Spot Pricing Framework for Loss Guaranteed
Internet Service Contracts

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Abstract—
We develop a spot pricing framework for intra-domain expected bandwidth contracts with loss based QoS guarantees. The framework accounts for both costs and risks associated with QoS delivery. A nonlinear pricing scheme is used for cost recovery and a utility based options pricing approach is developed for the risk related pricing. Application of options pricing techniques in Internet services provides a mechanism for fair risk sharing between the provider and the customer, and may be extended to price other uncertainties in QoS guarantees.

I. INTRODUCTION
The Internet today offers primarily a best-effort service. Research and technology development efforts are underway to allow provisioning of better Quality of Service (QoS) assurances. In this article, we develop a spot pricing framework for intra-domain expected bandwidth assured service with loss rate guarantees, which lays the foundation for a pricing framework for end-to-end, as well as more complex, QoS guaranteed bandwidth services for enterprise customers. The pricing framework consists of two components: (i) a nonlinear pricing scheme for cost recovery, and (ii) an options-based approach to price the risk of deviations in the loss based QoS experienced by the customer. The focus of this article is on pricing of risk. We develop an options-based approach to assign a price to the risk in providing loss-based QoS assurance. The framework also employs a nonlinear pricing scheme to recover the cost of providing bandwidth that supports the QoS guarantee, which was addressed in our earlier work [3].

The article proceeds as follows. In section II, we discuss the two-component approach to pricing QoS guaranteed services. Section III focuses on modeling for the options-based pricing approach for pricing the risks in QoS assured services. Finally, discussions of simulation results and conclusions are given in sections IV and V, respectively.

II. SPOT PRICING FRAMEWORK
Network performance can be defined in terms of a combination of its bandwidth, delay, delay-jitter and loss properties. Based on these performance measures, QoS guarantees can be stated in deterministic or probabilistic terms. In this article, we study pricing for an expected level of bandwidth with loss rate guarantees. The pricing framework consists of two components: pricing for cost recovery, and pricing of risk.

A. Pricing to Recover Cost
Nonlinear pricing refers to a pricing scheme where the tariff is not proportional to the quantity purchased and the marginal prices for successive purchases decrease [7]. In our pricing framework, a well known nonlinear pricing model, Ramsey pricing is employed to recover the cost in providing the expected bandwidth that is essential to support addition QoS assurances. The optimal price schedule \( p(q) \) from Ramsey pricing maximizes total customer surplus, and is given by the following Ramsey rule [7]:

\[
\frac{p(q) - c(q)}{p(q)} = \frac{\alpha}{\eta(p(q), q)},
\]

where \( c(q) \) is the marginal cost for the \( q \)th unit, and \( \eta(p(q), q) \) is the elasticity of the demand profile. The Ramsey number \( \alpha \) is the fraction of the monopoly profit margin that is needed for cost recovery, and is an indicator of the monopoly power of the provider.

B. Pricing the Risk

 Provision of a loss-based QoS guaranteed service is inherently risky, due to the uncertainties caused by competing traffic in the Internet. The future outcomes of a service may be in favor of or against the provider, i.e. the provider may or may not deliver the loss based QoS as promised. We use options pricing techniques to evaluate the risky nature of the service. In particular, we consider pricing from the provider’s perspective, and evaluate the monetary “reward” for the favorable risks to the provider. The options price is equal to the expectation of the payoff under the transformed risk neutral measure, \( Q \), defined by the state price density (SPD). If \( Y_t \) is the payoff from the loss process at time \( t \), the price of the risk in the loss guarantee is given by

\[
V = E_Q[\int_0^T Y_t dt].
\]
The SPD is a basic construct for an economic agent, and is used to describes an (representative) agent’s preferences for future outcomes. It defines the risk neutral measure, \(Q\), in the pricing equation (Eqn. 2), and is used for pricing assets governed by sources of uncertainty.

In our context, we construct an SPD to describe a representative provider’s preferences for future outcomes of the loss process. We consider SPD functions of 2 alternative forms: (i) A monotonously decreasing SPD, and (ii) a SPD peaking at a positive loss level, based on the following assumptions regarding the provider’s preference structure and the outcomes of the loss process:

- The provider would expect that losses are rare events during the contract duration, and that losses would more likely take small to moderate values, although there is a non-zero probability of extremely large losses to occur.
- The provider would not get rewarded for large losses.

In practice the SPD can be estimated from price data, in a similar manner as SPD estimation in finance [1]. We also discuss an aggregation method to derive SPD’s for different definitions of loss guarantees, which further reduces the needs of price data for SPD estimation. If \(q(s)\), \(s \in S\), and \(q^*(s^*)\), \(s^* \in S^*\) are SPD’s defined for differently defined loss guarantees, \(q^*(s^*)\) can be derived by “projecting” \(q(s)\) on to \(S^*\),

\[
q^*(s^*) = E_{S^*} \{ q(q(s)|s^*) \}, \tag{3}
\]

where \(g(\cdot)\) is a function appropriately chosen to make the prices generated by \(q(s)\) and \(q^*(s^*)\) comparable, also \(q^*(s^*)\) is a legitimate density function. For example, if we define \(q(s)\) as the SPD for per-minute loss rate, and \(q^*(s^*)\) for 2-minute loss rate, applying the aggregation method, we get Similarly,

\[
qu^*(s^*) = \frac{1}{c''} \int_{I_1'} \int_{s^1} \{ q(s^1) + q(s^2) \} f(s^1) f(I_1^2) dsdI_1^2, \tag{4}
\]

where \(c''\) is the normalization constant, \(I_1^2\) is the per-minute \(I_1\), \(s^1, s^2\) are the per-minute states (loss rates) in the first and second minute in a 2-minute interval, and \(f(s^1), f(I_1^2)\) are the distribution density functions of \(s^1\) and \(I_1^2\), respectively.

