	-1	0
U	U	BV	T2	V	V	-	-4	2	-2	0
U	U	BV	T2	V	BV	-	-4	2	-2	0
U	U	BV	T2	V	U	+	-3	-4	1	0
U	U	BV	T2	BV	T1	+	-3	-2	-1	0
U	U	BV	T2	BV	V	-	-4	2	-2	0
U	U	BV	T2	BV	BV	-	-4	2	-2	0
U	U	BV	T2	BV	U	+	-3	-4	1	0
U	U	BV	T2	T2	T1	-	-3	-1	0	0
U	U	BV	T2	T2	V	-	-3	1	-2	0

U	U	BV	T2	T2	BV	-	-3	1	-2	0
U	U	BV	T2	T2	U		0			
U	U	BV	T2	U	T1	-	-2	-2	0	0
U	U	BV	T2	U	V	+	-2	-2	-2	0
U	U	BV	T2	U	BV	+	-2	-2	-2	0
U	U	BV	U	T1	V	+	-2	-2	-2	0
U	U	BV	U	T1	BV	+	-2	-2	-2	0
U	U	BV	U	T1	T2	-	-4	-4	2	0
U	U	BV	U	V	T1	+	-3	0	-1	0
U	U	BV	U	V	V	-	-2	0	-2	0
U	U	BV	U	V	BV	-	-2	0	-2	0
U	U	BV	U	V	T2	+	-5	-2	1	0
U	U	BV	U	BV	T1	+	-3	0	-1	0
U	U	BV	U	BV	V	-	-2	0	-2	0
U	U	BV	U	BV	BV	-	-2	0	-2	0
U	U	BV	U	BV	T2	+	-5	-2	1	0
U	U	BV	U	T2	T1	-	-2	-2	0	0
U	U	BV	U	T2	V	+	-2	-2	-2	0
U	U	BV	U	T2	BV	+	-2	-2	-2	0
U	U	BV	U	U	A	+	-4	0	0	0
U	U	BV	U	U	T1	+	-4	-2	0	0
U	U	BV	U	U	V	+	-4	-2	-2	2
U	U	BV	U	U	BV	+	-4	-2	-2	2
U	U	BV	U	U	T2	+	-4	-4	-2	2
U	U	T2	A	T2	U	-	0	-3	0	0
U	U	T2	T1	V	V	-	-5	-2	1	0
U	U	T2	T1	V	BV	-	-5	-2	1	0
U	U	T2	T1	V	T2	-	-4	-4	2	0
U	U	T2	T1	V	U	-	-4	-4	2	0
U	U	T2	T1	BV	V	-	-5	-2	1	0

U	U	T2	T1	BV	BV	-	-5	-2	1	0
U	U	T2	T1	BV	T2	-	-4	-4	2	0
U	U	T2	T1	BV	U	-	-4	-4	2	0
U	U	T2	T1	U	V	-	-4	-4	2	0
U	U	T2	T1	U	BV	-	-4	-4	2	0
U	U	T2	T1	U	T2	+	0	-4	0	0
U	U	T2	V	T1	V	-	-5	-2	1	0
U	U	T2	V	T1	BV	-	-5	-2	1	0
U	U	T2	V	T1	T2	-	-4	-4	2	0
U	U	T2	V	T1	U	-	-4	-4	2	0
U	U	T2	V	V	T1	-	-5	-2	1	0
U	U	T2	V	V	V	-	-7	-2	1	0
U	U	T2	V	V	BV	-	-7	-2	1	0
U	U	T2	V	V	T2	+	-3	-4	1	0
U	U	T2	V	V	U	+	-3	-4	1	0
U	U	T2	V	BV	T1	-	-5	-2	1	0
U	U	T2	V	BV	V	-	-7	-2	1	0
U	U	T2	V	BV	BV	-	-7	-2	1	0
U	U	T2	V	BV	T2	+	-3	-4	1	0
U	U	T2	V	BV	U	+	-3	-4	1	0
U	U	T2	V	T2	T1	-	-5	-3	2	0
U	U	T2	V	T2	V	+	-6	-3	1	0
U	U	T2	V	T2	BV	+	-6	-3	1	0
U	U	T2	V	T2	U		0			
U	U	T2	V	U	T1		0			
U	U	T2	V	U	V	+	-5	-2	1	0
U	U	T2	V	U	BV	+	-5	-2	1	0
U	U	T2	V	U	T2	+	0	-4	0	0
U	U	T2	BV	T1	V	-	-5	-2	1	0
U	U	T2	BV	T1	BV	-	-5	-2	1	0

U	U	T2	BV	T1	T2	-	-4	-4	2	0
U	U	T2	BV	T1	U	-	-4	-4	2	0
U	U	T2	BV	V	T1	-	-5	-2	1	0
U	U	T2	BV	V	V	-	-7	-2	1	0
U	U	T2	BV	V	BV	-	-7	-2	1	0
U	U	T2	BV	V	T2	+	-3	-4	1	0
U	U	T2	BV	V	U	+	-3	-4	1	0
U	U	T2	BV	BV	T1	-	-5	-2	1	0
U	U	T2	BV	BV	V	-	-7	-2	1	0
U	U	T2	BV	BV	BV	-	-7	-2	1	0
U	U	T2	BV	BV	T2	+	-3	-4	1	0
U	U	T2	BV	BV	U	+	-3	-4	1	0
U	U	T2	BV	T2	T1	-	-5	-3	2	0
U	U	T2	BV	T2	V	+	-6	-3	1	0
U	U	T2	BV	T2	BV	+	-6	-3	1	0
U	U	T2	BV	T2	U		0			
U	U	T2	BV	U	T1		0			
U	U	T2	BV	U	V	+	-5	-2	1	0
U	U	T2	BV	U	BV	+	-5	-2	1	0
U	U	T2	BV	U	T2	+	0	-4	0	0
U	U	T2	T2	A	U	-	-2	-1	0	0
U	U	T2	T2	V	T1	-	-5	-3	2	0
U	U	T2	T2	V	V	+	-6	-3	1	0
U	U	T2	T2	V	BV	+	-6	-3	1	0
U	U	T2	T2	V	U	-	-1	-3	-2	0
U	U	T2	T2	BV	T1	-	-5	-3	2	0
U	U	T2	T2	BV	V	+	-6	-3	1	0
U	U	T2	T2	BV	BV	+	-6	-3	1	0
U	U	T2	T2	BV	U	-	-1	-3	-2	0
U	U	T2	T2	T2	T1	+	-5	-4	2	0

U	U	T2	T2	T2	V	−	−5	−4	2	0
U	U	T2	T2	T2	BV	−	−5	−4	2	0
U	U	T2	T2	T2	U	−	−3	−2	0	0
U	U	T2	U	T1	V	−	−4	−4	2	0
U	U	T2	U	T1	BV	−	−4	−4	2	0
U	U	T2	U	T1	T2	+	0	−4	0	0
U	U	T2	U	V	T1		0			
U	U	T2	U	V	V	+	−5	−2	1	0
U	U	T2	U	V	BV	+	−5	−2	1	0
U	U	T2	U	V	T2	+	0	−4	0	0
U	U	T2	U	BV	T1		0			
U	U	T2	U	BV	V	+	−5	−2	1	0
U	U	T2	U	BV	BV	+	−5	−2	1	0
U	U	T2	U	BV	T2	+	0	−4	0	0
U	U	T2	U	U	A	−	−4	0	0	0
U	U	T2	U	U	T1	+	−4	−4	0	2
U	U	T2	U	U	V	+	−4	−4	−2	2
U	U	T2	U	U	BV	+	−4	−4	−2	2
U	U	T2	U	U	T2	+	−4	−4	0	0

BU	V	T1	A	T1	V	–	0	–1	–1	0	
BU	V	T1	T1	T1	V	–	0	–2	–1	0	
BU	V	T1	T1	V	T1	+	0	0	–2	0	
BU	V	T1	T1	V	V		0				
BU	V	T1	V	T1	V	–	0	–1	–2	–1	
BU	V	T1	V	V	T1	+	0	–2	–2	0	
BU	V	T1	V	V	V		0				
BU	V	T1	BV	T1	V	–	–2	3	–5	1	
BU	V	T1	BV	V	T1	+	0	–2	–2	0	
BU	V	T1	BV	V	V	+	–3	–2	–3	–1	31
BU	V	T1	T2	V	T1	–	2	–2	–2	0	
BU	V	T1	T2	V	V	+	–2	–2	0	0	
BU	V	T1	U	V	T1	+	–4	–2	–2	2	
BU	V	T1	U	V	V	–	–6	0	0	0	
BU	V	T1	BU	V	T1	+	–4	–2	–2	2	
BU	V	T1	BU	V	V	–	–6	–2	0	0	
BU	V	V	A	V	V	+	0	0	–2	0	
BU	V	V	T1	T1	V		0				
BU	V	V	T1	V	T1		0				
BU	V	V	T1	V	V	+	2	–2	–2	0	
BU	V	V	V	T1	V		0				
BU	V	V	V	V	T1		0				
BU	V	V	V	V	V	+	2	0	–2	–2	
BU	V	V	BV	T1	V	+	–3	–2	–3	–1	31
BU	V	V	BV	V	T1	+	–3	–2	–3	–1	31
BU	V	V	BV	V	V	+	0	–2	–6	–2	11
BU	V	V	T2	T1	V	+	–2	–2	0	0	

BU	V	V	T2	V	T1	+	-2	-2	0	0	
BU	V	V	T2	V	V	+	2	-2	-2	0	
BU	V	V	U	T1	V	-	-6	0	0	0	
BU	V	V	U	V	T1	-	-6	0	0	0	
BU	V	V	U	V	V	-	-6	2	-2	0	
BU	V	V	BU	T1	V	-	-6	-2	0	0	
BU	V	V	BU	V	T1	-	-6	-2	0	0	
BU	V	V	BU	V	V	-	-6	-2	-2	0	
BU	BV	T1	A	T1	V	-	0	-1	-1	0	
BU	BV	T1	A	T1	BV	-	0	-1	-1	0	
BU	BV	T1	T1	T1	V	-	0	-2	-1	0	
BU	BV	T1	T1	T1	BV	-	0	-2	-1	0	
BU	BV	T1	T1	V	T1	+	0	0	-2	0	
BU	BV	T1	T1	V	V		0				
BU	BV	T1	T1	V	BV	+	1	-1	-2	0	
BU	BV	T1	T1	BV	T1	+	0	0	-2	0	
BU	BV	T1	T1	BV	V	-	1	-1	-2	0	
BU	BV	T1	T1	BV	BV	-	1	-1	-2	0	
BU	BV	T1	V	T1	V	-	-2	3	-5	1	
BU	BV	T1	V	T1	BV	-	0	-1	-2	-1	
BU	BV	T1	V	V	T1	+	0	-2	-2	0	
BU	BV	T1	V	V	V	+	-3	-2	-3	-1	31
BU	BV	T1	V	V	BV	+	1	0	-2	-1	
BU	BV	T1	V	BV	T1	+	0	-2	-2	0	
BU	BV	T1	V	BV	V	+	-3	0	-5	-1	29
BU	BV	T1	V	BV	BV	-	1	0	-2	-1	
BU	BV	T1	BV	T1	V	-	0	-1	-2	-1	
BU	BV	T1	BV	T1	BV	-	0	-1	-2	-1	
BU	BV	T1	BV	V	T1	+	0	-2	-2	0	
BU	BV	T1	BV	V	V		0				

BU	BV	T1	BV	V	BV	+	1	0	-2	-1
BU	BV	T1	BV	BV	T1	+	0	-2	-2	0
BU	BV	T1	BV	BV	V	-	1	0	-2	-1
BU	BV	T1	BV	BV	BV	-	1	0	-2	-1
BU	BV	T1	T2	V	T1	-	2	-2	-2	0
BU	BV	T1	T2	V	V	+	-2	-2	0	0
BU	BV	T1	T2	V	BV	+	-2	0	-2	0
BU	BV	T1	T2	BV	T1	-	2	-2	-2	0
BU	BV	T1	T2	BV	V	-	-2	0	-2	0
BU	BV	T1	T2	BV	BV	-	-2	0	-2	0
BU	BV	T1	U	V	T1	+	-4	-2	-2	2
BU	BV	T1	U	V	V	-	-6	0	0	0
BU	BV	T1	U	V	BV	-	-6	0	-2	0
BU	BV	T1	U	BV	T1	+	-4	-2	-2	2
BU	BV	T1	U	BV	V	+	-6	0	-2	0
BU	BV	T1	U	BV	BV	+	-6	0	-2	0
BU	BV	T1	BU	V	T1	+	-4	-2	-2	2
BU	BV	T1	BU	V	V	-	-6	0	0	0
BU	BV	T1	BU	V	BV	-	-6	0	-2	0
BU	BV	T1	BU	BV	T1	+	-4	-2	-2	2
BU	BV	T1	BU	BV	V	+	-6	0	-2	0
BU	BV	T1	BU	BV	BV	+	-6	0	-2	0
BU	BV	V	A	V	V		0			
BU	BV	V	A	V	BV	+	0	0	-2	0
BU	BV	V	A	BV	V	-	0	0	-2	0
BU	BV	V	A	BV	BV	-	0	0	-2	0
BU	BV	V	T1	T1	V	+	1	-1	-2	0
BU	BV	V	T1	T1	BV	+	1	-1	-2	0
BU	BV	V	T1	V	T1	-	1	-1	-2	0
BU	BV	V	T1	V	V		0			

BU	BV	V	T1	V	BV		0				
BU	BV	V	T1	BV	T1	-	1	-1	-2	0	
BU	BV	V	T1	BV	V		0				
BU	BV	V	T1	BV	BV		0				
BU	BV	V	V	T1	V	-	-3	0	-5	-1	29
BU	BV	V	V	T1	BV	+	1	0	-2	-1	
BU	BV	V	V	V	T1	-	1	0	-2	-1	
BU	BV	V	V	V	V	+	-4	0	-3	-2	31
BU	BV	V	V	V	BV	-	4	0	-2	-2	
BU	BV	V	V	BV	T1	+	-3	0	-5	-1	29
BU	BV	V	V	BV	V	+	0	0	-6	-2	53
BU	BV	V	V	BV	BV	+	-4	2	-5	-2	13
BU	BV	V	BV	T1	V	+	1	0	-2	-1	
BU	BV	V	BV	T1	BV	+	1	0	-2	-1	
BU	BV	V	BV	V	T1	+	-3	0	-5	-1	29
BU	BV	V	BV	V	V	-	-4	0	-3	-2	31
BU	BV	V	BV	V	BV	-	-4	2	-5	-2	13
BU	BV	V	BV	BV	T1	-	1	0	-2	-1	
BU	BV	V	BV	BV	V	+	4	0	-2	-2	
BU	BV	V	BV	BV	BV	+	4	0	-2	-2	
BU	BV	V	T2	T1	V	+	-2	0	-2	0	
BU	BV	V	T2	T1	BV	+	-2	0	-2	0	
BU	BV	V	T2	V	T1	-	-2	0	-2	0	
BU	BV	V	T2	V	V		0				
BU	BV	V	T2	V	BV		0				
BU	BV	V	T2	BV	T1	-	-2	0	-2	0	
BU	BV	V	T2	BV	V		0				
BU	BV	V	T2	BV	BV		0				
BU	BV	V	U	T1	V	-	-6	0	-2	0	
BU	BV	V	U	T1	BV	-	-6	0	-2	0	

BU	BV	V	U	V	T1	+	-6	0	-2	0	
BU	BV	V	U	V	V		0				
BU	BV	V	U	V	BV	-	-6	0	0	0	
BU	BV	V	U	BV	T1	+	-6	0	-2	0	
BU	BV	V	U	BV	V	+	-6	0	0	0	
BU	BV	V	U	BV	BV	+	-6	0	0	0	
BU	BV	V	BU	T1	V	+	-6	0	-2	0	
BU	BV	V	BU	T1	BV	+	-6	0	-2	0	
BU	BV	V	BU	V	T1	-	-6	0	0	0	
BU	BV	V	BU	V	V	+	-6	0	0	0	
BU	BV	V	BU	V	BV		0				
BU	BV	V	BU	BV	T1	+	-6	0	-2	0	
BU	BV	V	BU	BV	V	+	-6	0	0	0	
BU	BV	V	BU	BV	BV	+	-6	0	0	0	
BU	BV	BV	A	V	V		0				
BU	BV	BV	A	V	BV	+	0	0	-2	0	
BU	BV	BV	A	BV	V	-	0	0	-2	0	
BU	BV	BV	A	BV	BV	-	0	0	-2	0	
BU	BV	BV	T1	T1	V	+	1	-1	-2	0	
BU	BV	BV	T1	T1	BV	+	1	-1	-2	0	
BU	BV	BV	T1	V	T1	-	1	-1	-2	0	
BU	BV	BV	T1	V	V		0				
BU	BV	BV	T1	V	BV		0				
BU	BV	BV	T1	BV	T1	-	1	-1	-2	0	
BU	BV	BV	T1	BV	V		0				
BU	BV	BV	T1	BV	BV		0				
BU	BV	BV	V	T1	V	-	-3	0	-5	-1	29
BU	BV	BV	V	T1	BV	+	1	0	-2	-1	
BU	BV	BV	V	V	T1	+	-3	0	-5	-1	29
BU	BV	BV	V	V	V		0				

BU	BV	BV	V	V	BV	-	-4	2	-5	-2	13
BU	BV	BV	V	BV	T1	-	1	0	-2	-1	
BU	BV	BV	V	BV	V	+	-4	2	-5	-2	13
BU	BV	BV	V	BV	BV	+	4	0	-2	-2	
BU	BV	BV	BV	T1	V	+	1	0	-2	-1	
BU	BV	BV	BV	T1	BV	+	1	0	-2	-1	
BU	BV	BV	BV	V	T1	-	1	0	-2	-1	
BU	BV	BV	BV	V	V		0				
BU	BV	BV	BV	V	BV	-	4	0	-2	-2	
BU	BV	BV	BV	BV	T1	-	1	0	-2	-1	
BU	BV	BV	BV	BV	V	+	4	0	-2	-2	
BU	BV	BV	BV	BV	BV	+	4	0	-2	-2	
BU	BV	BV	T2	T1	V	+	-2	0	-2	0	
BU	BV	BV	T2	T1	BV	+	-2	0	-2	0	
BU	BV	BV	T2	V	T1	-	-2	0	-2	0	
BU	BV	BV	T2	V	V		0				
BU	BV	BV	T2	V	BV		0				
BU	BV	BV	T2	BV	T1	-	-2	0	-2	0	
BU	BV	BV	T2	BV	V		0				
BU	BV	BV	T2	BV	BV		0				
BU	BV	BV	U	T1	V	-	-6	0	-2	0	
BU	BV	BV	U	T1	BV	-	-6	0	-2	0	
BU	BV	BV	U	V	T1	+	-6	0	-2	0	
BU	BV	BV	U	V	V		0				
BU	BV	BV	U	V	BV	-	-6	0	0	0	
BU	BV	BV	U	BV	T1	+	-6	0	-2	0	
BU	BV	BV	U	BV	V	+	-6	0	0	0	
BU	BV	BV	U	BV	BV	+	-6	0	0	0	
BU	BV	BV	BU	T1	V	+	-6	0	-2	0	
BU	BV	BV	BU	T1	BV	+	-6	0	-2	0	

BU	BV	BV	BU	V	T1	−	−6	0	−2	0
BU	BV	BV	BU	V	V		0			
BU	BV	BV	BU	V	BV	+	−6	0	0	0
BU	BV	BV	BU	BV	T1	+	−6	0	−2	0
BU	BV	BV	BU	BV	V	+	−6	0	0	0
BU	BV	BV	BU	BV	BV	+	−6	0	0	0
BU	T2	T1	A	T1	T2	+	0	−2	0	0
BU	T2	T1	T1	T1	V	+	−1	−2	0	0
BU	T2	T1	T1	T1	BV	+	−1	−2	0	0
BU	T2	T1	T1	V	V	−	−2	−1	−1	0
BU	T2	T1	T1	V	BV	+	−2	−1	−1	0
BU	T2	T1	T1	BV	V	−	−2	−1	−1	0
BU	T2	T1	T1	BV	BV	−	−2	−1	−1	0
BU	T2	T1	V	T1	V	+	−2	−1	−1	0
BU	T2	T1	V	T1	BV	+	−2	−1	−1	0
BU	T2	T1	V	T1	T2	−	−2	−2	0	0
BU	T2	T1	V	V	T1	−	1	−2	−1	0
BU	T2	T1	V	V	V	−	−3	−2	−1	0
BU	T2	T1	V	V	BV	−	−3	0	−1	0
BU	T2	T1	V	V	T2	+	−2	−1	−1	0
BU	T2	T1	V	BV	T1	−	1	−2	−1	0
BU	T2	T1	V	BV	V	+	−3	0	−1	0
BU	T2	T1	V	BV	BV	+	−3	0	−1	0
BU	T2	T1	V	BV	T2	+	−2	−1	−1	0
BU	T2	T1	V	T2	T1		0			
BU	T2	T1	V	T2	V	+	1	−2	−1	0
BU	T2	T1	V	T2	BV	+	1	−2	−1	0
BU	T2	T1	BV	T1	V	+	−2	−1	−1	0
BU	T2	T1	BV	T1	BV	+	−2	−1	−1	0
BU	T2	T1	BV	T1	T2	−	−2	−2	0	0

BU	T2	T1	BV	V	T1	-	1	-2	-1	0
BU	T2	T1	BV	V	V	-	-3	-2	-1	0
BU	T2	T1	BV	V	BV	-	-3	0	-1	0
BU	T2	T1	BV	V	T2	+	-2	-1	-1	0
BU	T2	T1	BV	BV	T1	-	1	-2	-1	0
BU	T2	T1	BV	BV	V	+	-3	0	-1	0
BU	T2	T1	BV	BV	BV	+	-3	0	-1	0
BU	T2	T1	BV	BV	T2	+	-2	-1	-1	0
BU	T2	T1	BV	T2	T1		0			
BU	T2	T1	BV	T2	V	+	1	-2	-1	0
BU	T2	T1	BV	T2	BV	+	1	-2	-1	0
BU	T2	T1	T2	V	V	+	-2	-1	-1	0
BU	T2	T1	T2	V	BV	-	-2	-1	-1	0
BU	T2	T1	T2	V	T2	+	-1	-2	0	0
BU	T2	T1	T2	BV	V	+	-2	-1	-1	0
BU	T2	T1	T2	BV	BV	+	-2	-1	-1	0
BU	T2	T1	T2	BV	T2	+	-1	-2	0	0
BU	T2	T1	U	V	T1	-	-4	-2	0	0
BU	T2	T1	U	V	V	-	-5	-2	-1	2
BU	T2	T1	U	V	BV	-	-5	0	-1	0
BU	T2	T1	U	BV	T1	-	-4	-2	0	0
BU	T2	T1	U	BV	V	+	-5	0	-1	0
BU	T2	T1	U	BV	BV	+	-5	0	-1	0
BU	T2	T1	U	T2	T1	+	-4	0	0	0
BU	T2	T1	U	T2	V	-	-4	-2	0	0
BU	T2	T1	U	T2	BV	-	-4	-2	0	0
BU	T2	T1	BU	V	T1	-	-4	-2	0	0
BU	T2	T1	BU	V	V	-	-5	-2	-1	2
BU	T2	T1	BU	V	BV	-	-5	0	-1	0
BU	T2	T1	BU	BV	T1	-	-4	-2	0	0

BU	T2	T1	BU	BV	V	+	-5	0	-1	0	
BU	T2	T1	BU	BV	BV	+	-5	0	-1	0	
BU	T2	T1	BU	T2	T1	+	-4	0	0	0	
BU	T2	T1	BU	T2	V	-	-4	-2	0	0	
BU	T2	T1	BU	T2	BV	-	-4	-2	0	0	
BU	T2	V	A	V	T2	-	0	-1	-1	0	
BU	T2	V	A	BV	T2	-	0	-1	-1	0	
BU	T2	V	T1	T1	V	-	-1	-1	-2	0	
BU	T2	V	T1	T1	BV	-	-1	-1	-2	0	
BU	T2	V	T1	V	V	-	-2	-2	0	0	
BU	T2	V	T1	V	BV	-	-2	0	-2	0	
BU	T2	V	T1	BV	V	+	-2	0	-2	0	
BU	T2	V	T1	BV	BV	+	-2	0	-2	0	
BU	T2	V	T1	T2	V	-	2	-2	-2	0	
BU	T2	V	T1	T2	BV	-	2	-2	-2	0	
BU	T2	V	V	T1	V	-	-2	0	-2	0	
BU	T2	V	V	T1	BV	-	-2	0	-2	0	
BU	T2	V	V	T1	T2	+	-2	-1	-1	0	
BU	T2	V	V	V	T1	+	1	0	-2	-1	
BU	T2	V	V	V	V	-	-3	0	0	-1	
BU	T2	V	V	V	BV	+	-3	0	-2	-1	
BU	T2	V	V	V	T2	+	-2	-1	-2	1	
BU	T2	V	V	BV	T1	+	1	-2	-5	-1	17
BU	T2	V	V	BV	V	+	-1	0	-5	-1	23
BU	T2	V	V	BV	BV	+	-1	0	-5	-1	23
BU	T2	V	V	BV	T2	-	2	1	-5	-1	
BU	T2	V	V	T2	T1	+	1	-2	-1	0	
BU	T2	V	V	T2	V	+	0	-2	-2	0	
BU	T2	V	V	T2	BV	+	0	-2	-2	0	
BU	T2	V	BV	T1	V	-	-2	0	-2	0	

BU	T2	V	BV	T1	BV	-	-2	0	-2	0	
BU	T2	V	BV	T1	T2	+	-2	-1	-1	0	
BU	T2	V	BV	V	T1	+	1	-2	-5	-1	17
BU	T2	V	BV	V	V	+	-1	-2	-3	1	
BU	T2	V	BV	V	BV	-	-1	0	-5	-1	23
BU	T2	V	BV	V	T2	-	2	1	-5	-1	
BU	T2	V	BV	BV	T1	+	1	0	-2	-1	
BU	T2	V	BV	BV	V	-	-3	0	-2	-1	
BU	T2	V	BV	BV	BV	-	-3	0	-2	-1	
BU	T2	V	BV	BV	T2	+	-2	-1	-2	1	
BU	T2	V	BV	T2	T1	+	1	-2	-1	0	
BU	T2	V	BV	T2	V	+	0	-2	-2	0	
BU	T2	V	BV	T2	BV	+	0	-2	-2	0	
BU	T2	V	T2	T1	V	+	-1	-1	-2	0	
BU	T2	V	T2	T1	BV	+	-1	-1	-2	0	
BU	T2	V	T2	T1	T2	+	-1	-2	0	0	
BU	T2	V	T2	V	V		0				
BU	T2	V	T2	V	BV	+	1	-1	-2	0	
BU	T2	V	T2	V	T2	+	0	-2	-1	0	
BU	T2	V	T2	BV	V	-	1	-1	-2	0	
BU	T2	V	T2	BV	BV	-	1	-1	-2	0	
BU	T2	V	T2	BV	T2	+	0	-2	-1	0	
BU	T2	V	T2	T2	V	+	0	0	-2	0	
BU	T2	V	T2	T2	BV	+	0	0	-2	0	
BU	T2	V	U	T1	V	-	0	0	-2	0	
BU	T2	V	U	T1	BV	-	0	0	-2	0	
BU	T2	V	U	V	T1	+	-5	0	-1	0	
BU	T2	V	U	V	V	-	-6	0	0	0	
BU	T2	V	U	V	BV	-	-6	0	-2	0	
BU	T2	V	U	BV	T1	+	-5	0	-1	0	

BU	T2	V	U	BV	V	+	-6	0	-2	0	
BU	T2	V	U	BV	BV	+	-6	0	-2	0	
BU	T2	V	U	T2	T1	-	-4	-2	0	0	
BU	T2	V	U	T2	V	+	-4	-2	-2	2	
BU	T2	V	U	T2	BV	+	-4	-2	-2	2	
BU	T2	V	BU	T1	V	+	0	0	-2	0	
BU	T2	V	BU	T1	BV	+	0	0	-2	0	
BU	T2	V	BU	V	T1	-	-5	-2	-1	2	
BU	T2	V	BU	V	V	-	-6	-2	0	0	
BU	T2	V	BU	V	BV	+	-6	0	0	0	
BU	T2	V	BU	BV	T1	+	-5	0	-1	0	
BU	T2	V	BU	BV	V	+	-6	0	-2	0	
BU	T2	V	BU	BV	BV	+	-6	0	-2	0	
BU	T2	V	BU	T2	T1	-	-4	-2	0	0	
BU	T2	V	BU	T2	V	+	-4	-2	-2	2	
BU	T2	V	BU	T2	BV	+	-4	-2	-2	2	
BU	T2	BV	A	V	T2	-	0	-1	-1	0	
BU	T2	BV	A	BV	T2	-	0	-1	-1	0	
BU	T2	BV	T1	T1	V	-	-1	-1	-2	0	
BU	T2	BV	T1	T1	BV	-	-1	-1	-2	0	
BU	T2	BV	T1	V	V	-	-2	-2	0	0	
BU	T2	BV	T1	V	BV	-	-2	0	-2	0	
BU	T2	BV	T1	BV	V	+	-2	0	-2	0	
BU	T2	BV	T1	BV	BV	+	-2	0	-2	0	
BU	T2	BV	T1	T2	V	-	2	-2	-2	0	
BU	T2	BV	T1	T2	BV	-	2	-2	-2	0	
BU	T2	BV	V	T1	V	-	-2	0	-2	0	
BU	T2	BV	V	T1	BV	-	-2	0	-2	0	
BU	T2	BV	V	T1	T2	+	-2	-1	-1	0	
BU	T2	BV	V	V	T1	+	1	-2	-5	-1	17

BU	T2	BV	V	V	V	+	-1	-2	-3	1	
BU	T2	BV	V	V	BV	-	-1	0	-5	-1	23
BU	T2	BV	V	V	T2	-	2	1	-5	-1	
BU	T2	BV	V	BV	T1	+	1	0	-2	-1	
BU	T2	BV	V	BV	V	-	-3	0	-2	-1	
BU	T2	BV	V	BV	BV	-	-3	0	-2	-1	
BU	T2	BV	V	BV	T2	+	-2	-1	-2	1	
BU	T2	BV	V	T2	T1	+	1	-2	-1	0	
BU	T2	BV	V	T2	V	+	0	-2	-2	0	
BU	T2	BV	V	T2	BV	+	0	-2	-2	0	
BU	T2	BV	BV	T1	V	-	-2	0	-2	0	
BU	T2	BV	BV	T1	BV	-	-2	0	-2	0	
BU	T2	BV	BV	T1	T2	+	-2	-1	-1	0	
BU	T2	BV	BV	V	T1	+	1	0	-2	-1	
BU	T2	BV	BV	V	V	-	-3	0	0	-1	
BU	T2	BV	BV	V	BV	+	-3	0	-2	-1	
BU	T2	BV	BV	V	T2	+	-2	-1	-2	1	
BU	T2	BV	BV	BV	T1	+	1	0	-2	-1	
BU	T2	BV	BV	BV	V	-	-3	0	-2	-1	
BU	T2	BV	BV	BV	BV	-	-3	0	-2	-1	
BU	T2	BV	BV	BV	T2	+	-2	-1	-2	1	
BU	T2	BV	BV	T2	T1	+	1	-2	-1	0	
BU	T2	BV	BV	T2	V	+	0	-2	-2	0	
BU	T2	BV	BV	T2	BV	+	0	-2	-2	0	
BU	T2	BV	T2	T1	V	+	-1	-1	-2	0	
BU	T2	BV	T2	T1	BV	+	-1	-1	-2	0	
BU	T2	BV	T2	T1	T2	+	-1	-2	0	0	
BU	T2	BV	T2	V	V		0				
BU	T2	BV	T2	V	BV	+	1	-1	-2	0	
BU	T2	BV	T2	V	T2	+	0	-2	-1	0	

BU	T2	BV	T2	BV	V	-	1	-1	-2	0
BU	T2	BV	T2	BV	BV	-	1	-1	-2	0
BU	T2	BV	T2	BV	T2	+	0	-2	-1	0
BU	T2	BV	T2	T2	V	+	0	0	-2	0
BU	T2	BV	T2	T2	BV	+	0	0	-2	0
BU	T2	BV	U	T1	V	-	0	0	-2	0
BU	T2	BV	U	T1	BV	-	0	0	-2	0
BU	T2	BV	U	V	T1	+	-5	0	-1	0
BU	T2	BV	U	V	V	-	-6	0	0	0
BU	T2	BV	U	V	BV	-	-6	0	-2	0
BU	T2	BV	U	BV	T1	+	-5	0	-1	0
BU	T2	BV	U	BV	V	+	-6	0	-2	0
BU	T2	BV	U	BV	BV	+	-6	0	-2	0
BU	T2	BV	U	T2	T1	-	-4	-2	0	0
BU	T2	BV	U	T2	V	+	-4	-2	-2	2
BU	T2	BV	U	T2	BV	+	-4	-2	-2	2
BU	T2	BV	BU	T1	V	+	0	0	-2	0
BU	T2	BV	BU	T1	BV	+	0	0	-2	0
BU	T2	BV	BU	V	T1	-	-5	0	-1	0
BU	T2	BV	BU	V	V	+	-6	0	0	0
BU	T2	BV	BU	V	BV	+	-6	0	-2	0
BU	T2	BV	BU	BV	T1	+	-5	0	-1	0
BU	T2	BV	BU	BV	V	+	-6	0	-2	0
BU	T2	BV	BU	BV	BV	+	-6	0	-2	0
BU	T2	BV	BU	T2	T1	-	-4	-2	0	0
BU	T2	BV	BU	T2	V	+	-4	-2	-2	2
BU	T2	BV	BU	T2	BV	+	-4	-2	-2	2
BU	U	A	A	A	U	-	-2	0	0	0
BU	U	A	T1	T1	V	-	-2	-1	0	0
BU	U	A	T1	T1	BV	-	-2	-1	0	0

BU	U	A	T1	T1	T2	+	-2	-1	0	0
BU	U	A	T1	T1	U	+	-2	-1	0	0
BU	U	A	V	V	T1	+	-2	0	-1	0
BU	U	A	V	V	V		0			
BU	U	A	V	V	BV	+	-2	0	-1	0
BU	U	A	V	V	T2	-	-2	0	-1	0
BU	U	A	V	V	U	-	-2	0	-1	0
BU	U	A	V	BV	T1	+	-2	0	-1	0
BU	U	A	V	BV	V	-	-2	0	-1	0
BU	U	A	V	BV	BV	-	-2	0	-1	0
BU	U	A	V	BV	T2	-	-2	0	-1	0
BU	U	A	V	BV	U	-	-2	0	-1	0
BU	U	A	BV	V	T1	+	-2	0	-1	0
BU	U	A	BV	V	V		0			
BU	U	A	BV	V	BV	+	-2	0	-1	0
BU	U	A	BV	V	T2	-	-2	0	-1	0
BU	U	A	BV	V	U	-	-2	0	-1	0
BU	U	A	BV	BV	T1	+	-2	0	-1	0
BU	U	A	BV	BV	V	-	-2	0	-1	0
BU	U	A	BV	BV	BV	-	-2	0	-1	0
BU	U	A	BV	BV	T2	-	-2	0	-1	0
BU	U	A	BV	BV	U	-	-2	0	-1	0
BU	U	A	T2	T2	T1	-	-2	-1	0	0
BU	U	A	T2	T2	V	+	-2	-1	0	0
BU	U	A	T2	T2	BV	+	-2	-1	0	0
BU	U	A	T2	T2	U	+	0	-3	0	0
BU	U	A	U	U	A	+	-4	0	0	0
BU	U	A	U	U	T1	-	-4	0	0	0
BU	U	A	U	U	V	+	-4	0	0	0
BU	U	A	U	U	BV	+	-4	0	0	0

BU	U	A	U	U	T2	−	−4	0	0	0
BU	U	A	BU	U	A	+	−4	0	0	0
BU	U	A	BU	U	T1	−	−4	0	0	0
BU	U	A	BU	U	V	+	−4	0	0	0
BU	U	A	BU	U	BV	+	−4	0	0	0
BU	U	A	BU	U	T2	−	−4	0	0	0
BU	U	A	BU	U	U	−	−4	−2	0	0
BU	U	T1	A	T1	U	+	−2	−1	0	0
BU	U	T1	T1	A	U	+	−2	−1	0	0
BU	U	T1	T1	T1	V	+	−3	−2	0	0
BU	U	T1	T1	T1	BV	+	−3	−2	0	0
BU	U	T1	T1	T1	T2	−	−3	−2	0	0
BU	U	T1	T1	T1	U	+	−1	−2	0	0
BU	U	T1	T1	V	V	+	−4	1	−1	0
BU	U	T1	T1	V	BV	+	−4	−1	−1	0
BU	U	T1	T1	V	T2	−	−3	−1	0	0
BU	U	T1	T1	V	U		0			
BU	U	T1	T1	BV	V	−	−4	−1	−1	0
BU	U	T1	T1	BV	BV	−	−4	−1	−1	0
BU	U	T1	T1	BV	T2	−	−3	−1	0	0
BU	U	T1	T1	BV	U		0			
BU	U	T1	V	T1	V	+	−4	−1	−1	0
BU	U	T1	V	T1	BV	+	−4	−1	−1	0
BU	U	T1	V	T1	T2	−	−3	−1	0	0
BU	U	T1	V	T1	U		0			
BU	U	T1	V	V	T1	−	−1	−2	−1	0
BU	U	T1	V	V	V	+	−5	0	−1	0
BU	U	T1	V	V	BV	−	−5	0	−1	0
BU	U	T1	V	V	T2	−	−3	0	−1	0
BU	U	T1	V	V	U	+	−1	−2	−1	0

BU	U	T1	V	BV	T1	-	-1	-2	-1	0
BU	U	T1	V	BV	V	+	-5	0	-1	0
BU	U	T1	V	BV	BV	+	-5	0	-1	0
BU	U	T1	V	BV	T2	-	-3	0	-1	0
BU	U	T1	V	BV	U	+	-1	-2	-1	0
BU	U	T1	V	T2	T1	-	-2	-2	0	0
BU	U	T1	V	T2	V	-	-3	0	-1	0
BU	U	T1	V	T2	BV	-	-3	0	-1	0
BU	U	T1	V	T2	U	+	-4	-4	2	0
BU	U	T1	V	U	T1	+	-2	-2	0	0
BU	U	T1	V	U	V	+	-3	0	-1	0
BU	U	T1	V	U	BV	+	-3	0	-1	0
BU	U	T1	V	U	T2		0			
BU	U	T1	V	U	U		0			
BU	U	T1	BV	T1	V	+	-4	-1	-1	0
BU	U	T1	BV	T1	BV	+	-4	-1	-1	0
BU	U	T1	BV	T1	T2	-	-3	-1	0	0
BU	U	T1	BV	T1	U		0			
BU	U	T1	BV	V	T1	-	-1	-2	-1	0
BU	U	T1	BV	V	V	+	-5	0	-1	0
BU	U	T1	BV	V	BV	-	-5	0	-1	0
BU	U	T1	BV	V	T2	-	-3	0	-1	0
BU	U	T1	BV	V	U	+	-1	-2	-1	0
BU	U	T1	BV	BV	T1	-	-1	-2	-1	0
BU	U	T1	BV	BV	V	+	-5	0	-1	0
BU	U	T1	BV	BV	BV	+	-5	0	-1	0
BU	U	T1	BV	BV	T2	-	-3	0	-1	0
BU	U	T1	BV	BV	U	+	-1	-2	-1	0
BU	U	T1	BV	T2	T1	-	-2	-2	0	0
BU	U	T1	BV	T2	V	-	-3	0	-1	0

BU	U	T1	BV	T2	BV	−	−3	0	−1	0
BU	U	T1	BV	T2	U	+	−4	−4	2	0
BU	U	T1	BV	U	T1	+	−2	−2	0	0
BU	U	T1	BV	U	V	+	−3	0	−1	0
BU	U	T1	BV	U	BV	+	−3	0	−1	0
BU	U	T1	BV	U	T2		0			
BU	U	T1	BV	U	U		0			
BU	U	T1	T2	V	T1	−	−2	−2	0	0
BU	U	T1	T2	V	V	−	−3	−2	−1	0
BU	U	T1	T2	V	BV	+	−3	0	−1	0
BU	U	T1	T2	V	U	+	−4	−4	2	0
BU	U	T1	T2	BV	T1	−	−2	−2	0	0
BU	U	T1	T2	BV	V	−	−3	0	−1	0
BU	U	T1	T2	BV	BV	−	−3	0	−1	0
BU	U	T1	T2	BV	U	+	−4	−4	2	0
BU	U	T1	T2	U	T1	+	−2	−2	0	0
BU	U	T1	T2	U	V	−	−2	−2	0	0
BU	U	T1	T2	U	BV	−	−2	−2	0	0
BU	U	T1	T2	U	U	+	−4	−4	0	0
BU	U	T1	U	V	T1	+	−2	−2	0	0
BU	U	T1	U	V	V	+	−3	−2	−1	0
BU	U	T1	U	V	BV	−	−3	0	−1	0
BU	U	T1	U	V	T2		0			
BU	U	T1	U	BV	T1	+	−2	−2	0	0
BU	U	T1	U	BV	V	+	−3	0	−1	0
BU	U	T1	U	BV	BV	+	−3	0	−1	0
BU	U	T1	U	BV	T2		0			
BU	U	T1	U	T2	T1	+	−2	−2	0	0
BU	U	T1	U	T2	V	−	−2	−2	0	0
BU	U	T1	U	T2	BV	−	−2	−2	0	0

BU	U	T1	U	U	A	−	−4	0	0	0
BU	U	T1	U	U	T1	−	−4	−2	0	0
BU	U	T1	U	U	V	+	−4	−2	0	0
BU	U	T1	U	U	BV	+	−4	−2	0	0
BU	U	T1	U	U	T2	+	−4	−4	0	2
BU	U	T1	BU	V	T1	+	−2	−2	0	0
BU	U	T1	BU	V	V	+	−3	−2	−1	0
BU	U	T1	BU	V	BV	−	−3	0	−1	0
BU	U	T1	BU	V	T2		0			
BU	U	T1	BU	V	U		0			
BU	U	T1	BU	BV	T1	+	−2	−2	0	0
BU	U	T1	BU	BV	V	+	−3	0	−1	0
BU	U	T1	BU	BV	BV	+	−3	0	−1	0
BU	U	T1	BU	BV	T2		0			
BU	U	T1	BU	BV	U		0			
BU	U	T1	BU	T2	T1	+	−2	−2	0	0
BU	U	T1	BU	T2	V	−	−2	−2	0	0
BU	U	T1	BU	T2	BV	−	−2	−2	0	0
BU	U	T1	BU	T2	U	−	−4	−4	0	0
BU	U	T1	BU	U	A	−	−4	0	0	0
BU	U	T1	BU	U	T1	−	−4	−2	0	0
BU	U	T1	BU	U	V	+	−4	−2	0	0
BU	U	T1	BU	U	BV	+	−4	−2	0	0
BU	U	T1	BU	U	T2	+	−4	−4	0	2
BU	U	T1	BU	U	U	+	−4	−2	0	0
BU	U	V	A	V	U	−	−2	0	−1	0
BU	U	V	A	BV	U	−	−2	0	−1	0
BU	U	V	T1	T1	V	−	−3	1	−2	0
BU	U	V	T1	T1	BV	−	−3	1	−2	0
BU	U	V	T1	T1	T2	−	−3	−1	0	0

BU	U	V	T1	T1	U		0				
BU	U	V	T1	V	V	+	-4	-2	0	0	
BU	U	V	T1	V	BV	-	-4	2	-2	0	
BU	U	V	T1	V	T2	+	-3	-2	-1	0	
BU	U	V	T1	V	U	+	-1	-2	-1	0	
BU	U	V	T1	BV	V	+	-4	2	-2	0	
BU	U	V	T1	BV	BV	+	-4	2	-2	0	
BU	U	V	T1	BV	T2	+	-3	-2	-1	0	
BU	U	V	T1	BV	U	+	-1	-2	-1	0	
BU	U	V	T1	T2	V	+	2	-2	-2	0	
BU	U	V	T1	T2	BV	+	2	-2	-2	0	
BU	U	V	T1	T2	U	+	-4	-4	2	0	
BU	U	V	T1	U	V	+	-2	-2	-2	0	
BU	U	V	T1	U	BV	+	-2	-2	-2	0	
BU	U	V	T1	U	T2	-	-4	-4	2	0	
BU	U	V	T1	U	U		0				
BU	U	V	V	A	U	-	-2	0	-1	0	
BU	U	V	V	T1	V	-	-4	2	-2	0	
BU	U	V	V	T1	BV	-	-4	2	-2	0	
BU	U	V	V	T1	T2	+	-3	-2	-1	0	
BU	U	V	V	T1	U	+	-1	-2	-1	0	
BU	U	V	V	V	T1	+	-1	2	-2	-1	
BU	U	V	V	V	V	+	-5	0	0	-1	
BU	U	V	V	V	BV	+	-5	2	-2	-1	
BU	U	V	V	V	T2	-	-3	0	-2	-1	
BU	U	V	V	V	U	+	1	0	-2	-1	
BU	U	V	V	BV	T1	-	-3	-2	-5	-1	53
BU	U	V	V	BV	V	-	-5	0	-5	-1	17
BU	U	V	V	BV	BV	-	-5	0	-5	-1	17
BU	U	V	V	BV	T2	-	1	-2	-5	-1	43

BU	U	V	V	BV	U	+	1	-2	-5	-1	17
BU	U	V	V	T2	T1	+	-3	-2	-1	0	
BU	U	V	V	T2	V	-	-4	2	-2	0	
BU	U	V	V	T2	BV	-	-4	2	-2	0	
BU	U	V	V	T2	U	-	-3	-4	1	0	
BU	U	V	V	U	T1	+	-3	0	-1	0	
BU	U	V	V	U	V	-	-2	0	-2	0	
BU	U	V	V	U	BV	-	-2	0	-2	0	
BU	U	V	V	U	T2	+	-5	-2	1	0	
BU	U	V	V	U	U		0				
BU	U	V	BV	A	U	-	-2	0	-1	0	
BU	U	V	BV	T1	V	-	-4	2	-2	0	
BU	U	V	BV	T1	BV	-	-4	2	-2	0	
BU	U	V	BV	T1	T2	+	-3	-2	-1	0	
BU	U	V	BV	T1	U	+	-1	-2	-1	0	
BU	U	V	BV	V	T1	-	-3	-2	-5	-1	53
BU	U	V	BV	V	V	-	-5	2	-1	-1	
BU	U	V	BV	V	BV	+	-5	0	-5	-1	17
BU	U	V	BV	V	T2	-	1	-2	-5	-1	43
BU	U	V	BV	V	U	+	1	-2	-5	-1	17
BU	U	V	BV	BV	T1	+	-1	2	-2	-1	
BU	U	V	BV	BV	V	-	-5	2	-2	-1	
BU	U	V	BV	BV	BV	-	-5	2	-2	-1	
BU	U	V	BV	BV	T2	-	-3	0	-2	-1	
BU	U	V	BV	BV	U	+	1	0	-2	-1	
BU	U	V	BV	T2	T1	+	-3	-2	-1	0	
BU	U	V	BV	T2	V	-	-4	2	-2	0	
BU	U	V	BV	T2	BV	-	-4	2	-2	0	
BU	U	V	BV	T2	U	-	-3	-4	1	0	
BU	U	V	BV	U	T1	+	-3	0	-1	0	

BU	U	V	BV	U	V	−	−2	0	−2	0
BU	U	V	BV	U	BV	−	−2	0	−2	0
BU	U	V	BV	U	T2	+	−5	−2	1	0
BU	U	V	BV	U	U		0			
BU	U	V	T2	T1	V	−	2	−2	−2	0
BU	U	V	T2	T1	BV	−	2	−2	−2	0
BU	U	V	T2	T1	U	+	−4	−4	2	0
BU	U	V	T2	V	T1	+	−3	−2	−1	0
BU	U	V	T2	V	V	+	−4	−2	0	0
BU	U	V	T2	V	BV	+	−4	2	−2	0
BU	U	V	T2	V	U	−	−3	−4	1	0
BU	U	V	T2	BV	T1	+	−3	−2	−1	0
BU	U	V	T2	BV	V	−	−4	2	−2	0
BU	U	V	T2	BV	BV	−	−4	2	−2	0
BU	U	V	T2	BV	U	−	−3	−4	1	0
BU	U	V	T2	T2	T1	−	−3	−1	0	0
BU	U	V	T2	T2	V	−	−3	1	−2	0
BU	U	V	T2	T2	BV	−	−3	1	−2	0
BU	U	V	T2	T2	U		0			
BU	U	V	T2	U	T1	−	−2	−2	0	0
BU	U	V	T2	U	V	+	−2	−2	−2	0
BU	U	V	T2	U	BV	+	−2	−2	−2	0
BU	U	V	T2	U	U	−	−4	−2	−2	0
BU	U	V	U	T1	V	−	−2	−2	−2	0
BU	U	V	U	T1	BV	−	−2	−2	−2	0
BU	U	V	U	T1	T2	+	−4	−4	2	0
BU	U	V	U	V	T1	+	−3	0	−1	0
BU	U	V	U	V	V		0			
BU	U	V	U	V	BV	+	−2	0	−2	0
BU	U	V	U	V	T2	−	−5	−2	1	0

BU	U	V	U	BV	T1	+	-3	0	-1	0
BU	U	V	U	BV	V	-	-2	0	-2	0
BU	U	V	U	BV	BV	-	-2	0	-2	0
BU	U	V	U	BV	T2	-	-5	-2	1	0
BU	U	V	U	T2	T1	-	-2	-2	0	0
BU	U	V	U	T2	V	+	-2	-2	-2	0
BU	U	V	U	T2	BV	+	-2	-2	-2	0
BU	U	V	U	U	A	+	-4	0	0	0
BU	U	V	U	U	T1	+	-4	-2	0	0
BU	U	V	U	U	V	+	-4	-2	-2	2
BU	U	V	U	U	BV	+	-4	-2	-2	2
BU	U	V	U	U	T2	+	-4	-4	-2	2
BU	U	V	BU	T1	V	+	-2	-2	-2	0
BU	U	V	BU	T1	BV	+	-2	-2	-2	0
BU	U	V	BU	T1	T2	-	-4	-4	2	0
BU	U	V	BU	T1	U		0			
BU	U	V	BU	V	T1	+	-3	-2	-1	0
BU	U	V	BU	V	V	+	-2	-2	0	0
BU	U	V	BU	V	BV		0			
BU	U	V	BU	V	T2	+	-5	-4	1	0
BU	U	V	BU	V	U		0			
BU	U	V	BU	BV	T1	+	-3	0	-1	0
BU	U	V	BU	BV	V	-	-2	0	-2	0
BU	U	V	BU	BV	BV	-	-2	0	-2	0
BU	U	V	BU	BV	T2	-	-5	-2	1	0
BU	U	V	BU	BV	U		0			
BU	U	V	BU	T2	T1	-	-2	-2	0	0
BU	U	V	BU	T2	V	+	-2	-2	-2	0
BU	U	V	BU	T2	BV	+	-2	-2	-2	0
BU	U	V	BU	T2	U	+	-4	-4	-2	2

BU	U	V	BU	U	A	+	-4	0	0	0	
BU	U	V	BU	U	T1	+	-4	-2	0	0	
BU	U	V	BU	U	V	+	-4	-2	-2	2	
BU	U	V	BU	U	BV	+	-4	-2	-2	2	
BU	U	V	BU	U	T2	+	-4	-4	-2	2	
BU	U	V	BU	U	U	+	-4	-4	-2	0	13
BU	U	BV	A	V	U	-	-2	0	-1	0	
BU	U	BV	A	BV	U	-	-2	0	-1	0	
BU	U	BV	T1	T1	V	-	-3	1	-2	0	
BU	U	BV	T1	T1	BV	-	-3	1	-2	0	
BU	U	BV	T1	T1	T2	-	-3	-1	0	0	
BU	U	BV	T1	T1	U		0				
BU	U	BV	T1	V	V	+	-4	-2	0	0	
BU	U	BV	T1	V	BV	-	-4	2	-2	0	
BU	U	BV	T1	V	T2	+	-3	-2	-1	0	
BU	U	BV	T1	V	U	+	-1	-2	-1	0	
BU	U	BV	T1	BV	V	+	-4	2	-2	0	
BU	U	BV	T1	BV	BV	+	-4	2	-2	0	
BU	U	BV	T1	BV	T2	+	-3	-2	-1	0	
BU	U	BV	T1	BV	U	+	-1	-2	-1	0	
BU	U	BV	T1	T2	V	+	2	-2	-2	0	
BU	U	BV	T1	T2	BV	+	2	-2	-2	0	
BU	U	BV	T1	T2	U	+	-4	-4	2	0	
BU	U	BV	T1	U	V	+	-2	-2	-2	0	
BU	U	BV	T1	U	BV	+	-2	-2	-2	0	
BU	U	BV	T1	U	T2	-	-4	-4	2	0	
BU	U	BV	T1	U	U		0				
BU	U	BV	V	A	U	-	-2	0	-1	0	
BU	U	BV	V	T1	V	-	-4	2	-2	0	
BU	U	BV	V	T1	BV	-	-4	2	-2	0	

BU	U	BV	V	T1	T2	+	-3	-2	-1	0	
BU	U	BV	V	T1	U	+	-1	-2	-1	0	
BU	U	BV	V	V	T1	-	-3	-2	-5	-1	53
BU	U	BV	V	V	V	-	-5	2	-1	-1	
BU	U	BV	V	V	BV	+	-5	0	-5	-1	17
BU	U	BV	V	V	T2	-	1	-2	-5	-1	43
BU	U	BV	V	V	U	+	1	-2	-5	-1	17
BU	U	BV	V	BV	T1	+	-1	2	-2	-1	
BU	U	BV	V	BV	V	-	-5	2	-2	-1	
BU	U	BV	V	BV	BV	-	-5	2	-2	-1	
BU	U	BV	V	BV	T2	-	-3	0	-2	-1	
BU	U	BV	V	BV	U	+	1	0	-2	-1	
BU	U	BV	V	T2	T1	+	-3	-2	-1	0	
BU	U	BV	V	T2	V	-	-4	2	-2	0	
BU	U	BV	V	T2	BV	-	-4	2	-2	0	
BU	U	BV	V	T2	U	-	-3	-4	1	0	
BU	U	BV	V	U	T1	+	-3	0	-1	0	
BU	U	BV	V	U	V	-	-2	0	-2	0	
BU	U	BV	V	U	BV	-	-2	0	-2	0	
BU	U	BV	V	U	T2	+	-5	-2	1	0	
BU	U	BV	V	U	U		0				
BU	U	BV	BV	A	U	-	-2	0	-1	0	
BU	U	BV	BV	T1	V	-	-4	2	-2	0	
BU	U	BV	BV	T1	BV	-	-4	2	-2	0	
BU	U	BV	BV	T1	T2	+	-3	-2	-1	0	
BU	U	BV	BV	T1	U	+	-1	-2	-1	0	
BU	U	BV	BV	V	T1	+	-1	2	-2	-1	
BU	U	BV	BV	V	V	+	-5	0	0	-1	
BU	U	BV	BV	V	BV	+	-5	2	-2	-1	
BU	U	BV	BV	V	T2	-	-3	0	-2	-1	

BU	U	BV	BV	V	U	+	1	0	-2	-1
BU	U	BV	BV	BV	T1	+	-1	2	-2	-1
BU	U	BV	BV	BV	V	-	-5	2	-2	-1
BU	U	BV	BV	BV	BV	-	-5	2	-2	-1
BU	U	BV	BV	BV	T2	-	-3	0	-2	-1
BU	U	BV	BV	BV	U	+	1	0	-2	-1
BU	U	BV	BV	T2	T1	+	-3	-2	-1	0
BU	U	BV	BV	T2	V	-	-4	2	-2	0
BU	U	BV	BV	T2	BV	-	-4	2	-2	0
BU	U	BV	BV	T2	U	-	-3	-4	1	0
BU	U	BV	BV	U	T1	+	-3	0	-1	0
BU	U	BV	BV	U	V	-	-2	0	-2	0
BU	U	BV	BV	U	BV	-	-2	0	-2	0
BU	U	BV	BV	U	T2	+	-5	-2	1	0
BU	U	BV	BV	U	U		0			
BU	U	BV	T2	T1	V	-	2	-2	-2	0
BU	U	BV	T2	T1	BV	-	2	-2	-2	0
BU	U	BV	T2	T1	U	+	-4	-4	2	0
BU	U	BV	T2	V	T1	+	-3	-2	-1	0
BU	U	BV	T2	V	V	+	-4	-2	0	0
BU	U	BV	T2	V	BV	+	-4	2	-2	0
BU	U	BV	T2	V	U	-	-3	-4	1	0
BU	U	BV	T2	BV	T1	+	-3	-2	-1	0
BU	U	BV	T2	BV	V	-	-4	2	-2	0
BU	U	BV	T2	BV	BV	-	-4	2	-2	0
BU	U	BV	T2	BV	U	-	-3	-4	1	0
BU	U	BV	T2	T2	T1	-	-3	-1	0	0
BU	U	BV	T2	T2	V	-	-3	1	-2	0
BU	U	BV	T2	T2	BV	-	-3	1	-2	0
BU	U	BV	T2	T2	U		0			

BU	U	BV	T2	U	T1	−	−2	−2	0	0
BU	U	BV	T2	U	V	+	−2	−2	−2	0
BU	U	BV	T2	U	BV	+	−2	−2	−2	0
BU	U	BV	T2	U	U	−	−4	−2	−2	0
BU	U	BV	U	T1	V	−	−2	−2	−2	0
BU	U	BV	U	T1	BV	−	−2	−2	−2	0
BU	U	BV	U	T1	T2	+	−4	−4	2	0
BU	U	BV	U	V	T1	+	−3	0	−1	0
BU	U	BV	U	V	V		0			
BU	U	BV	U	V	BV	+	−2	0	−2	0
BU	U	BV	U	V	T2	−	−5	−2	1	0
BU	U	BV	U	BV	T1	+	−3	0	−1	0
BU	U	BV	U	BV	V	−	−2	0	−2	0
BU	U	BV	U	BV	BV	−	−2	0	−2	0
BU	U	BV	U	BV	T2	−	−5	−2	1	0
BU	U	BV	U	T2	T1	−	−2	−2	0	0
BU	U	BV	U	T2	V	+	−2	−2	−2	0
BU	U	BV	U	T2	BV	+	−2	−2	−2	0
BU	U	BV	U	U	A	+	−4	0	0	0
BU	U	BV	U	U	T1	+	−4	−2	0	0
BU	U	BV	U	U	V	+	−4	−2	−2	2
BU	U	BV	U	U	BV	+	−4	−2	−2	2
BU	U	BV	U	U	T2	+	−4	−4	−2	2
BU	U	BV	BU	T1	V	+	−2	−2	−2	0
BU	U	BV	BU	T1	BV	+	−2	−2	−2	0
BU	U	BV	BU	T1	T2	−	−4	−4	2	0
BU	U	BV	BU	T1	U		0			
BU	U	BV	BU	V	T1	−	−3	0	−1	0
BU	U	BV	BU	V	V		0			
BU	U	BV	BU	V	BV	−	−2	0	−2	0

BU	U	BV	BU	V	T2	+	-5	-2	1	0	
BU	U	BV	BU	V	U		0				
BU	U	BV	BU	BV	T1	+	-3	0	-1	0	
BU	U	BV	BU	BV	V	-	-2	0	-2	0	
BU	U	BV	BU	BV	BV	-	-2	0	-2	0	
BU	U	BV	BU	BV	T2	-	-5	-2	1	0	
BU	U	BV	BU	BV	U		0				
BU	U	BV	BU	T2	T1	-	-2	-2	0	0	
BU	U	BV	BU	T2	V	+	-2	-2	-2	0	
BU	U	BV	BU	T2	BV	+	-2	-2	-2	0	
BU	U	BV	BU	T2	U	+	-4	-4	-2	2	
BU	U	BV	BU	U	A	+	-4	0	0	0	
BU	U	BV	BU	U	T1	+	-4	-2	0	0	
BU	U	BV	BU	U	V	+	-4	-2	-2	2	
BU	U	BV	BU	U	BV	+	-4	-2	-2	2	
BU	U	BV	BU	U	T2	+	-4	-4	-2	2	
BU	U	BV	BU	U	U	+	-4	-4	-2	0	13
BU	U	T2	A	T2	U	+	0	-3	0	0	
BU	U	T2	T1	V	V	+	-5	-4	1	0	
BU	U	T2	T1	V	BV	+	-5	-2	1	0	
BU	U	T2	T1	V	T2	+	-4	-4	2	0	
BU	U	T2	T1	V	U	+	-4	-4	2	0	
BU	U	T2	T1	BV	V	-	-5	-2	1	0	
BU	U	T2	T1	BV	BV	-	-5	-2	1	0	
BU	U	T2	T1	BV	T2	+	-4	-4	2	0	
BU	U	T2	T1	BV	U	+	-4	-4	2	0	
BU	U	T2	T1	U	V	-	-4	-4	2	0	
BU	U	T2	T1	U	BV	-	-4	-4	2	0	
BU	U	T2	T1	U	T2	+	0	-4	0	0	
BU	U	T2	T1	U	U	+	-4	-4	0	0	

BU	U	T2	V	T1	V	+	-5	-2	1	0
BU	U	T2	V	T1	BV	+	-5	-2	1	0
BU	U	T2	V	T1	T2	+	-4	-4	2	0
BU	U	T2	V	T1	U	+	-4	-4	2	0
BU	U	T2	V	V	T1	-	-5	-2	1	0
BU	U	T2	V	V	V	+	-7	-2	1	0
BU	U	T2	V	V	BV	+	-7	-2	1	0
BU	U	T2	V	V	T2	-	-3	-4	1	0
BU	U	T2	V	V	U	-	-3	-4	1	0
BU	U	T2	V	BV	T1	-	-5	-2	1	0
BU	U	T2	V	BV	V	-	-7	-2	1	0
BU	U	T2	V	BV	BV	-	-7	-2	1	0
BU	U	T2	V	BV	T2	-	-3	-4	1	0
BU	U	T2	V	BV	U	-	-3	-4	1	0
BU	U	T2	V	T2	T1	-	-5	-3	2	0
BU	U	T2	V	T2	V	+	-6	-3	1	0
BU	U	T2	V	T2	BV	+	-6	-3	1	0
BU	U	T2	V	T2	U		0			
BU	U	T2	V	U	T1		0			
BU	U	T2	V	U	V	+	-5	-2	1	0
BU	U	T2	V	U	BV	+	-5	-2	1	0
BU	U	T2	V	U	T2	+	0	-4	0	0
BU	U	T2	V	U	U	+	-4	-4	0	0
BU	U	T2	BV	T1	V	+	-5	-2	1	0
BU	U	T2	BV	T1	BV	+	-5	-2	1	0
BU	U	T2	BV	T1	T2	+	-4	-4	2	0
BU	U	T2	BV	T1	U	+	-4	-4	2	0
BU	U	T2	BV	V	T1	-	-5	-2	1	0
BU	U	T2	BV	V	V	+	-7	-2	1	0
BU	U	T2	BV	V	BV	+	-7	-2	1	0

BU	U	T2	BV	V	T2	-	-3	-4	1	0
BU	U	T2	BV	V	U	-	-3	-4	1	0
BU	U	T2	BV	BV	T1	-	-5	-2	1	0
BU	U	T2	BV	BV	V	-	-7	-2	1	0
BU	U	T2	BV	BV	BV	-	-7	-2	1	0
BU	U	T2	BV	BV	T2	-	-3	-4	1	0
BU	U	T2	BV	BV	U	-	-3	-4	1	0
BU	U	T2	BV	T2	T1	-	-5	-3	2	0
BU	U	T2	BV	T2	V	+	-6	-3	1	0
BU	U	T2	BV	T2	BV	+	-6	-3	1	0
BU	U	T2	BV	T2	U		0			
BU	U	T2	BV	U	T1		0			
BU	U	T2	BV	U	V	+	-5	-2	1	0
BU	U	T2	BV	U	BV	+	-5	-2	1	0
BU	U	T2	BV	U	T2	+	0	-4	0	0
BU	U	T2	BV	U	U	+	-4	-4	0	0
BU	U	T2	T2	A	U	+	-2	-1	0	0
BU	U	T2	T2	V	T1	-	-5	-3	2	0
BU	U	T2	T2	V	V	+	-6	-1	1	0
BU	U	T2	T2	V	BV	-	-6	-3	1	0
BU	U	T2	T2	V	U	+	-1	-3	-2	0
BU	U	T2	T2	BV	T1	-	-5	-3	2	0
BU	U	T2	T2	BV	V	+	-6	-3	1	0
BU	U	T2	T2	BV	BV	+	-6	-3	1	0
BU	U	T2	T2	BV	U	+	-1	-3	-2	0
BU	U	T2	T2	T2	T1	+	-5	-4	2	0
BU	U	T2	T2	T2	V	-	-5	-4	2	0
BU	U	T2	T2	T2	BV	-	-5	-4	2	0
BU	U	T2	T2	T2	U	+	-3	-2	0	0
BU	U	T2	U	T1	V	+	-4	-4	2	0

BU	U	T2	U	T1	BV	+	-4	-4	2	0
BU	U	T2	U	T1	T2	-	0	-4	0	0
BU	U	T2	U	V	T1		0			
BU	U	T2	U	V	V	-	-5	-4	1	0
BU	U	T2	U	V	BV	-	-5	-2	1	0
BU	U	T2	U	V	T2	-	0	-4	0	0
BU	U	T2	U	BV	T1		0			
BU	U	T2	U	BV	V	+	-5	-2	1	0
BU	U	T2	U	BV	BV	+	-5	-2	1	0
BU	U	T2	U	BV	T2	-	0	-4	0	0
BU	U	T2	U	U	A	-	-4	0	0	0
BU	U	T2	U	U	T1	+	-4	-4	0	2
BU	U	T2	U	U	V	+	-4	-4	-2	2
BU	U	T2	U	U	BV	+	-4	-4	-2	2
BU	U	T2	U	U	T2	+	-4	-4	0	0
BU	U	T2	BU	T1	V	-	-4	-4	2	0
BU	U	T2	BU	T1	BV	-	-4	-4	2	0
BU	U	T2	BU	T1	T2	+	0	-4	0	0
BU	U	T2	BU	T1	U	-	-4	-4	0	0
BU	U	T2	BU	V	T1		0			
BU	U	T2	BU	V	V	+	-5	-4	1	0
BU	U	T2	BU	V	BV	+	-5	-2	1	0
BU	U	T2	BU	V	T2	+	0	-4	0	0
BU	U	T2	BU	V	U	+	-4	-4	-2	2
BU	U	T2	BU	BV	T1		0			
BU	U	T2	BU	BV	V	-	-5	-2	1	0
BU	U	T2	BU	BV	BV	-	-5	-2	1	0
BU	U	T2	BU	BV	T2	+	0	-4	0	0
BU	U	T2	BU	BV	U	+	-4	-4	-2	2
BU	U	T2	BU	U	A	-	-4	0	0	0

BU	U	T2	BU	U	T1	+	-4	-4	0	2
BU	U	T2	BU	U	V	+	-4	-4	-2	2
BU	U	T2	BU	U	BV	+	-4	-4	-2	2
BU	U	T2	BU	U	T2	+	-4	-4	0	0
BU	U	T2	BU	U	U	+	-2	-4	0	0
BU	U	U	A	U	U	+	-4	0	0	0
BU	U	U	T1	V	V		0			
BU	U	U	T1	V	BV		0			
BU	U	U	T1	V	T2		0			
BU	U	U	T1	V	U		0			
BU	U	U	T1	BV	V		0			
BU	U	U	T1	BV	BV		0			
BU	U	U	T1	BV	T2		0			
BU	U	U	T1	BV	U		0			
BU	U	U	T1	T2	V		0			
BU	U	U	T1	T2	BV		0			
BU	U	U	T1	T2	U	+	-4	-4	0	0
BU	U	U	T1	U	V		0			
BU	U	U	T1	U	BV		0			
BU	U	U	T1	U	T2	+	-4	-4	0	0
BU	U	U	T1	U	U	+	-4	-2	0	0
BU	U	U	V	T1	V		0			
BU	U	U	V	T1	BV		0			
BU	U	U	V	T1	T2		0			
BU	U	U	V	T1	U		0			
BU	U	U	V	V	T1		0			
BU	U	U	V	V	V		0			
BU	U	U	V	V	BV		0			
BU	U	U	V	V	T2		0			
BU	U	U	V	V	U		0			

BU	U	U	V	BV	T1		0			
BU	U	U	V	BV	V		0			
BU	U	U	V	BV	BV		0			
BU	U	U	V	BV	T2		0			
BU	U	U	V	BV	U		0			
BU	U	U	V	T2	T1		0			
BU	U	U	V	T2	V		0			
BU	U	U	V	T2	BV		0			
BU	U	U	V	T2	U	+	-4	-4	0	0
BU	U	U	V	U	T1		0			
BU	U	U	V	U	V		0			
BU	U	U	V	U	BV		0			
BU	U	U	V	U	T2	+	-4	-4	0	0
BU	U	U	V	U	U	-	-4	-2	0	0
BU	U	U	BV	T1	V		0			
BU	U	U	BV	T1	BV		0			
BU	U	U	BV	T1	T2		0			
BU	U	U	BV	T1	U		0			
BU	U	U	BV	V	T1		0			
BU	U	U	BV	V	V		0			
BU	U	U	BV	V	BV		0			
BU	U	U	BV	V	T2		0			
BU	U	U	BV	V	U		0			
BU	U	U	BV	BV	T1		0			
BU	U	U	BV	BV	V		0			
BU	U	U	BV	BV	BV		0			
BU	U	U	BV	BV	T2		0			
BU	U	U	BV	BV	U		0			
BU	U	U	BV	T2	T1		0			
BU	U	U	BV	T2	V		0			

BU	U	U	BV	T2	BV		0			
BU	U	U	BV	T2	U	+	-4	-4	0	0
BU	U	U	BV	U	T1		0			
BU	U	U	BV	U	V		0			
BU	U	U	BV	U	BV		0			
BU	U	U	BV	U	T2	+	-4	-4	0	0
BU	U	U	BV	U	U	-	-4	-2	0	0
BU	U	U	T2	T1	V		0			
BU	U	U	T2	T1	BV		0			
BU	U	U	T2	T1	U	+	-4	-4	0	0
BU	U	U	T2	V	T1		0			
BU	U	U	T2	V	V		0			
BU	U	U	T2	V	BV		0			
BU	U	U	T2	V	U	-	-4	-2	-2	0
BU	U	U	T2	BV	T1		0			
BU	U	U	T2	BV	V		0			
BU	U	U	T2	BV	BV		0			
BU	U	U	T2	BV	U	-	-4	-2	-2	0
BU	U	U	T2	U	T1	+	-4	-4	0	0
BU	U	U	T2	U	V	-	-4	-2	-2	0
BU	U	U	T2	U	BV	-	-4	-2	-2	0
BU	U	U	T2	U	U	+	-2	-4	0	0
BU	U	U	U	T1	V		0			
BU	U	U	U	T1	BV		0			
BU	U	U	U	T1	T2	+	-4	-4	0	0
BU	U	U	U	V	T1		0			
BU	U	U	U	V	V		0			
BU	U	U	U	V	BV		0			
BU	U	U	U	V	T2	-	-4	-4	-2	2
BU	U	U	U	BV	T1		0			

BU	U	U	U	BV	V		0				
BU	U	U	U	BV	BV		0				
BU	U	U	U	BV	T2	-	-4	-4	-2	2	
BU	U	U	U	T2	T1	+	-4	-4	0	0	
BU	U	U	U	T2	V	-	-4	-4	-2	2	
BU	U	U	U	T2	BV	-	-4	-4	-2	2	
BU	U	U	BU	A	U	-	-4	-2	0	0	
BU	U	U	BU	T1	V		0				
BU	U	U	BU	T1	BV		0				
BU	U	U	BU	T1	T2	-	-4	-4	0	0	
BU	U	U	BU	T1	U	+	-4	-2	0	0	
BU	U	U	BU	V	T1		0				
BU	U	U	BU	V	V		0				
BU	U	U	BU	V	BV		0				
BU	U	U	BU	V	T2	+	-4	-4	-2	2	
BU	U	U	BU	V	U	+	-4	-4	-2	0	13
BU	U	U	BU	BV	T1		0				
BU	U	U	BU	BV	V		0				
BU	U	U	BU	BV	BV		0				
BU	U	U	BU	BV	T2	+	-4	-4	-2	2	
BU	U	U	BU	BV	U	+	-4	-4	-2	0	13
BU	U	U	BU	T2	T1	-	-4	-4	0	0	
BU	U	U	BU	T2	V	+	-4	-4	-2	2	
BU	U	U	BU	T2	BV	+	-4	-4	-2	2	
BU	U	U	BU	T2	U	+	-2	-4	0	0	
BU	U	U	BU	U	A	-	-4	-2	0	0	
BU	U	U	BU	U	T1	+	-4	-2	0	0	
BU	U	U	BU	U	V	+	-4	-4	-2	0	13
BU	U	U	BU	U	BV	+	-4	-4	-2	0	13
BU	U	U	BU	U	T2	+	-2	-4	0	0	

BU	U	U	BU	U	U	+	-2	-2	0	0
BU	BU	A	A	A	U	-	-2	0	0	0
BU	BU	A	A	A	BU	+	-2	0	0	0
BU	BU	A	T1	T1	V	-	-2	-1	0	0
BU	BU	A	T1	T1	BV	-	-2	-1	0	0
BU	BU	A	T1	T1	T2	+	-2	-1	0	0
BU	BU	A	T1	T1	U	+	-2	-1	0	0
BU	BU	A	T1	T1	BU	-	-2	-1	0	0
BU	BU	A	V	V	T1	-	-2	0	-1	0
BU	BU	A	V	V	V	+	-2	0	-1	0
BU	BU	A	V	V	BV	+	-2	0	-1	0
BU	BU	A	V	V	T2	-	-2	0	-1	0
BU	BU	A	V	V	U	-	-2	0	-1	0
BU	BU	A	V	V	BU	+	-2	0	-1	0
BU	BU	A	V	BV	T1	-	-2	0	-1	0
BU	BU	A	V	BV	V		0			
BU	BU	A	V	BV	BV	-	-2	0	-1	0
BU	BU	A	V	BV	T2	-	-2	0	-1	0
BU	BU	A	V	BV	U	-	-2	0	-1	0
BU	BU	A	V	BV	BU	+	-2	0	-1	0
BU	BU	A	BV	V	T1	-	-2	0	-1	0
BU	BU	A	BV	V	V		0			
BU	BU	A	BV	V	BV	-	-2	0	-1	0
BU	BU	A	BV	V	T2	-	-2	0	-1	0
BU	BU	A	BV	V	U	-	-2	0	-1	0
BU	BU	A	BV	V	BU	+	-2	0	-1	0
BU	BU	A	BV	BV	T1	-	-2	0	-1	0
BU	BU	A	BV	BV	V	+	-2	0	-1	0
BU	BU	A	BV	BV	BV	+	-2	0	-1	0
BU	BU	A	BV	BV	T2	-	-2	0	-1	0

BU	BU	A	BV	BV	U	-	-2	0	-1	0
BU	BU	A	BV	BV	BU	+	-2	0	-1	0
BU	BU	A	T2	T2	T1	+	-2	-1	0	0
BU	BU	A	T2	T2	V	-	-2	-1	0	0
BU	BU	A	T2	T2	BV	-	-2	-1	0	0
BU	BU	A	T2	T2	U	+	0	-3	0	0
BU	BU	A	T2	T2	BU	-	0	-3	0	0
BU	BU	A	U	U	A	-	-4	0	0	0
BU	BU	A	U	U	T1	+	-4	0	0	0
BU	BU	A	U	U	V	-	-4	0	0	0
BU	BU	A	U	U	BV	-	-4	0	0	0
BU	BU	A	U	U	T2	+	-2	-2	0	0
BU	BU	A	U	U	U	+	-4	0	0	0
BU	BU	A	U	BU	A	+	-4	0	0	0
BU	BU	A	U	BU	T1	-	-4	0	0	0
BU	BU	A	U	BU	V	+	-4	0	0	0
BU	BU	A	U	BU	BV	+	-4	0	0	0
BU	BU	A	U	BU	T2	-	-2	-2	0	0
BU	BU	A	BU	U	A	+	-4	0	0	0
BU	BU	A	BU	U	T1	-	-4	0	0	0
BU	BU	A	BU	U	V	+	-4	0	0	0
BU	BU	A	BU	U	BV	+	-4	0	0	0
BU	BU	A	BU	U	T2	-	-4	0	0	0
BU	BU	A	BU	BU	A	+	-4	0	0	0
BU	BU	A	BU	BU	T1	-	-4	0	0	0
BU	BU	A	BU	BU	V	+	-4	0	0	0
BU	BU	A	BU	BU	BV	+	-4	0	0	0
BU	BU	A	BU	BU	T2	-	-4	0	0	0
BU	BU	T1	A	T1	U	+	-2	-1	0	0
BU	BU	T1	A	T1	BU	-	-2	-1	0	0

BU	BU	T1	T1	A	U	+	-2	-1	0	0
BU	BU	T1	T1	A	BU	-	-2	-1	0	0
BU	BU	T1	T1	T1	V	+	-3	-2	0	0
BU	BU	T1	T1	T1	BV	+	-3	-2	0	0
BU	BU	T1	T1	T1	T2	-	-3	-2	0	0
BU	BU	T1	T1	T1	U	+	-1	-2	0	0
BU	BU	T1	T1	T1	BU	-	-1	-2	0	0
BU	BU	T1	T1	V	V	+	-4	1	-1	0
BU	BU	T1	T1	V	BV	+	-4	-1	-1	0
BU	BU	T1	T1	V	T2	-	-3	-1	0	0
BU	BU	T1	T1	V	U		0			
BU	BU	T1	T1	V	BU		0			
BU	BU	T1	T1	BV	V	-	-4	-1	-1	0
BU	BU	T1	T1	BV	BV	-	-4	-1	-1	0
BU	BU	T1	T1	BV	T2	-	-3	-1	0	0
BU	BU	T1	T1	BV	U		0			
BU	BU	T1	T1	BV	BU		0			
BU	BU	T1	V	T1	V	+	-4	1	-1	0
BU	BU	T1	V	T1	BV	+	-4	-1	-1	0
BU	BU	T1	V	T1	T2	-	-3	-1	0	0
BU	BU	T1	V	T1	U		0			
BU	BU	T1	V	T1	BU		0			
BU	BU	T1	V	V	T1	+	-1	-2	-1	0
BU	BU	T1	V	V	V	+	-5	-2	1	0
BU	BU	T1	V	V	BV	-	-5	0	-1	0
BU	BU	T1	V	V	T2	-	-3	0	-1	0
BU	BU	T1	V	V	U	+	-1	-2	-1	0
BU	BU	T1	V	V	BU	-	-1	-2	-1	0
BU	BU	T1	V	BV	T1	+	-1	-2	-1	0
BU	BU	T1	V	BV	V	-	-5	0	-1	0

BU	BU	T1	V	BV	BV	+	-5	0	-1	0
BU	BU	T1	V	BV	T2	-	-3	0	-1	0
BU	BU	T1	V	BV	U	+	-1	-2	-1	0
BU	BU	T1	V	BV	BU	-	-1	-2	-1	0
BU	BU	T1	V	T2	T1	+	-2	-2	0	0
BU	BU	T1	V	T2	V	+	-3	-2	-1	0
BU	BU	T1	V	T2	BV	-	-3	0	-1	0
BU	BU	T1	V	T2	U	+	-4	-4	2	0
BU	BU	T1	V	T2	BU	-	-4	-4	2	0
BU	BU	T1	V	U	T1	-	-2	-2	0	0
BU	BU	T1	V	U	V	-	-3	-2	-1	0
BU	BU	T1	V	U	BV	+	-3	0	-1	0
BU	BU	T1	V	U	T2		0			
BU	BU	T1	V	U	U		0			
BU	BU	T1	V	BU	T1	+	-2	-2	0	0
BU	BU	T1	V	BU	V	+	-3	-2	-1	0
BU	BU	T1	V	BU	BV	-	-3	0	-1	0
BU	BU	T1	V	BU	T2		0			
BU	BU	T1	BV	T1	V	-	-4	-1	-1	0
BU	BU	T1	BV	T1	BV	-	-4	-1	-1	0
BU	BU	T1	BV	T1	T2	-	-3	-1	0	0
BU	BU	T1	BV	T1	U		0			
BU	BU	T1	BV	T1	BU		0			
BU	BU	T1	BV	V	T1	+	-1	-2	-1	0
BU	BU	T1	BV	V	V	-	-5	0	-1	0
BU	BU	T1	BV	V	BV	+	-5	0	-1	0
BU	BU	T1	BV	V	T2	-	-3	0	-1	0
BU	BU	T1	BV	V	U	+	-1	-2	-1	0
BU	BU	T1	BV	V	BU	-	-1	-2	-1	0
BU	BU	T1	BV	BV	T1	+	-1	-2	-1	0

BU	BU	T1	BV	BV	V	-	-5	0	-1	0
BU	BU	T1	BV	BV	BV	-	-5	0	-1	0
BU	BU	T1	BV	BV	T2	-	-3	0	-1	0
BU	BU	T1	BV	BV	U	+	-1	-2	-1	0
BU	BU	T1	BV	BV	BU	-	-1	-2	-1	0
BU	BU	T1	BV	T2	T1	+	-2	-2	0	0
BU	BU	T1	BV	T2	V	+	-3	0	-1	0
BU	BU	T1	BV	T2	BV	+	-3	0	-1	0
BU	BU	T1	BV	T2	U	+	-4	-4	2	0
BU	BU	T1	BV	T2	BU	-	-4	-4	2	0
BU	BU	T1	BV	U	T1	-	-2	-2	0	0
BU	BU	T1	BV	U	V	-	-3	0	-1	0
BU	BU	T1	BV	U	BV	-	-3	0	-1	0
BU	BU	T1	BV	U	T2		0			
BU	BU	T1	BV	U	U		0			
BU	BU	T1	BV	BU	T1	+	-2	-2	0	0
BU	BU	T1	BV	BU	V	+	-3	0	-1	0
BU	BU	T1	BV	BU	BV	+	-3	0	-1	0
BU	BU	T1	BV	BU	T2		0			
BU	BU	T1	T2	V	T1	+	-2	-2	0	0
BU	BU	T1	T2	V	V	+	-3	-2	-1	0
BU	BU	T1	T2	V	BV	-	-3	0	-1	0
BU	BU	T1	T2	V	U	+	-4	-4	2	0
BU	BU	T1	T2	V	BU	-	-4	-4	2	0
BU	BU	T1	T2	BV	T1	+	-2	-2	0	0
BU	BU	T1	T2	BV	V	+	-3	0	-1	0
BU	BU	T1	T2	BV	BV	+	-3	0	-1	0
BU	BU	T1	T2	BV	U	+	-4	-4	2	0
BU	BU	T1	T2	BV	BU	-	-4	-4	2	0
BU	BU	T1	T2	U	T1	-	-2	-2	0	0

BU	BU	T1	T2	U	V	+	-2	-2	0	0
BU	BU	T1	T2	U	BV	+	-2	-2	0	0
BU	BU	T1	T2	U	U	+	-4	-4	0	0
BU	BU	T1	T2	BU	T1	+	-2	-2	0	0
BU	BU	T1	T2	BU	V	-	-2	-2	0	0
BU	BU	T1	T2	BU	BV	-	-2	-2	0	0
BU	BU	T1	U	V	T1	-	-2	-2	0	0
BU	BU	T1	U	V	V	-	-3	-2	-1	0
BU	BU	T1	U	V	BV	+	-3	0	-1	0
BU	BU	T1	U	V	T2		0			
BU	BU	T1	U	V	U		0			
BU	BU	T1	U	BV	T1	-	-2	-2	0	0
BU	BU	T1	U	BV	V	-	-3	0	-1	0
BU	BU	T1	U	BV	BV	-	-3	0	-1	0
BU	BU	T1	U	BV	T2		0			
BU	BU	T1	U	BV	U		0			
BU	BU	T1	U	T2	T1	-	-2	-2	0	0
BU	BU	T1	U	T2	V	+	-2	-2	0	0
BU	BU	T1	U	T2	BV	+	-2	-2	0	0
BU	BU	T1	U	T2	U	+	-4	-4	0	0
BU	BU	T1	U	U	A	+	-4	0	0	0
BU	BU	T1	U	U	T1	+	-4	-2	0	0
BU	BU	T1	U	U	V	-	-4	-2	0	0
BU	BU	T1	U	U	BV	-	-4	-2	0	0
BU	BU	T1	U	U	T2	-	-2	-2	0	0
BU	BU	T1	U	U	U	-	-4	-4	0	0
BU	BU	T1	U	BU	A	-	-4	0	0	0
BU	BU	T1	U	BU	T1	-	-4	-2	0	0
BU	BU	T1	U	BU	V	+	-4	-2	0	0
BU	BU	T1	U	BU	BV	+	-4	-2	0	0

BU	BU	T1	U	BU	T2	+	-2	-2	0	0
BU	BU	T1	BU	V	T1	+	-2	-2	0	0
BU	BU	T1	BU	V	V	+	-3	-2	-1	0
BU	BU	T1	BU	V	BV	-	-3	0	-1	0
BU	BU	T1	BU	V	T2		0			
BU	BU	T1	BU	BV	T1	+	-2	-2	0	0
BU	BU	T1	BU	BV	V	+	-3	0	-1	0
BU	BU	T1	BU	BV	BV	+	-3	0	-1	0
BU	BU	T1	BU	BV	T2		0			
BU	BU	T1	BU	T2	T1	+	-2	-2	0	0
BU	BU	T1	BU	T2	V	-	-2	-2	0	0
BU	BU	T1	BU	T2	BV	-	-2	-2	0	0
BU	BU	T1	BU	U	A	-	-4	0	0	0
BU	BU	T1	BU	U	T1	-	-4	-2	0	0
BU	BU	T1	BU	U	V	+	-4	-2	0	0
BU	BU	T1	BU	U	BV	+	-4	-2	0	0
BU	BU	T1	BU	U	T2	+	-4	-4	0	2
BU	BU	T1	BU	BU	A	-	-4	0	0	0
BU	BU	T1	BU	BU	T1	-	-4	-2	0	0
BU	BU	T1	BU	BU	V	+	-4	-2	0	0
BU	BU	T1	BU	BU	BV	+	-4	-2	0	0
BU	BU	T1	BU	BU	T2	+	-4	-4	0	2
BU	BU	V	A	V	U	-	-2	0	-1	0
BU	BU	V	A	V	BU	+	-2	0	-1	0
BU	BU	V	A	BV	U	-	-2	0	-1	0
BU	BU	V	A	BV	BU	+	-2	0	-1	0
BU	BU	V	T1	T1	V	-	-3	1	-2	0
BU	BU	V	T1	T1	BV	-	-3	1	-2	0
BU	BU	V	T1	T1	T2	-	-3	-1	0	0
BU	BU	V	T1	T1	U		0			

BU	BU	V	T1	T1	BU		0			
BU	BU	V	T1	V	V	+	-4	-2	0	0
BU	BU	V	T1	V	BV	-	-4	2	-2	0
BU	BU	V	T1	V	T2	+	-3	-2	-1	0
BU	BU	V	T1	V	U	+	-1	-2	-1	0
BU	BU	V	T1	V	BU	-	-1	-2	-1	0
BU	BU	V	T1	BV	V	+	-4	2	-2	0
BU	BU	V	T1	BV	BV	+	-4	2	-2	0
BU	BU	V	T1	BV	T2	+	-3	-2	-1	0
BU	BU	V	T1	BV	U	+	-1	-2	-1	0
BU	BU	V	T1	BV	BU	-	-1	-2	-1	0
BU	BU	V	T1	T2	V	+	2	-2	-2	0
BU	BU	V	T1	T2	BV	+	2	-2	-2	0
BU	BU	V	T1	T2	U	+	-4	-4	2	0
BU	BU	V	T1	T2	BU	-	-4	-4	2	0
BU	BU	V	T1	U	V	+	-2	-2	-2	0
BU	BU	V	T1	U	BV	+	-2	-2	-2	0
BU	BU	V	T1	U	T2	-	-4	-4	2	0
BU	BU	V	T1	U	U		0			
BU	BU	V	T1	BU	V	+	-2	-2	-2	0
BU	BU	V	T1	BU	BV	+	-2	-2	-2	0
BU	BU	V	T1	BU	T2	-	-4	-4	2	0
BU	BU	V	V	A	U	-	-2	0	-1	0
BU	BU	V	V	A	BU	+	-2	0	-1	0
BU	BU	V	V	T1	V	+	-4	-2	0	0
BU	BU	V	V	T1	BV	-	-4	2	-2	0
BU	BU	V	V	T1	T2	+	-3	-2	-1	0
BU	BU	V	V	T1	U	+	-1	-2	-1	0
BU	BU	V	V	T1	BU	-	-1	-2	-1	0
BU	BU	V	V	V	T1	-	-1	2	-2	-1

BU	BU	V	V	V	V	−	−5	0	0	−1	
BU	BU	V	V	V	BV	+	−5	2	−2	−1	
BU	BU	V	V	V	T2	−	−3	0	−2	−1	
BU	BU	V	V	V	U	+	1	0	−2	−1	
BU	BU	V	V	V	BU	−	1	0	−2	−1	
BU	BU	V	V	BV	T1	+	−3	−2	−5	−1	53
BU	BU	V	V	BV	V	+	−5	2	−1	−1	
BU	BU	V	V	BV	BV	−	−5	0	−5	−1	17
BU	BU	V	V	BV	T2	−	1	−2	−5	−1	43
BU	BU	V	V	BV	U	+	1	−2	−5	−1	17
BU	BU	V	V	BV	BU	−	1	−2	−5	−1	17
BU	BU	V	V	T2	T1	−	−3	−2	−1	0	
BU	BU	V	V	T2	V	−	−4	−2	0	0	
BU	BU	V	V	T2	BV	−	−4	2	−2	0	
BU	BU	V	V	T2	U	−	−3	−4	1	0	
BU	BU	V	V	T2	BU	+	−3	−4	1	0	
BU	BU	V	V	U	T1	−	−3	0	−1	0	
BU	BU	V	V	U	V		0				
BU	BU	V	V	U	BV	−	−2	0	−2	0	
BU	BU	V	V	U	T2	+	−5	−2	1	0	
BU	BU	V	V	U	U		0				
BU	BU	V	V	BU	T1	+	−3	−2	−1	0	
BU	BU	V	V	BU	V	+	−2	−2	0	0	
BU	BU	V	V	BU	BV		0				
BU	BU	V	V	BU	T2	+	−5	−4	1	0	
BU	BU	V	BV	A	U	−	−2	0	−1	0	
BU	BU	V	BV	A	BU	+	−2	0	−1	0	
BU	BU	V	BV	T1	V	+	−4	2	−2	0	
BU	BU	V	BV	T1	BV	+	−4	2	−2	0	
BU	BU	V	BV	T1	T2	+	−3	−2	−1	0	

BU	BU	V	BV	T1	U	+	-1	-2	-1	0	
BU	BU	V	BV	T1	BU	−	-1	-2	-1	0	
BU	BU	V	BV	V	T1	+	-3	-2	-5	-1	53
BU	BU	V	BV	V	V	+	-5	2	-1	-1	
BU	BU	V	BV	V	BV	−	-5	0	-5	-1	17
BU	BU	V	BV	V	T2	−	1	-2	-5	-1	43
BU	BU	V	BV	V	U	+	1	-2	-5	-1	17
BU	BU	V	BV	V	BU	−	1	-2	-5	-1	17
BU	BU	V	BV	BV	T1	−	-1	2	-2	-1	
BU	BU	V	BV	BV	V	+	-5	2	-2	-1	
BU	BU	V	BV	BV	BV	+	-5	2	-2	-1	
BU	BU	V	BV	BV	T2	−	-3	0	-2	-1	
BU	BU	V	BV	BV	U	+	1	0	-2	-1	
BU	BU	V	BV	BV	BU	−	1	0	-2	-1	
BU	BU	V	BV	T2	T1	−	-3	-2	-1	0	
BU	BU	V	BV	T2	V	+	-4	2	-2	0	
BU	BU	V	BV	T2	BV	+	-4	2	-2	0	
BU	BU	V	BV	T2	U	−	-3	-4	1	0	
BU	BU	V	BV	T2	BU	+	-3	-4	1	0	
BU	BU	V	BV	U	T1	−	-3	0	-1	0	
BU	BU	V	BV	U	V	+	-2	0	-2	0	
BU	BU	V	BV	U	BV	+	-2	0	-2	0	
BU	BU	V	BV	U	T2	+	-5	-2	1	0	
BU	BU	V	BV	U	U		0				
BU	BU	V	BV	BU	T1	+	-3	0	-1	0	
BU	BU	V	BV	BU	V	−	-2	0	-2	0	
BU	BU	V	BV	BU	BV	−	-2	0	-2	0	
BU	BU	V	BV	BU	T2	−	-5	-2	1	0	
BU	BU	V	T2	T1	V	+	2	-2	-2	0	
BU	BU	V	T2	T1	BV	+	2	-2	-2	0	

BU	BU	V	T2	T1	U	+	-4	-4	2	0
BU	BU	V	T2	T1	BU	-	-4	-4	2	0
BU	BU	V	T2	V	T1	-	-3	-2	-1	0
BU	BU	V	T2	V	V	-	-4	-2	0	0
BU	BU	V	T2	V	BV	-	-4	2	-2	0
BU	BU	V	T2	V	U	-	-3	-4	1	0
BU	BU	V	T2	V	BU	+	-3	-4	1	0
BU	BU	V	T2	BV	T1	-	-3	-2	-1	0
BU	BU	V	T2	BV	V	+	-4	2	-2	0
BU	BU	V	T2	BV	BV	+	-4	2	-2	0
BU	BU	V	T2	BV	U	-	-3	-4	1	0
BU	BU	V	T2	BV	BU	+	-3	-4	1	0
BU	BU	V	T2	T2	T1	+	-3	-1	0	0
BU	BU	V	T2	T2	V	+	-3	1	-2	0
BU	BU	V	T2	T2	BV	+	-3	1	-2	0
BU	BU	V	T2	T2	U		0			
BU	BU	V	T2	T2	BU		0			
BU	BU	V	T2	U	T1	+	-2	-2	0	0
BU	BU	V	T2	U	V	-	-2	-2	-2	0
BU	BU	V	T2	U	BV	-	-2	-2	-2	0
BU	BU	V	T2	U	U	-	-4	-2	-2	0
BU	BU	V	T2	BU	T1	-	-2	-2	0	0
BU	BU	V	T2	BU	V	+	-2	-2	-2	0
BU	BU	V	T2	BU	BV	+	-2	-2	-2	0
BU	BU	V	U	T1	V	+	-2	-2	-2	0
BU	BU	V	U	T1	BV	+	-2	-2	-2	0
BU	BU	V	U	T1	T2	-	-4	-4	2	0
BU	BU	V	U	T1	U		0			
BU	BU	V	U	V	T1	-	-3	0	-1	0
BU	BU	V	U	V	V		0			

BU	BU	V	U	V	BV	−	−2	0	−2	0
BU	BU	V	U	V	T2	+	−5	−2	1	0
BU	BU	V	U	V	U		0			
BU	BU	V	U	BV	T1	−	−3	0	−1	0
BU	BU	V	U	BV	V	+	−2	0	−2	0
BU	BU	V	U	BV	BV	+	−2	0	−2	0
BU	BU	V	U	BV	T2	+	−5	−2	1	0
BU	BU	V	U	BV	U		0			
BU	BU	V	U	T2	T1	+	−2	−2	0	0
BU	BU	V	U	T2	V	−	−2	−2	−2	0
BU	BU	V	U	T2	BV	−	−2	−2	−2	0
BU	BU	V	U	T2	U	−	−4	−2	−2	0
BU	BU	V	U	U	A	−	−4	0	0	0
BU	BU	V	U	U	T1	−	−4	−2	0	0
BU	BU	V	U	U	V	−	−4	−2	−2	2
BU	BU	V	U	U	BV	−	−4	−2	−2	2
BU	BU	V	U	U	T2	−	−2	0	−2	0
BU	BU	V	U	U	U	−	−4	−4	−2	2
BU	BU	V	U	BU	A	+	−4	0	0	0
BU	BU	V	U	BU	T1	+	−4	−2	0	0
BU	BU	V	U	BU	V	+	−4	−2	−2	2
BU	BU	V	U	BU	BV	+	−4	−2	−2	2
BU	BU	V	U	BU	T2	+	−2	0	−2	0
BU	BU	V	BU	T1	V	+	−2	−2	−2	0
BU	BU	V	BU	T1	BV	+	−2	−2	−2	0
BU	BU	V	BU	T1	T2	−	−4	−4	2	0
BU	BU	V	BU	V	T1	+	−3	−2	−1	0
BU	BU	V	BU	V	V	+	−2	−2	0	0
BU	BU	V	BU	V	BV		0			
BU	BU	V	BU	V	T2	+	−5	−4	1	0

BU	BU	V	BU	BV	T1	+	-3	0	-1	0
BU	BU	V	BU	BV	V	-	-2	0	-2	0
BU	BU	V	BU	BV	BV	-	-2	0	-2	0
BU	BU	V	BU	BV	T2	-	-5	-2	1	0
BU	BU	V	BU	T2	T1	-	-2	-2	0	0
BU	BU	V	BU	T2	V	+	-2	-2	-2	0
BU	BU	V	BU	T2	BV	+	-2	-2	-2	0
BU	BU	V	BU	U	A	+	-4	0	0	0
BU	BU	V	BU	U	T1	+	-4	-2	0	0
BU	BU	V	BU	U	V	+	-4	-2	-2	2
BU	BU	V	BU	U	BV	+	-4	-2	-2	2
BU	BU	V	BU	U	T2	+	-4	-4	-2	2
BU	BU	V	BU	BU	A	+	-4	0	0	0
BU	BU	V	BU	BU	T1	+	-4	-2	0	0
BU	BU	V	BU	BU	V	+	-4	-2	-2	2
BU	BU	V	BU	BU	BV	+	-4	-2	-2	2
BU	BU	V	BU	BU	T2	+	-4	-4	-2	2
BU	BU	BV	A	V	U	-	-2	0	-1	0
BU	BU	BV	A	V	BU	+	-2	0	-1	0
BU	BU	BV	A	BV	U	-	-2	0	-1	0
BU	BU	BV	A	BV	BU	+	-2	0	-1	0
BU	BU	BV	T1	T1	V	-	-3	1	-2	0
BU	BU	BV	T1	T1	BV	-	-3	1	-2	0
BU	BU	BV	T1	T1	T2	-	-3	-1	0	0
BU	BU	BV	T1	T1	U		0			
BU	BU	BV	T1	T1	BU		0			
BU	BU	BV	T1	V	V	+	-4	-2	0	0
BU	BU	BV	T1	V	BV	-	-4	2	-2	0
BU	BU	BV	T1	V	T2	+	-3	-2	-1	0
BU	BU	BV	T1	V	U	+	-1	-2	-1	0

W COEFFICIENTS FOR THE ICOSAHEDRAL GROUP

BU	BU	BV	A	V	U	-	-2	0	-1	0
BU	BU	BV	A	V	BU	+	-2	0	-1	0
BU	BU	BV	A	BV	U	-	-2	0	-1	0
BU	BU	BV	A	BV	BU	+	-2	0	-1	0
BU	BU	BV	T1	T1	V	-	-3	1	-2	0
BU	BU	BV	T1	T1	BV	-	-3	1	-2	0
BU	BU	BV	T1	T1	T2	-	-3	-1	0	0
BU	BU	BV	T1	T1	U		0			
BU	BU	BV	T1	T1	BU		0			
BU	BU	BV	T1	V	V	+	-4	-2	0	0
BU	BU	BV	T1	V	BV	-	-4	2	-2	0
BU	BU	BV	T1	V	T2	+	-3	-2	-1	0
BU	BU	BV	T1	V	U	+	-1	-2	-1	0
BU	BU	BV	T1	V	BU	-	-1	-2	-1	0
BU	BU	BV	T1	BV	V	+	-4	2	-2	0
BU	BU	BV	T1	BV	BV	+	-4	2	-2	0
BU	BU	BV	T1	BV	T2	+	-3	-2	-1	0
BU	BU	BV	T1	BV	U	+	-1	-2	-1	0
BU	BU	BV	T1	BV	BU	-	-1	-2	-1	0
BU	BU	BV	T1	T2	V	+	2	-2	-2	0
BU	BU	BV	T1	T2	BV	+	2	-2	-2	0
BU	BU	BV	T1	T2	U	+	-4	-4	2	0
BU	BU	BV	T1	T2	BU	-	-4	-4	2	0
BU	BU	BV	T1	U	V	+	-2	-2	-2	0
BU	BU	BV	T1	U	BV	+	-2	-2	-2	0
BU	BU	BV	T1	U	T2	-	-4	-4	2	0
BU	BU	BV	T1	U	U		0			

BU	BU	BV	T1	BU	V	+	-2	-2	-2	0	
BU	BU	BV	T1	BU	BV	+	-2	-2	-2	0	
BU	BU	BV	T1	BU	T2	-	-4	-4	2	0	
BU	BU	BV	V	A	U	-	-2	0	-1	0	
BU	BU	BV	V	A	BU	+	-2	0	-1	0	
BU	BU	BV	V	T1	V	+	-4	-2	0	0	
BU	BU	BV	V	T1	BV	-	-4	2	-2	0	
BU	BU	BV	V	T1	T2	+	-3	-2	-1	0	
BU	BU	BV	V	T1	U	+	-1	-2	-1	0	
BU	BU	BV	V	T1	BU	-	-1	-2	-1	0	
BU	BU	BV	V	V	T1	+	-3	-2	-5	-1	53
BU	BU	BV	V	V	V	-	-5	-2	-3	-1	17
BU	BU	BV	V	V	BV	+	-5	0	-5	-1	17
BU	BU	BV	V	V	T2	-	1	-2	-5	-1	43
BU	BU	BV	V	V	U	+	1	-2	-5	-1	17
BU	BU	BV	V	V	BU	-	1	-2	-5	-1	17
BU	BU	BV	V	BV	T1	-	-1	2	-2	-1	
BU	BU	BV	V	BV	V	-	-5	0	0	-1	
BU	BU	BV	V	BV	BV	-	-5	2	-2	-1	
BU	BU	BV	V	BV	T2	-	-3	0	-2	-1	
BU	BU	BV	V	BV	U	+	1	0	-2	-1	
BU	BU	BV	V	BV	BU	-	1	0	-2	-1	
BU	BU	BV	V	T2	T1	-	-3	-2	-1	0	
BU	BU	BV	V	T2	V	-	-4	-2	0	0	
BU	BU	BV	V	T2	BV	-	-4	2	-2	0	
BU	BU	BV	V	T2	U	-	-3	-4	1	0	
BU	BU	BV	V	T2	BU	+	-3	-4	1	0	
BU	BU	BV	V	U	T1	-	-3	0	-1	0	
BU	BU	BV	V	U	V		0				
BU	BU	BV	V	U	BV	-	-2	0	-2	0	

BU	BU	BV	V	U	T2	+	−5	−2	1	0
BU	BU	BV	V	U	U		0			
BU	BU	BV	V	BU	T1	−	−3	0	−1	0
BU	BU	BV	V	BU	V		0			
BU	BU	BV	V	BU	BV	−	−2	0	−2	0
BU	BU	BV	V	BU	T2	+	−5	−2	1	0
BU	BU	BV	BV	A	U	−	−2	0	−1	0
BU	BU	BV	BV	A	BU	+	−2	0	−1	0
BU	BU	BV	BV	T1	V	+	−4	2	−2	0
BU	BU	BV	BV	T1	BV	+	−4	2	−2	0
BU	BU	BV	BV	T1	T2	+	−3	−2	−1	0
BU	BU	BV	BV	T1	U	+	−1	−2	−1	0
BU	BU	BV	BV	T1	BU	−	−1	−2	−1	0
BU	BU	BV	BV	V	T1	−	−1	2	−2	−1
BU	BU	BV	BV	V	V	−	−5	0	0	−1
BU	BU	BV	BV	V	BV	−	−5	2	−2	−1
BU	BU	BV	BV	V	T2	−	−3	0	−2	−1
BU	BU	BV	BV	V	U	+	1	0	−2	−1
BU	BU	BV	BV	V	BU	−	1	0	−2	−1
BU	BU	BV	BV	BV	T1	−	−1	2	−2	−1
BU	BU	BV	BV	BV	V	+	−5	2	−2	−1
BU	BU	BV	BV	BV	BV	+	−5	2	−2	−1
BU	BU	BV	BV	BV	T2	−	−3	0	−2	−1
BU	BU	BV	BV	BV	U	+	1	0	−2	−1
BU	BU	BV	BV	BV	BU	−	1	0	−2	−1
BU	BU	BV	BV	T2	T1	−	−3	−2	−1	0
BU	BU	BV	BV	T2	V	+	−4	2	−2	0
BU	BU	BV	BV	T2	BV	+	−4	2	−2	0
BU	BU	BV	BV	T2	U	−	−3	−4	1	0
BU	BU	BV	BV	T2	BU	+	−3	−4	1	0

BU	BU	BV	BV	U	T1	−	−3	0	−1	0
BU	BU	BV	BV	U	V	+	−2	0	−2	0
BU	BU	BV	BV	U	BV	+	−2	0	−2	0
BU	BU	BV	BV	U	T2	+	−5	−2	1	0
BU	BU	BV	BV	U	U		0			
BU	BU	BV	BV	BU	T1	+	−3	0	−1	0
BU	BU	BV	BV	BU	V	−	−2	0	−2	0
BU	BU	BV	BV	BU	BV	−	−2	0	−2	0
BU	BU	BV	BV	BU	T2	−	−5	−2	1	0
BU	BU	BV	T2	T1	V	+	2	−2	−2	0
BU	BU	BV	T2	T1	BV	+	2	−2	−2	0
BU	BU	BV	T2	T1	U	+	−4	−4	2	0
BU	BU	BV	T2	T1	BU	−	−4	−4	2	0
BU	BU	BV	T2	V	T1	−	−3	−2	−1	0
BU	BU	BV	T2	V	V	−	−4	−2	0	0
BU	BU	BV	T2	V	BV	−	−4	2	−2	0
BU	BU	BV	T2	V	U	−	−3	−4	1	0
BU	BU	BV	T2	V	BU	+	−3	−4	1	0
BU	BU	BV	T2	BV	T1	−	−3	−2	−1	0
BU	BU	BV	T2	BV	V	+	−4	2	−2	0
BU	BU	BV	T2	BV	BV	+	−4	2	−2	0
BU	BU	BV	T2	BV	U	−	−3	−4	1	0
BU	BU	BV	T2	BV	BU	+	−3	−4	1	0
BU	BU	BV	T2	T2	T1	+	−3	−1	0	0
BU	BU	BV	T2	T2	V	+	−3	1	−2	0
BU	BU	BV	T2	T2	BV	+	−3	1	−2	0
BU	BU	BV	T2	T2	U		0			
BU	BU	BV	T2	T2	BU		0			
BU	BU	BV	T2	U	T1	+	−2	−2	0	0
BU	BU	BV	T2	U	V	−	−2	−2	−2	0

BU	BU	BV	T2	U	BV	−	−2	−2	−2	0
BU	BU	BV	T2	U	U	−	−4	−2	−2	0
BU	BU	BV	T2	BU	T1	−	−2	−2	0	0
BU	BU	BV	T2	BU	V	+	−2	−2	−2	0
BU	BU	BV	T2	BU	BV	+	−2	−2	−2	0
BU	BU	BV	U	T1	V	+	−2	−2	−2	0
BU	BU	BV	U	T1	BV	+	−2	−2	−2	0
BU	BU	BV	U	T1	T2	−	−4	−4	2	0
BU	BU	BV	U	T1	U		0			
BU	BU	BV	U	V	T1	−	−3	0	−1	0
BU	BU	BV	U	V	V		0			
BU	BU	BV	U	V	BV	−	−2	0	−2	0
BU	BU	BV	U	V	T2	+	−5	−2	1	0
BU	BU	BV	U	V	U		0			
BU	BU	BV	U	BV	T1	−	−3	0	−1	0
BU	BU	BV	U	BV	V	+	−2	0	−2	0
BU	BU	BV	U	BV	BV	+	−2	0	−2	0
BU	BU	BV	U	BV	T2	+	−5	−2	1	0
BU	BU	BV	U	BV	U		0			
BU	BU	BV	U	T2	T1	+	−2	−2	0	0
BU	BU	BV	U	T2	V	−	−2	−2	−2	0
BU	BU	BV	U	T2	BV	−	−2	−2	−2	0
BU	BU	BV	U	T2	U	−	−4	−2	−2	0
BU	BU	BV	U	U	A	−	−4	0	0	0
BU	BU	BV	U	U	T1	−	−4	−2	0	0
BU	BU	BV	U	U	V	−	−4	−2	−2	2
BU	BU	BV	U	U	BV	−	−4	−2	−2	2
BU	BU	BV	U	U	T2	−	−2	0	−2	0
BU	BU	BV	U	U	U	−	−4	−4	−2	2
BU	BU	BV	U	BU	A	+	−4	0	0	0

BU	BU	BV	U	BU	T1	+	-4	-2	0	0
BU	BU	BV	U	BU	V	+	-4	-2	-2	2
BU	BU	BV	U	BU	BV	+	-4	-2	-2	2
BU	BU	BV	U	BU	T2	+	-2	0	-2	0
BU	BU	BV	BU	T1	V	+	-2	-2	-2	0
BU	BU	BV	BU	T1	BV	+	-2	-2	-2	0
BU	BU	BV	BU	T1	T2	-	-4	-4	2	0
BU	BU	BV	BU	V	T1	-	-3	0	-1	0
BU	BU	BV	BU	V	V		0			
BU	BU	BV	BU	V	BV	-	-2	0	-2	0
BU	BU	BV	BU	V	T2	+	-5	-2	1	0
BU	BU	BV	BU	BV	T1	+	-3	0	-1	0
BU	BU	BV	BU	BV	V	-	-2	0	-2	0
BU	BU	BV	BU	BV	BV	-	-2	0	-2	0
BU	BU	BV	BU	BV	T2	-	-5	-2	1	0
BU	BU	BV	BU	T2	T1	-	-2	-2	0	0
BU	BU	BV	BU	T2	V	+	-2	-2	-2	0
BU	BU	BV	BU	T2	BV	+	-2	-2	-2	0
BU	BU	BV	BU	U	A	+	-4	0	0	0
BU	BU	BV	BU	U	T1	+	-4	-2	0	0
BU	BU	BV	BU	U	V	+	-4	-2	-2	2
BU	BU	BV	BU	U	BV	+	-4	-2	-2	2
BU	BU	BV	BU	U	T2	+	-4	-4	-2	2
BU	BU	BV	BU	BU	A	+	-4	0	0	0
BU	BU	BV	BU	BU	T1	+	-4	-2	0	0
BU	BU	BV	BU	BU	V	+	-4	-2	-2	2
BU	BU	BV	BU	BU	BV	+	-4	-2	-2	2
BU	BU	BV	BU	BU	T2	+	-4	-4	-2	2
BU	BU	T2	A	T2	U	+	0	-3	0	0
BU	BU	T2	A	T2	BU	-	0	-3	0	0

BU	BU	T2	T1	V	V	+	-5	-4	1	0
BU	BU	T2	T1	V	BV	+	-5	-2	1	0
BU	BU	T2	T1	V	T2	+	-4	-4	2	0
BU	BU	T2	T1	V	U	+	-4	-4	2	0
BU	BU	T2	T1	V	BU	-	-4	-4	2	0
BU	BU	T2	T1	BV	V	-	-5	-2	1	0
BU	BU	T2	T1	BV	BV	-	-5	-2	1	0
BU	BU	T2	T1	BV	T2	+	-4	-4	2	0
BU	BU	T2	T1	BV	U	+	-4	-4	2	0
BU	BU	T2	T1	BV	BU	-	-4	-4	2	0
BU	BU	T2	T1	U	V	-	-4	-4	2	0
BU	BU	T2	T1	U	BV	-	-4	-4	2	0
BU	BU	T2	T1	U	T2	+	0	-4	0	0
BU	BU	T2	T1	U	U	+	-4	-4	0	0
BU	BU	T2	T1	BU	V	-	-4	-4	2	0
BU	BU	T2	T1	BU	BV	-	-4	-4	2	0
BU	BU	T2	T1	BU	T2	+	0	-4	0	0
BU	BU	T2	V	T1	V	+	-5	-4	1	0
BU	BU	T2	V	T1	BV	+	-5	-2	1	0
BU	BU	T2	V	T1	T2	+	-4	-4	2	0
BU	BU	T2	V	T1	U	+	-4	-4	2	0
BU	BU	T2	V	T1	BU	-	-4	-4	2	0
BU	BU	T2	V	V	T1	+	-5	-2	1	0
BU	BU	T2	V	V	V	-	-7	-4	3	0
BU	BU	T2	V	V	BV	+	-7	-2	1	0
BU	BU	T2	V	V	T2	-	-3	-4	1	0
BU	BU	T2	V	V	U	-	-3	-4	1	0
BU	BU	T2	V	V	BU	+	-3	-4	1	0
BU	BU	T2	V	BV	T1	+	-5	-2	1	0
BU	BU	T2	V	BV	V	-	-7	-2	1	0

BU	BU	T2	V	BV	BV	-	-7	-2	1	0
BU	BU	T2	V	BV	T2	-	-3	-4	1	0
BU	BU	T2	V	BV	U	-	-3	-4	1	0
BU	BU	T2	V	BV	BU	+	-3	-4	1	0
BU	BU	T2	V	T2	T1	+	-5	-3	2	0
BU	BU	T2	V	T2	V	-	-6	-1	1	0
BU	BU	T2	V	T2	BV	+	-6	-3	1	0
BU	BU	T2	V	T2	U		0			
BU	BU	T2	V	T2	BU		0			
BU	BU	T2	V	U	T1		0			
BU	BU	T2	V	U	V	+	-5	-4	1	0
BU	BU	T2	V	U	BV	+	-5	-2	1	0
BU	BU	T2	V	U	T2	+	0	-4	0	0
BU	BU	T2	V	U	U	+	-4	-4	0	0
BU	BU	T2	V	BU	T1		0			
BU	BU	T2	V	BU	V	+	-5	-4	1	0
BU	BU	T2	V	BU	BV	+	-5	-2	1	0
BU	BU	T2	V	BU	T2	+	0	-4	0	0
BU	BU	T2	BV	T1	V	-	-5	-2	1	0
BU	BU	T2	BV	T1	BV	-	-5	-2	1	0
BU	BU	T2	BV	T1	T2	+	-4	-4	2	0
BU	BU	T2	BV	T1	U	+	-4	-4	2	0
BU	BU	T2	BV	T1	BU	-	-4	-4	2	0
BU	BU	T2	BV	V	T1	+	-5	-2	1	0
BU	BU	T2	BV	V	V	-	-7	-2	1	0
BU	BU	T2	BV	V	BV	-	-7	-2	1	0
BU	BU	T2	BV	V	T2	-	-3	-4	1	0
BU	BU	T2	BV	V	U	-	-3	-4	1	0
BU	BU	T2	BV	V	BU	+	-3	-4	1	0
BU	BU	T2	BV	BV	T1	+	-5	-2	1	0

BU	BU	T2	BV	BV	V	+	−7	−2	1	0
BU	BU	T2	BV	BV	BV	+	−7	−2	1	0
BU	BU	T2	BV	BV	T2	−	−3	−4	1	0
BU	BU	T2	BV	BV	U	−	−3	−4	1	0
BU	BU	T2	BV	BV	BU	+	−3	−4	1	0
BU	BU	T2	BV	T2	T1	+	−5	−3	2	0
BU	BU	T2	BV	T2	V	−	−6	−3	1	0
BU	BU	T2	BV	T2	BV	−	−6	−3	1	0
BU	BU	T2	BV	T2	U		0			
BU	BU	T2	BV	T2	BU		0			
BU	BU	T2	BV	U	T1		0			
BU	BU	T2	BV	U	V	−	−5	−2	1	0
BU	BU	T2	BV	U	BV	−	−5	−2	1	0
BU	BU	T2	BV	U	T2	+	0	−4	0	0
BU	BU	T2	BV	U	U	+	−4	−4	0	0
BU	BU	T2	BV	BU	T1		0			
BU	BU	T2	BV	BU	V	−	−5	−2	1	0
BU	BU	T2	BV	BU	BV	−	−5	−2	1	0
BU	BU	T2	BV	BU	T2	+	0	−4	0	0
BU	BU	T2	T2	A	U	+	−2	−1	0	0
BU	BU	T2	T2	A	BU	−	−2	−1	0	0
BU	BU	T2	T2	V	T1	+	−5	−3	2	0
BU	BU	T2	T2	V	V	−	−6	−1	1	0
BU	BU	T2	T2	V	BV	+	−6	−3	1	0
BU	BU	T2	T2	V	U	+	−1	−3	−2	0
BU	BU	T2	T2	V	BU	−	−1	−3	−2	0
BU	BU	T2	T2	BV	T1	+	−5	−3	2	0
BU	BU	T2	T2	BV	V	−	−6	−3	1	0
BU	BU	T2	T2	BV	BV	−	−6	−3	1	0
BU	BU	T2	T2	BV	U	+	−1	−3	−2	0

BU	BU	T2	T2	BV	BU	-	-1	-3	-2	0	
BU	BU	T2	T2	T2	T1	-	-5	-4	2	0	
BU	BU	T2	T2	T2	V	+	-5	-4	2	0	
BU	BU	T2	T2	T2	BV	+	-5	-4	2	0	
BU	BU	T2	T2	T2	U	+	-3	-2	0	0	
BU	BU	T2	T2	T2	BU	-	-3	-2	0	0	
BU	BU	T2	U	T1	V	-	-4	-4	2	0	
BU	BU	T2	U	T1	BV	-	-4	-4	2	0	
BU	BU	T2	U	T1	T2	+	-2	-2	0	0	
BU	BU	T2	U	T1	U	+	-4	-4	0	0	
BU	BU	T2	U	V	T1		0				
BU	BU	T2	U	V	V	+	-5	-4	1	0	
BU	BU	T2	U	V	BV	+	-5	-2	1	0	
BU	BU	T2	U	V	T2	+	-2	-4	-2	0	11
BU	BU	T2	U	V	U	+	-4	-4	0	0	
BU	BU	T2	U	BV	T1		0				
BU	BU	T2	U	BV	V	-	-5	-2	1	0	
BU	BU	T2	U	BV	BV	-	-5	-2	1	0	
BU	BU	T2	U	BV	T2	+	-2	-4	-2	0	11
BU	BU	T2	U	BV	U	+	-4	-4	0	0	
BU	BU	T2	U	U	A	+	-4	0	0	0	
BU	BU	T2	U	U	T1	-	-4	-4	0	2	
BU	BU	T2	U	U	V	-	-4	-4	-2	2	
BU	BU	T2	U	U	BV	-	-4	-4	-2	2	
BU	BU	T2	U	U	T2		0				
BU	BU	T2	U	U	U	-	-2	-4	0	0	
BU	BU	T2	U	BU	A	-	-2	-2	0	0	
BU	BU	T2	U	BU	T1	+	-2	-2	0	0	
BU	BU	T2	U	BU	V	+	-2	0	-2	0	
BU	BU	T2	U	BU	BV	+	-2	0	-2	0	

BU	BU	T2	U	BU	T2	+	-4	-4	0	0
BU	BU	T2	BU	T1	V	-	-4	-4	2	0
BU	BU	T2	BU	T1	BV	-	-4	-4	2	0
BU	BU	T2	BU	T1	T2	+	0	-4	0	0
BU	BU	T2	BU	V	T1		0			
BU	BU	T2	BU	V	V	+	-5	-4	1	0
BU	BU	T2	BU	V	BV	+	-5	-2	1	0
BU	BU	T2	BU	V	T2	+	0	-4	0	0
BU	BU	T2	BU	BV	T1		0			
BU	BU	T2	BU	BV	V	-	-5	-2	1	0
BU	BU	T2	BU	BV	BV	-	-5	-2	1	0
BU	BU	T2	BU	BV	T2	+	0	-4	0	0
BU	BU	T2	BU	U	A	-	-4	0	0	0
BU	BU	T2	BU	U	T1	+	-4	-4	0	2
BU	BU	T2	BU	U	V	+	-4	-4	-2	2
BU	BU	T2	BU	U	BV	+	-4	-4	-2	2
BU	BU	T2	BU	U	T2	+	-4	-4	0	0
BU	BU	T2	BU	BU	A	-	-4	0	0	0
BU	BU	T2	BU	BU	T1	+	-4	-4	0	2
BU	BU	T2	BU	BU	V	+	-4	-4	-2	2
BU	BU	T2	BU	BU	BV	+	-4	-4	-2	2
BU	BU	T2	BU	BU	T2	+	-4	-4	0	0

FOUR: THE DIHEDRAL GROUPS D^*_n.

The z-axis is the n-fold axis and the y-axis is the two-fold axis.

TABLE 4.1: THE ICR BASIS VECTORS FOR D^*_n WITH n EVEN

Component	Expression	J	M
$\vert A_1 a_1>$	$\vert J0>$	even	-
$\vert A_1 a_1>$	$\frac{1}{\sqrt{2}} \vert JM> + \frac{1}{\sqrt{2}} \vert J - M>$	even	kn
$\vert A_1 a_1>$	$\frac{1}{\sqrt{2}} \vert JM> - \frac{1}{\sqrt{2}} \vert J - M>$	odd	kn
$\vert A_2 a_2>$	$i \vert j0>$	odd	-
$\vert A_2 a_2>$	$\frac{i}{\sqrt{2}} \vert JM> - \frac{i}{\sqrt{2}} \vert J - M>$	even	kn
$\vert A_2 a_2>$	$\frac{i}{\sqrt{2}} \vert JM> + \frac{i}{\sqrt{2}} \vert J - M>$	odd	kn
$\vert B_1 b_1>$	$\frac{1}{\sqrt{2}} \vert JM> + \frac{1}{\sqrt{2}} \vert J - M>$	even	(2k + 1)n/2
$\vert B_1 b_1>$	$\frac{1}{\sqrt{2}} \vert JM> - \frac{1}{\sqrt{2}} \vert J - M>$	odd	(2k + 1)n/2
$\vert B_2 b_2>$	$\frac{i}{\sqrt{2}} \vert JM> - \frac{i}{\sqrt{2}} \vert J - M>$	even	(2k + 1)n/2
$\vert B_2 b_2>$	$\frac{i}{\sqrt{2}} \vert JM> + \frac{i}{\sqrt{2}} \vert J - M>$	odd	(2k + 1)n/2
$\vert E\ x>$	$\vert JM>$	-	l + kn
$\vert E_l\, y>$	$(-1)^{J-l} \vert J - M>$	-	l + kn
$\vert E'_l\, \alpha>$	$\vert JM >$	-	l + kn
$\vert E'_l\, \beta>$	$(-1)^{J-l}\ J - M$	-	l + kn

TABLE 4.2: THE WIGNER TENSOR FOR D^*_n WITH n EVEN

j	m	n	value
A_1	a_1	a_1	1
A_2	a_2	a_2	1
B_1	b_1	b_1	1
B_2	b_2	b_2	1
E_l	x	y	-1
	y	x	-1
E'_l	α	β	1
	β	α	-1

TABLE 4.3: $\bar{V}$ COEFFICIENTS FOR D^*_n WITH n EVEN

j_1	j_2	j_3	m_1	m_2	m_3	$\bar{V}$	Parity
A_1	A_1	A_1	a_1	a_1	a_1	1	even
A_2	A_2	A_1	a_2	a_2	a_1	1	even
B_1	B_1	A_1	b_1	b_1	a_1	1	even
B_2	B_2	A_1	b_2	b_2	a_1	1	even
B_2	B_1	A_2	b_2	b_2	a_2	-1	odd
E_k	E_k	A_1	k	-k	a_1	$1/\sqrt{2}$	even
E_k	E_k	A_2	k	-k	a_2	$i/\sqrt{2}$	odd
E_k	E_l	E_{k+l}	k	l	$-(k + l)$	$1/\sqrt{2}$	even
E_k	$E_{n/2-k}$	B_1	k	$n/2 - k$	b_1	$1/\sqrt{2}$	even
E_k	$E_{n/2-k}$	B_2	k	$n/2 - k$	b_2	$i/\sqrt{2}$	even
E'_k	E'_k	A_1	k	-k	a_1	$1/\sqrt{2}$	odd
E'_k	E'_k	A_2	k	-k	a_2	$-i/\sqrt{2}$	even
E'_k	E'_l	E_{k+l}	k	l	$-(k + l)$	$-1/\sqrt{2}$	even
E'_k	E'_l	E_{k-l}	k	$-l$	$-(k - l)$	$-1/\sqrt{2}$	even
E'_k	$E'_{n/2-k}$	B_1	k	$n/2 - k$	b_1	$-1/\sqrt{2}$	even
E'_k	$E'_{n/2-k}$	B_2	k	$n/2 - k$	b_2	$-i/\sqrt{2}$	even

TABLE 4.4 : BASIS VECTORS FOR D^*_n WITH n ODD

Component	Expression	J	M
$\|A_1a_1>$	$\|J0>$	even	-
$\|A_1a_1>$	$\frac{1}{\sqrt{2}}\|JM> + \frac{(-1)^J}{\sqrt{2}}\|J-M>$	-	2kn
$\|A_1a_1>$	$\frac{1}{\sqrt{2}}\|JM> - \frac{(-1)^J}{\sqrt{2}}\|J-M>$	-	(2k + 1)n
$\|A_1a_2>$	$i\|J0>$	odd	-
$\|A_2a_2>$	$\frac{i}{\sqrt{2}}\|JM> - \frac{(-1)^J}{\sqrt{2}}\|J-M>$	-	2kn
$\|A_2a_2>$	$\frac{i}{\sqrt{2}}\|JM> + i\frac{(-1)^J}{\sqrt{2}}\|J-M>$	-	(2k + 1)n
$\|E_l x>$	$\|JM>$	-	l + 2kn
$\|E_l y>$	$(-1)^{J-l}\|J-M>$	-	(l + 2kn)
$\|E_l x>$	$i\|JM>$	-	l + (2k + 1)n
$\|E_l y>$	$(-1)^{J-l}\|J-M>$	-	l + (2k + 1)n
$\|E'_l\alpha>$	$\|JM>$	-	l + 2kn
$\|E'_l\beta>$	$(-1)^{J-l}\|J-M>$	-	l + 2kn
$\|E'_l\alpha>$	$i\|JM>$	-	l + (2k + 1)n
$\|E'_l\beta>$	$(-1)^{-l}\| -M>$	-	l + (2k + 1)n
$\|E'\alpha>$	$\frac{1}{\sqrt{2}}\|JM> + (-1)^{J-n/2}\frac{i}{\sqrt{2}}\|J-M>>$	-	$\frac{4k+1}{2}n$
$\|E'\beta>$	$\frac{i}{\sqrt{2}}\|JM> + (-1)^{J-n/2}\frac{1}{\sqrt{2}}\|J-M>$	-	$\frac{4k+1}{2}n$
$\|E'\alpha>$	$\frac{1}{\sqrt{2}}\|JM> - (1)^{J-n/2}\frac{i}{\sqrt{2}}\|J-M>$	-	$\frac{4k+3}{2}n$
$\|E'\beta>$	$\frac{i}{\sqrt{2}}\|JM> - (-1)^{J-n/2}\frac{1}{\sqrt{2}}\|JM-M>$	-	$\frac{4k+3}{2}n$

TABLE 4.5: THE WIGNER TENSOR FOR D^*_n WITH n ODD

j	m	$\dot{n}$	value
A_1	a_1	a_1	1
A_2	a_2	a_2	1
E_ℓ	x	y	-1
	y	x	-1
E'_l	α	β	1
	β	α	-1
E'	α	β	1
	β	α	-1

TABLE 4.6: $\bar{V}$ COEFFICIENTS FOR D^*_n WITH n ODD

j_1	j_2	j_3	m_1	m_2	m_3	$\bar{V}$	Parity
A_1	A_1	A_1	a_1	a_1	a_1	1	even
A_2	A_2	A_1	a_2	a_2	a_1	1	even
E_k	E_k	A_1	k	-k	a_1	$1/\sqrt{2}$	even
E_k	E_k	A_2	k	-k	a_2	$i/\sqrt{2}$	odd
E_k	E_l	E_{k+l}	k	l	$-(k + l)$	$1/\sqrt{2}$	even
E'_k	E'_k	A_1	k	-k	a_1	$1/\sqrt{2}$	odd
E'_k	E'_k	A_2	k	-k	a_2	$-i/\sqrt{2}$	even
E'_k	E'_l	E_{k+l}	k	l	$-(k + l)$	$-1/\sqrt{2}$	even
E'_k	E'_l	E_{k-l}	k	$-l$	$-(k - l)$	$-1/\sqrt{2}$	even
E'	E'	A_1	α	β	a_1	$1/\sqrt{2}$	odd
E'	E'	A_1	α	β	a_1	$i/\sqrt{2}$	even
E'	E'	A_2	α	α	a_2	$-1/\sqrt{2}$	even
E'	E'	A_2	α	α	a_2	$-i/\sqrt{2}$	even
E'	E'_k	$aE_{n/2+k}$	n/2	k	n/2 - k	$-1/\sqrt{2}$	even
E'	E'_k	$bE_{n/2-k}$	n/2	-k	-n/2 + k	$-1/\sqrt{2}$	odd

TABLE 4.7: BASIS VECTORS FOR D^*_3

J = 0	$\|A_1a_1\rangle = \|00\rangle$
J = 1	$\|A_2a_2\rangle = i\|10\rangle$
	$\|E_1x\rangle = \|11\rangle$
	$\|E_1y\rangle = \|1\ -1\rangle$
J = 2	$\|A_1a_1\rangle = \|20\rangle$
	$\|aE_1x\rangle = \|21\rangle$
	$\|aE_1y\rangle = -\|2\ -1\rangle$
	$\|bE_1x\rangle = \|2\ -2\rangle$
	$\|bE_1y\rangle = \|22\rangle$
J = 3	$\|A_1a_1\rangle = \frac{1}{\sqrt{2}}\|33\rangle + \frac{1}{\sqrt{2}}\|3\ -3\rangle$
	$\|aA_2a_2\rangle = i\|30\rangle$
	$\|bA_2a_2\rangle = \frac{i}{\sqrt{2}}\|33\rangle - \frac{i}{\sqrt{2}}\|3\ -3\rangle$
	$\|aE_1x\rangle = \|31\rangle$
	$\|aE_1y\rangle = \|3\ -1\rangle$
	$\|bE_1x\rangle = i\|3\ -2\rangle$
	$\|bE_1y\rangle = i\|32\rangle$
J = 1/2	$\|E'_{1/2}\alpha\rangle = \|1/2\ 1/2\rangle$
	$\|E'_{1/2}\beta\rangle = \|1/2\ -1/2\rangle$
J = 3/2	$\|E'_{1/2}\alpha\rangle = \|3/2\ 1/2\rangle$
	$\|E'_{1/2}\beta\rangle = -\|3/2\ -1/2\rangle$
	$\|E'\alpha\rangle = \frac{1}{\sqrt{2}}\|3/2\ 3/2\rangle + \frac{i}{\sqrt{2}}\|3/2\ -3/2\rangle$
	$\|E'\beta\rangle = \frac{i}{\sqrt{2}}\|3/2\ 3/2\rangle + \frac{1}{\sqrt{2}}\|3/2\ -3/2\rangle$

$J = 5/2 \quad |aE'_{1/2}\alpha> \; = \; |5/2 \;\; 1/2>$

$|aE'_{1/2}\beta> \; = \; |5/2 \; - \; 1/2>$

$|bE'_{1/2}\alpha> \; = \; |5/2 \; - \; 5/2>$

$|bE'_{1/2}\alpha> \; = \; -|5/2 \;\; 5/2>$

$|E'\alpha> \; = \; \frac{1}{\sqrt{2}} \; |5/3 \;\; 3/2> \; + \; \frac{i}{\sqrt{2}} \; |5/2 \; - \; 3/2>$

$|E'\beta> \; = \; - \; \frac{i}{\sqrt{2}} \; |5/2 \;\; 3/2> \; - \; \frac{1}{\sqrt{2}} \; |5/2 \; - \; 3/2>$

TABLE 4.8: $\bar{V}$ COEFFICIENTS FOR D^*_3

j_1	j_2	j_3	m_1	m_2	m_3	$\bar{V}$
A_1	A_1	A_1	a_1	a_1	a_1	1
A_2	A_2	A_1	a_2	a_2	a_1	1
E_1	E_1	A_2	x	y	a_1	$1/\sqrt{2}$
E_1	E_1	A_2	x	y	a_2	$i/\sqrt{2}$
E_1	E_1	E_1	x	x	x	1/2
$E'_{1/2}$	$E'_{1/2}$	A_1	α	β	a_1	$1/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	A_2	α	β	a_2	$-i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	E'	α	α	y	$-1/\sqrt{2}$
E'	E'	A_1	α	β	a_1	$-1/\sqrt{2}$
E'	E'	$3A_1$	α	β	a_1	$i/\sqrt{2}$
E'	E'	A_2	α	α	a_2	$-1/\sqrt{2}$
E'	E'	$3A_2$	α	α	a	$-i/\sqrt{2}$
E'	$E'_{1/2}$	E_1	α	α	x	$-1/\sqrt{2}$
E'	$E'_{1/2}$	E_1	α	β	y	$-1/\sqrt{2}$

TABLE 4.9: BASIS VECTORS FOR D^*_4

J = 0 $|A_1a_1\rangle = |00\rangle$

J = 1 $|A_2a_2\rangle = i|10\rangle$

$|E_1x\rangle = |11\rangle$

$|E_1y\rangle = |1\ -1\rangle$

J = 2 $|A_1a_1\rangle = |20\rangle$

$|B_1b_1\rangle = \frac{1}{\sqrt{2}}|22\rangle + \frac{1}{\sqrt{2}}|2\ -2\rangle$

$|B_2b_2\rangle = \frac{i}{\sqrt{2}}|22\rangle - \frac{i}{\sqrt{2}}|2\ -2\rangle$

$|E_1x\rangle = |21\rangle$

$|E_1y\rangle = -|2\ -1\rangle$

J = 3 $|A_2a_2\rangle = i|30\rangle$

$|B_1b_1\rangle = \frac{1}{\sqrt{2}}|32\rangle - \frac{1}{\sqrt{2}}|3\ -2\rangle$

$|B_2b_2\rangle = \frac{i}{\sqrt{2}}|32\rangle + \frac{i}{\sqrt{2}}|3\ -2\rangle$

$|aE_1x\rangle = |31\rangle$

$|aE_1y\rangle = -|3\ -1\rangle$

$|bE_1x\rangle = |3\ -3\rangle$

$|bE_1y\rangle = |33\rangle$

J = 4 $|aA_1a_1\rangle = |40\rangle$

$|bA_1a_1\rangle = \frac{1}{\sqrt{2}}|44\rangle + \frac{1}{\sqrt{2}}|4\ -4\rangle$

$|A_2a_2\rangle = \frac{i}{\sqrt{2}}|44\rangle - \frac{i}{\sqrt{2}}|4\ -4\rangle$

$|B_1b_1\rangle = \frac{1}{\sqrt{2}}|42\rangle + \frac{1}{\sqrt{2}}|4\ -2\rangle$

$|B_2b_2\rangle = \frac{i}{\sqrt{2}}|42\rangle - \frac{i}{\sqrt{2}}|4\ -2\rangle$

$|aE_1x\rangle = |41\rangle$

$|aE_1y\rangle = -|4\ -1\rangle$

$|bE_1x\rangle = |4\ -3\rangle$

$|bE_1 y\rangle = |43\rangle$

J = 1/2 $|E'_{1/2}\alpha\rangle = |1/2\ 1/2\rangle$

$|E'_{1/2}\beta\rangle = |1/2\ -1/2\rangle$

J = 3/2 $|E'_{1/2}\alpha\rangle = |3/2\ 1/2\rangle$

$|E'_{1/2}\beta\rangle = -|3/2\ -1/2\rangle$

$|E'_{3/2}\alpha\rangle = |3/2\ 3/2\rangle$

$|E'_{3/2}\beta\rangle = |3/2\ -3/2\rangle$

J = 5/2 $|E'_{1/2}\alpha\rangle = |5/2\ 1/2\rangle$

$|E'_{1/2}\beta\rangle = |5/2\ -1/2\rangle$

$|aE'_{3/2}\alpha\rangle = |5/2\ 3/2\rangle$

$|aE'_{3/2}\beta\rangle = -|5/2\ -3/2\rangle$

$|bE'_{5/2}\alpha\rangle = |5/2\ -5/2\rangle$

$|bE'_{5/2}\beta\rangle = |5/2\ 5/2\rangle$

TABLE 4.10: $\bar{V}$ COEFFICIENTS FOR D^*_4

j_1	j_2	j_3	m_1	m_2	m_3	$\bar{V}$
A_1	A_1	A_1	a_1	a_1	a_1	1
A_2	A_2	A_1	a_2	a_2	a_2	1
B_1	B_1	A_1	b_1	b_1	b_1	1
B_2	B_2	A_1	b_2	b_2	b_2	1
B_2	B_1	A_2	b_2	b_1	a_2	-1
E_1	E_1	A	x	y	a_1	$1/\sqrt{2}$
E_1	E_1	A_2	x	y	a_2	$i/\sqrt{2}$
E_1	E_1	B_1	x	x	b_1	$1/\sqrt{2}$
E_1	E_1	B_2	x	x	b_2	$i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	A_1	α	β	a_1	$1/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	A_2	α	β	a_2	$-i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	E_1	α	α	y	$-1/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	A_1	α	β	a	$-i/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	E_2	α	β	x	$-1/\sqrt{2}$
$E'_{3/2}$	$E'_{1/2}$	B_1	α	α	b_1	$-1/\sqrt{2}$
$E'_{3/2}$	$E'_{1/2}$	B_2	α	α	b_2	$-i\sqrt{2}$
$E'_{3/2}$	$E'_{1/2}$	E_1	α	β	y	$-1/\sqrt{2}$

TABLE 4.11: BASIS VECTORS FOR D^*_5

$J = 0$	$\|A_1a_1\rangle = \|00\rangle$
$J = 1$	$\|A_2a_2\rangle = i\|10\rangle$
	$\|E_1x\rangle = \|11\rangle$
	$\|E_1y\rangle = \|1\ -1\rangle$
$J = 2$	$\|A_1a_1\rangle = \|20\rangle$
	$\|E_1x\rangle = \|21\rangle$
	$\|E_1y\rangle = -\|2\ -1\rangle$
	$\|E_2x\rangle = \|22\rangle$
	$\|E_2y\rangle = \|2\ -2\rangle$
$J = 3$	$\|A_2a_2\rangle = i\|30\rangle$
	$\|E_1x\rangle = \|31\rangle$
	$\|E_1y\rangle = \|3\ -1\rangle$
	$\|aE_2x\rangle = \|32\rangle$
	$\|aE_2y\rangle = -\|3\ -2\rangle$
	$\|bE_2x\rangle = \|3\ -3\rangle$
	$\|bE_2y\rangle = \|33\rangle$
$J = 5$	$\|5aA_1a_1\rangle = \frac{1}{\sqrt{2}}\|55\rangle + \frac{1}{\sqrt{2}}\|5\ -5\rangle$
	$\|5aA_2a_2\rangle = \frac{i}{\sqrt{2}}\|55\rangle - \frac{i}{\sqrt{2}}\|5\ -5\rangle$
$J = 1/2$	$\|E'_{1/2}\alpha\rangle = \|1/2\ 1/2\rangle$
	$\|E'_{1/2}\beta\rangle = \|1/2\ -1/2\rangle$
$J = 3/2$	$\|E'_{1/2}\alpha\rangle = \|3/2\ 1/2\rangle$
	$\|E'_{1/2}\beta\rangle = -\|3/2\ -1/2\rangle$
	$\|E'_{3/2}\alpha\rangle = \|3/2\ 3/2\rangle$

$|E'_{3/2}\beta\rangle = |3/2\ -3/2\rangle$

J = 5/2 $\quad |E'_{1/2}\alpha\rangle = |5/2\ 1/2\rangle$

$|E'_{1/2}\beta\rangle = |5/2\ -1/2\rangle$

$|E'_{3/2}\alpha\rangle = |5/2\ 3/2\rangle$

$|E'_{3/2}\beta\rangle = -|5/2\ 3/2\rangle$

$|E'\alpha\rangle = \frac{1}{\sqrt{2}}|5/2\ 5/2\rangle + \frac{i}{\sqrt{2}}|5/2\ -5/2\rangle$

$|E'\beta\rangle = \frac{i}{\sqrt{2}}|5/2\ 5/2\rangle + \frac{1}{\sqrt{2}}|5/2\ -5/2\rangle$

TABLE 4.12: $\overline{V}$ COEFFICIENTS FOR D^*_5

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$
A_1	A_1	A_1	a_1	a_1	a_1	1
A_2	A_2	A_1	a_2	a_2	a_1	1
E_1	E_1	A_1	x	y	a_1	$1/\sqrt{2}$
E_1	E_1	A_2	x	y	a_2	$i/\sqrt{2}$
E_1	E_1	E_2	x	x	y	$1/\sqrt{2}$
E_2	E_2	E_1	x	x	x	$1/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	A_1	α	β	a_1	$1/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	A_2	α	β	a_2	$-i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	E_1	α	α	y	$-1/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	A_1	α	β	a_1	$1/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	A_2	α	β	a_2	$-i/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	$3bE_2$	α	α	x	$-1/\sqrt{2}$
$E'_{1/2}$	$E'_{3/2}$	E_2	α	α	y	$-1/\sqrt{2}$
$E'_{1/2}$	$E'_{3/2}$	E_1	α	β	x	$-1/\sqrt{2}$
E'	E'	A_1	α	β	a_1	$-1/\sqrt{2}$
E'	E'	$5A_1$	α	β	a_1	$i/\sqrt{2}$
E'	E'	A_2	α	α	a_2	$-1/\sqrt{2}$
E'	E'	$5A_2$	α	α	a_2	$-i/\sqrt{2}$

E'	$E'_{1/2}$	$3bE_2$	α	α	x	$-1/\sqrt{2}$
E'	$E'_{1/2}$	$3aE_2$	α	β	y	$-1/\sqrt{2}$
E'	$E'_{3/2}$	$4E_1$	α	α	x	$-1/\sqrt{2}$
E'	$E'_{3/2}$	$1E_1$	α	β	y	$-1/\sqrt{2}$

FIVE: THE CYCLIC GROUPS C^*_n

The z-axis is the n-fold axis of symmetry.

Upon descent in symmetry each ICR of D^*_n is also an ICR of C^*_n (time-reversal is similar to a C_2 rotation) and so in one sense the following tables are superfluous: the tables for D^*_n may be used. However, not only does the removal of the C_2 operator give greater freedom in the phases of the basis vectors but also increase the multiplicity of A_1 in the triple product. For this reason we include tables with the phases chosen so that all possible independent $\bar{V}$ coefficients are given.

TABLE 5.1: THE ICR BASIS VECTORS FOR C^*_n WITH n EVEN

$|Aa\rangle = |J\ 0\rangle$ J even

$|Aa\rangle = i|J\ 0\rangle$ J odd

$|Aa\rangle = \frac{1}{\sqrt{2}}|J\ kn\rangle + \frac{1}{\sqrt{2}}|J\ -kn\rangle$ J even

$|Aa\rangle = \frac{i}{\sqrt{2}}|J\ kn\rangle + \frac{i}{\sqrt{2}}|J\ -kn\rangle$ J odd

$|Aa\rangle = \frac{i}{\sqrt{2}}|J\ kn\rangle - \frac{i}{\sqrt{2}}|J\ -kn\rangle$ J even

$|Aa\rangle = \frac{1}{\sqrt{2}}|J\ kn\rangle - \frac{1}{\sqrt{2}}|J\ -kn\rangle$ J odd

$|E_{m/2}x\rangle = |J\ m/2 + kn\rangle$ $(J - m/2)$ even

$|E_{m/2}x\rangle = i|J\ m/2 + kn\rangle$ $(J - m/2)$ odd

$|E_{m/2}y\rangle = |J\ -m/2 - kn)\rangle$ $(J - m/2)$ even

$|E_{m/2}y\rangle = i|J\ -m/2 - kn)\rangle$ $(J - m/2)$ odd

$|A_2a_2\rangle = \frac{1}{\sqrt{2}}|J\ n/2 + kn\rangle + \frac{1}{\sqrt{2}}|J\ -n/2 - kn\rangle$ $(J - n/2)$ even

$|A_2a_2\rangle = \frac{i}{\sqrt{2}}|J\ n/2 + kn\rangle + \frac{i}{\sqrt{2}}|J\ -n/2 - kn\rangle$ $(J - n/2)$ odd

$|A_2a_2\rangle = \frac{i}{\sqrt{2}}|J\ n/2 + kn\rangle - \frac{i}{\sqrt{2}}|J\ -n/2 - kn\rangle$ $(J - n/2)$ even

$|A_2a_2\rangle = \frac{1}{\sqrt{2}}|J\ n/2 + kn\rangle - \frac{1}{\sqrt{2}}|J\ -n/2 - kn\rangle$ $(J - n/2)$ odd

The basis vectors for $|E'_{m/2}\alpha\rangle$ and $|E'_{m/2}\beta\rangle$ follow the same pattern as $|E_{m/2}{}^{x}\rangle$ and $|E_{m/2}{}^{y}\rangle$ respectively.

TABLE 5.2: THE WIGNER TENSOR FOR C^*_n WITH n EVEN

j	j	n	value
A	a	a	1
$E_{m/2}$	x	y	1
	y	x	1
A_2	a_2	a_2	1
$E'_{m/2}$	α	β	1
	β	α	-1

TABLE 5.3: $\overline{V}$ COEFFICIENTS FOR C^*_n WITH n EVEN

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$
A	A	A	a	a	a	1
E_k	E_k	OA	k	-k	a	$1/\sqrt{2}$
E_k	E_k	1A	k	-k	a	$i/\sqrt{2}$
E_k	E_l	$E_{(k+l)}$	k	l	$-(k + l)$	1/2 if ΣJ is even
E_k	E_l	$E_{(k+l)}$	k	l	$-(k + l)$	$-i/2$ if ΣJ is odd
E_k	$E_{(n-k)}$	naA_2	k	n - k	a_2	$1/\sqrt{2}$
E_k	$E_{(n-k)}$	nbA_2	k	n - k	a_2	$i/\sqrt{2}$
E'_k	E'_k	OA	k	-k	a	$1/\sqrt{2}$
E'_k	E'_k	1A	k	-k	a	$-i/\sqrt{2}$
E'_k	E'_l	$E_{(k+l)}$	k	l	$-(k + l)$	-1/2 if ΣJ is even
E'_k	E'_l	$E_{(k+l)}$	k	l	$-(k + l)$	$i/2$ if ΣJ is even
E'_k	$E'_{(n-k)}$	naA_2	k	n - k	a_2	$-1/\sqrt{2}$
E'_k	$E'_{(n-k)}$	nbA_2	k	n - k	a_2	$-i/\sqrt{2}$

TABLE 5.4: THE ICR BASIS VECTORS FOR C^*_n WITH n ODD

$|Aa\rangle = |J\ 0\rangle$ J even

$|Aa\rangle = i|J\ 0\rangle$ J odd

$|Aa\rangle = \frac{1}{\sqrt{2}}|J\ 2nk\rangle + \frac{1}{\sqrt{2}}|J - 2nk\rangle$ J even

$|Aa\rangle = \frac{i}{\sqrt{2}}|J\ 2nk\rangle + \frac{i}{\sqrt{2}}|J - 2nk\rangle$ J odd

$|Aa\rangle = \frac{i}{\sqrt{2}}|J\ 2nk\rangle - \frac{i}{\sqrt{2}}|J - 2nk\rangle$ J even

$|Aa\rangle = \frac{1}{\sqrt{2}}|J\ 2nk\rangle - \frac{1}{\sqrt{2}}|J - 2nk\rangle$ J odd

$|Aa\rangle = \frac{i}{\sqrt{2}}|J\ (2k+1)n\rangle + \frac{i}{\sqrt{2}}|J - (2k+1)n\rangle$ J even

$|Aa\rangle = \frac{1}{\sqrt{2}}|J\ (2k+1)n\rangle + \frac{1}{\sqrt{2}}|J - (2k+1)n\rangle$ J odd

$|Aa\rangle = \frac{1}{\sqrt{2}}|J\ (2k+1)n\rangle - \frac{1}{\sqrt{2}}|J - (2k+1)n\rangle$ J even

$|Aa\rangle = \frac{i}{\sqrt{2}}|J\ (2k+1)n\rangle - \frac{i}{\sqrt{2}}|J - (2k+1)n\rangle$ J odd

$|E_{M/2}x\rangle = |J\ M/2 + 2nk\rangle$ (J - M/2) even

$|E_{M/2}x\rangle = i|J\ M/2 + 2nk\rangle$ (J - M/2) odd

$|E_{M/2}x\rangle = i|J\ M/2 + (2k+1)n\rangle$ (J - M/2) even

$|E_{M/2}x\rangle = |J + M/2 + (2k+1)n\rangle$ (J - M/2) odd

$|E_{M/2}y\rangle = |J - M/2 - 2nk\rangle$ (J - M/2) even

$|E_{M/2}y\rangle = i|J - M/2 - 2nk\rangle$ (J - M/2) odd

$|E_{M/2}y\rangle = i|J - M/2 - (2k+1)n\rangle$ (J - M/2) even

$|E_{M/2}y\rangle = |J - M/2 - (2k+1)n\rangle$ (J - M/2) odd

The basis vectors for $|E'_{M/2}\alpha\rangle$ and $|E'_{M/2}\beta\rangle$ follow the same pattern as $|E_{M/2}x\rangle$ and $|E_{M/2}y\rangle$ respectively.

TABLE 5.5: THE WIGNER TENSOR FOR C^*_n WITH n ODD

j	m	$\dot{n}$	value
A	a	a	1
$E_{m/2}$	x	y	1
$E_{m/2}$	y	x	1
$E'_{m/2}$	α	β	1
$E'_{m/2}$	β	α	-1
E'	α	β	1
E'	β	α	-1

TABLE 5.6: $\overline{V}$ COEFFICIENTS FOR C^*_n WITH n ODD

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$
A	A	A	a	a	a	1
E_k	E_k	OA	k	-k	a	$1/\sqrt{2}$
E_k	E_k	1A	k	-k	a	$i/\sqrt{2}$
E_k	E_l	$E_{(k+l)}$	k	l	$-(k+l)$	1/2 if ΣJ is even
E_k	E_l	$E_{(K+l)}$	k	l	$-(k+l)$	$i/2$ if ΣJ is odd
E'_k	E'_k	OA	k	-k	a	$1/\sqrt{2}$
E'_k	E'_k	1A	k	-k	a	$-i/\sqrt{2}$
E'_k	E'_l	$E_{(k+l)}$	k	l	$-(k+l)$	-1/2 if ΣJ is even
E'_k	E'_l	$E_{(k+l)}$	k	l	$-(k+l)$	$\frac{i}{2}$ if ΣJ is odd
E'	E'_k	$E_{(n/2+k)}$	α	k	$-(n/2+k)$	-1/2 if ΣJ is even
E'	E'_k	$E_{(n/2+k)}$	α	k	$-(n/2+k)$	$\frac{i}{2}$ if ΣJ is odd
E'	E'	OA	α	β	a	$-/\sqrt{2}$
E'	E'	1A	α	β	a	$-i/\sqrt{2}$
E'	E'	naA	α	α	a	$1/\sqrt{2}$
E'	E'	nbA	α	α	a	$i/\sqrt{2}$

TABLE 5.7: ICR BASIS VECTORS FOR C^*_3

$J = 0$	$\|Aa\rangle = \|00\rangle$
$J = 1$	$\|Aa\rangle = i\|10\rangle$
	$\|E_1x\rangle = \|11\rangle$
	$\|E_1y\rangle = \|1 - 1\rangle$
$J = 2$	$\|Aa\rangle = \|20\rangle$
	$\|aE_1x\rangle = i\|21\rangle$
	$\|aE_1y\rangle = i\|2 - 1\rangle$
	$\|bE_1x\rangle = \|2 - 2\rangle$
	$\|bE_1y\rangle = \|22\rangle$
$J = 3$	$\|aAa\rangle = i\|30\rangle$
	$\|bAa\rangle = \frac{1}{\sqrt{2}}\|33\rangle + \frac{1}{\sqrt{2}}\|3 - 3\rangle$
	$\|cAa\rangle = \frac{i}{\sqrt{2}}\|33\rangle - \frac{i}{\sqrt{2}}\|3 - 3\rangle$
	$\|aE_1x\rangle = \|31\rangle$
	$\|aE_1y\rangle = \|3 - 1\rangle$
	$\|bE_1x\rangle = i\|3 - 2\rangle$
	$\|bE_1y\rangle = i\|32\rangle$
$J = 1/2$	$\|E'_{1/2}\alpha\rangle = \|1/2\ 1/2\rangle$
	$\|E'_{1/2}\beta\rangle = \|1/2 - 1/2\rangle$
$J = 3/2$	$\|E'_{1/2}\alpha\rangle = i\|3/2\ 1/2\rangle$
	$\|E'_{1/2}\beta\rangle = i\|3/2 - 1/2\rangle$
	$\|E'\alpha\rangle = \|3/2\ 3/2\rangle$
	$\|E'\beta\rangle = \|3/2 - 3/2\rangle$

$J = 5/2$

$|aE'_{1/2}\alpha\rangle = |5/2\ 1/2\rangle$

$|aE'_{1/2}\beta\rangle = |5/2\ -1/2\rangle$

$|bE'_{1/2}\alpha\rangle = i|5/2\ -5/2\rangle$

$|bE'_{1/2}\beta\rangle = i|5/2\ 5/2\rangle$

$|E'\alpha\rangle = i|5/2\ 3/2\rangle$

$|E'\beta\rangle = i|5/2\ -3/2\rangle$

TABLE 5.8: $\overline{V}$ COEFFICIENTS FOR C^*_3

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$
A	A	A	a	a	a	11
E_1	E_1	0A	x	y	a	$1/\sqrt{2}$
E_1	E_1	1A	x	y	a	$i/\sqrt{2}$
E_1	E_1	$2bE_1$	x	x	x	1/2
E_1	$2aE_1$	$2bE_1$	x	x	x	$-i/2$
$E'_{1/2}$	$E'_{1/2}$	A	α	β	a	$1/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	E_1	α	α	y	-1/2
$E'_{1/2}$	$3/2E'_{1/2}$	E_1	α	α	y	$i/2$
$E'_{1/2}$	$E'_{1/2}$	1A	α	β	a	$-i/\sqrt{2}$
E'	E'	A	α	β	a	$-1/\sqrt{2}$
E'	E'	1A	α	β	a	$-i/\sqrt{2}$
E'	E'	3bA	α	α	a	$1/\sqrt{2}$
E'	E'	30A	α	α	a	$i/\sqrt{2}$

TABLE 5.9 : ICR BASIS VECTORS FOR C^*_4

J = 0	$\|Aa\rangle = \|00\rangle$
J = 1	$\|Aa\rangle = i\|10\rangle$
	$\|E\ x\rangle = \|11\rangle$
	$\|E_1 y\rangle = \|1\ -\ 1\rangle$
J = 2	$\|Aa\rangle = \|20\rangle$
	$\|aA_2a_2\rangle = \frac{1}{\sqrt{2}}\|22\rangle + \frac{1}{\sqrt{2}}\|2\ -\ 2\rangle$
	$\|bA_2a_2\rangle = \frac{i}{\sqrt{2}}\|22\rangle - \frac{i}{\sqrt{2}}\|2\ -\ 2\rangle$
	$\|E_1x\rangle = i\|21\rangle$
	$\|E_1y\rangle = i\|2\ -\ 1\rangle$
J = 3	$\|Aa\rangle = \|30\rangle$
	$\|aA_2a_2\rangle = \frac{i}{\sqrt{2}}\|32\rangle + \frac{i}{\sqrt{2}}\|3\ -\ 2\rangle$
	$\|bA_2a_2\rangle = \frac{1}{\sqrt{2}}\|32\rangle - \frac{1}{\sqrt{2}}\|3\ -\ 2\rangle$
	$\|aE_1x\rangle = \|31\rangle$
	$\|aE_1y\rangle = \|3\ -\ 1\rangle$
	$\|bE_1x\rangle = \|3\ -\ 3\rangle$
	$\|bE_1y\rangle = \|33\rangle$
J = 4	$\|aAa\rangle = \|40\rangle$
	$\|bAa\rangle = \frac{1}{\sqrt{2}}\|44\rangle + \frac{1}{\sqrt{2}}\|4\ -\ 4\rangle$
	$\|cAa\rangle = \frac{i}{\sqrt{2}}\|44\rangle - \frac{i}{\sqrt{2}}\|4\ -\ 4\rangle$
	$\|aE_1x\rangle = i\|41\rangle$
	$\|aE_1y\rangle = i\|4\ -\ 1\rangle$
	$\|bE_1x\rangle = i\|4\ -\ 3\rangle$
	$\|bE_1y\rangle = i\|43\rangle$

J = 1/2

$$|E'_{1/2}\alpha\rangle = |1/2\ \ 1/2\rangle$$

$$||E'_{1/2}\beta\rangle = |1/2\ -1/2\rangle$$

J = 3/2

$$|E'_{1/2}\beta\rangle = i|3/2\ \ 1/2\rangle$$

$$|E'_{1/2}\beta\rangle = i|3/2\ -1/2\rangle$$

$$|E'_{3/2}\alpha\rangle = |3/2\ \ 3/2\rangle$$

$$|E'_{3/2}\beta\rangle = |3/2\ -3/2\rangle$$

J = 5/2

$$|E'_{1/2}\alpha\rangle = |5/2\ \ 1/2\rangle$$

$$|E'_{1/2}\beta\rangle = |5/2\ -1/2\rangle$$

$$|aE'_{3/2}\alpha\rangle = i|5/2\ \ 3/2\rangle$$

$$|aE'_{3/2}\beta\rangle = i|5/2\ -3/2\rangle$$

$$|bE'_{3/2}\alpha\rangle = i|5/2\ -5/2\rangle$$

$$|bE'_{3/2}\beta\rangle = i|5/2\ \ 5/2\rangle$$

J = 7/2

$$|aE'_{1/2}\alpha\rangle = i|7/2\ \ 1/2\rangle$$

$$|aE'_{1/2}\beta\rangle = i|7/2\ -1/2\rangle$$

$$|bE'_{1/2}\alpha\rangle = i|7/2\ -7/2\rangle$$

$$|bE'_{1/2}\beta\rangle = i|7/2\ \ 7/2\rangle$$

$$|aE'_{3/2}\alpha\rangle = |7/2\ \ 3/2\rangle$$

$$|aE'_{3/2}\beta\rangle = |7/2\ -3/2\rangle$$

$$|bE'_{3/2}\alpha\rangle = |7/2\ -5/2\rangle$$

$$|bE'_{3/2}\beta\rangle = |7/2\ \ 5/2\rangle$$

TABLE 5.10: $\bar{V}$ COEFFICIENTS FOR C^*_4

j_1	j_2	j_3	m_1	m_2	m_3	$\bar{V}$
A	A	A	a	a	a	1
A_2	A_2	A	a_2	a_2	a	1
E_1	E_1	A	x	y	a	$1/\sqrt{2}$
E_1	E_1	1A	x	y	a	$i/\sqrt{2}$
E_1	E_1	$2aA_2$	x	x	a_2	$1/\sqrt{2}$
E_1	E_1	$2bA_2$	x	x	a_2	$i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	A	α	β	a	$1/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	1A	α	β	a	$-i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	E_1	α	α	y	$-1/2$
$E'_{1/2}$	$3/2E'_{1/2}$	E_1	α	α	y	$i/2$
$E'_{3/2}$	$E'_{3/2}$	A	α	β	a	$1/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	1A	α	β	a	$-i/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	$3bE_1$	α	α	x	$-1/2$
$E'_{3/2}$	$3/2E'_{3/2}$	3bE	α	α	x	$i/2$
$E'_{1/2}$	$E'_{3/2}$	$2aA_2$	α	α	a_2	$-1/\sqrt{2}$
$E'_{1/2}$	$E'_{3/2}$	$2bA_2$	α	α	a_2	$-i/$ 2

TABLE 5.11: ICR BASIS VECTORS FOR $C^*{}_5$

$J = 0$	$\lvert Aa\rangle = \lvert 00\rangle$
$J = 1$	$\lvert Aa\rangle = i\lvert 10\rangle$
	$\lvert E_1x\rangle = \lvert\,\lvert 11\rangle$
	$\lvert E_1y\rangle = \lvert 1 - 1\rangle$
$J = 2$	$\lvert Aa\rangle = \lvert 20\rangle$
	$\lvert E_1x\rangle = i\lvert 21\rangle$
	$\lvert E_1y\rangle = i\lvert 2 - 1\rangle$
	$\lvert E_2x\rangle = \lvert 22\rangle$
	$\lvert E_2y\rangle = \lvert 2 - 2\rangle$
$J = 3$	$\lvert Aa\rangle = i\lvert 30\rangle$
	$\lvert E_1x\rangle = \lvert 31\rangle$
	$\lvert E_1y\rangle = \lvert 3 - 1\rangle$
	$\lvert aE_2x\rangle = i\lvert 32\rangle$
	$\lvert aE_2y\rangle = i\lvert 3 - 2\rangle$
	$\lvert bE_2x\rangle = \lvert 3 - 3\rangle$
	$\lvert bE_2y\rangle = \lvert 33\rangle$
$J = 4$	$\lvert Aa\rangle = \lvert 40\rangle$
	$\lvert bE_1x\rangle = i\lvert 41\rangle$
	$\lvert aE_1y\rangle = i\lvert 4 - 1\rangle$
	$\lvert bE_1x\rangle = \lvert 44\rangle$
	$\lvert bE_1y\rangle = \lvert 4 - 4\rangle$
	$\lvert aE_2x\rangle = \lvert 42\rangle$
	$\lvert aE_2y\rangle = \lvert 4 - 2\rangle$
	$\lvert bE_2x\rangle = i\lvert 4 - 3\rangle$
	$\lvert bE_2y\rangle = i\lvert 43\rangle$

J = 1/2

$$|E'_{1/2}\alpha\rangle = |1/2\ 1/2\rangle$$

$$|E'_{1/2}\beta\rangle = |1/2\ -\ 1/2\rangle$$

J = 3/2

$$|E'_{1/2}\alpha\rangle = i|3/2\ 1/2\rangle$$

$$|E'_{1/2}\beta\rangle = i|3/2\ -\ 1/2\rangle$$

$$|E'_{3/2}\alpha\rangle = |3/2\ 3/2\rangle$$

$$|E'_{3/2}\beta\rangle = |3/2\ -\ 3/2\rangle$$

J = 5/2

$$|E'_{1/2}\alpha\rangle = |5/2\ 1/2\rangle$$

$$|E'_{1/2}\beta\rangle = |5/2\ -\ 1/2\rangle$$

$$|E'_{3/2}\alpha\rangle = i|5/2\ 3/2\rangle$$

$$|E'_{3/2}\beta\rangle = i|5/2\ -\ 3/2\rangle$$

$$|E'\alpha\rangle = |5/2\ 5/2\rangle$$

$$|E'\beta\rangle = |5/2\ -\ 5/2\rangle$$

J = 7/2

$$|E'_{1/2}\alpha\rangle = i|7/2\ 1/2\rangle$$

$$|E'_{1/2}\beta\rangle = i|7/2\ -\ 1/2\rangle$$

$$|aE'_{3/2}\alpha\rangle = |7/2\ 3/2\rangle$$

$$|aE'_{3/2}\alpha\rangle = |7/2\ -\ 3/2\rangle$$

$$|bE'_{3/2}\alpha\rangle = i|7/2\ -\ 7/2\rangle$$

$$|bE'_{3/2}\beta\rangle = i|7/2\ 7/2\rangle$$

$$|E'\alpha\rangle = i|7/2\ 5/2\rangle$$

$$|E'\beta\rangle = i|7/2\ -\ 5/2\rangle$$

TABLE 5.12: $\overline{V}$ COEFFICIENTS FOR C^*_5

j_1	j_2	j_3	m_1	m_2	m_3	$\overline{V}$
A	A	A	a	a	a	11
E_1	E_1	A	x	y	a	$1/\sqrt{2}$
E_1	E_1	1A	x	y	a	$i/\sqrt{2}$
E_1	E_1	E_2	x	x	y	1/2
E_1	$2E_1$	E_2	x	x	y	$-i/2$
E_1	E_2	$3aE_2$	x	x	x	1/2
E_1	$3E_2$	$3aE_2$	x	x	x	$-i/2$
E_2	E_2	A	x	y	a	$1/\sqrt{2}$
E_2	E_2	1A	x	y	a	$i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	A	α	β	a	$1/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	1A	α	β	a	$-i/\sqrt{2}$
$E'_{1/2}$	$E'_{1/2}$	E_1	α	α	y	-1/2
$E'_{1/2}$	$3/2E'_{1/2}$	E_1	α	α	y	$i/\sqrt{2}$
$E'_{1/2}$	$E'_{3/2}$	E_2	α	α	y	-1/2
$E'_{1/2}$	$5/2E'_{3/2}$	E_2	α	α	y	$i/2$
$E'_{3/2}$	$E'_{3/2}$	A	α	β	a	$1/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	1A	α	β	a	$-i/\sqrt{2}$
$E'_{3/2}$	$E'_{3/2}$	$3bE_2$	α	α	x	-1/2
$E'_{3/2}$	$5/2E'_{3/2}$	$4bE_2$			x	$-i/2$

E'	E'	A	α	β	a	$-1/\sqrt{2}$
E'	E'	1A	α	β	a	$-i/\sqrt{2}$
E'	E'	5aA	α	α	a	$-1/\sqrt{2}$
E'	E'	5bA	α	α	a	$-i/\sqrt{2}$
E'	$E'_{1/2}$	$3bE_2$	α	α	x	$-1/\sqrt{2}$
E'	$3/2E'_{1/2}$	E_2	α	β	x	$-i/2$
E'	$E'_{1/2}$	$3aE_2$	α	α	y	$-1/2$
E'	$E'_{3/2}$	$4bE_1$	α	α	y	$-1/2$
E'	$3/2E'_{3/2}$	$5E_1$	α	β	y	$-i/2$
E'	$E'_{3/2}$	E_1	α	β	y	$-1/2$
E'	$3/2E'_{3/2}$	E_1	α	β	y	$-i/2$
E_2	E_2	$4bE_1$	x	x	y	$1/2$
E_2	$3aE_2$	$4bE_1$	x	x	y	$i/2$

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