We have demonstrated the pricing of risk from the provider’s perspective. Following similar arguments, prices may be determined from the customer’s perspective using the options pricing framework. This, however, would require considering the customer’s preferences as well as the negotiation power of the two parties.

III. MODEL DEFINITION AND ASSUMPTIONS

Losses of a customer’s data are essentially determined by the customer’s own traffic and its interactions with the background traffic in the network. In our network modeling for the options pricing of risk, the customer’s traffic is modeled separately from the aggregate background traffic. For pricing purpose the network is abstracted by a single pipe with a fixed capacity.

Traffic from the customer is modeled on a flow basis. Following literature on Internet traffic analysis [2] [4], the arrivals of files from the customer is modeled by a time-dependent Poisson process with a rate of \(\lambda = 5/min\) averaged over a day, and file sizes and transfer times are modeled by Pareto distributions of shape \((a)\) and scale \((b)\) parameters appropriately chosen. At a given time \(t\), we define data in transit, \(I_t\), as the amount of the customer’s data in the network, i.e. the data susceptible to loss, at time \(t\).

The aggregate background traffic is modeled by a single process, the Aggregate [5]. The Aggregate depicts the current state of the network, and is intended to capture two significant characteristics of the aggregated Internet traffic [5] [6], i.e. diurnal pattern and self-similarity. We use a sinusoidal curve with a period of 24 hours to model the diurnal pattern, and fractional Gaussian Noise (FGN) process to model the self-similarity. Therefore, at any given time \(t\), we define the Aggregate process, \(A_t\) as,

\[
A_t = R \sin(2\pi ft + \theta) + \overline{A}_t + Z_t, \tag{5}
\]

where \(R, f, \theta\) and \(\overline{A}_t\) are parameters of the sinusoidal curve, and \(Z_t\) is the FGN process. The Hurst Parameter of a process describes the degree of self-similarity of the process; for a self-similar process, \(0.5 < H < 1\). We simulated different values of \(H\) of the FGN in the range of \(0.7 - 0.95\).

Given \(I_t\) and \(A_t\), the loss process is modeled as a 2-state Markov process, 1 representing a state where losses happen and 0 representing a loss free state. The transition probabilities depend on the total amount of data in the network, i.e. \(I_t + A_t\), and \(I_t\). Two threshold levels, \(T^U\) and \(T^L\), for the total amount of data in the network, as well as an upper threshold, \(T^U_i\), for \(I_t\) are set. Therefore, the transition matrix \(P_{ij}\), \((i,j = 0, 1)\) is given by

\[
P_{ij} = \begin{cases}
1 & \text{if } A_t + I_t \leq T^L;
0 & \text{if } T^L < A_t + I_t \leq T^U;
0 & \text{if } A_t + I_t > T^U;\tag{6}
0 & \text{if } A_t + I_t > T^U \text{ and } I_t \geq T^U_i;
\end{cases}
\]

For simplicity, it is further assumed that when \(I_t\) is in a loss state, the customer’s data in transit, \(I_t\), is lost, i.e. \(L_t = I_t\) when \(L_t\) is in state 1. A realization of the \(L_t\) process in a

![Fig. 1: 24 Hour Variation of \(L_t\)](image-url)
24 hour period is given in Fig. 1. $L_t$ shows high burstiness. As expected, losses happen more frequent when $A_t$ is high; a comparison of $L_t$ and the corresponding $I_t$ indicates a positive correlation between $L_t$ and large values of $I_t$.

IV. SIMULATION ANALYSIS AND DISCUSSION

The spot pricing framework described in the above 2 sections is simulated using a demonstrative contract for per minute loss-rate guarantee, with different choices of SPD’s and network settings. The results of price variations are shown in Fig. 3.

4 sample SPD functions (Fig. 2–a), $q(s)$, are selected for pricing: an exponential distribution ($\mu = 0.02$) for the monotonically decreasing SPD, and 3 beta distributions for SPD’s peaking at different positive loss rates (0.5%, 0.3%, and 0.05%, respectively). We simulated the aggregation procedure described in section II-B on the sample SPD’s, and obtained the SPD’s defined for 2-minute loss-rate guarantee corresponding to each $q(s)$ (Fig. 2–b). It is clear that the aggregated SPD’s keep the key characteristics of the original SPD’s.

Fig. 3–a shows the differences between prices from the decreasing SPD (SPD 1) and from the prices from the beta SPD’s peaking at positive losses (SPD 2 to 4). Prices from SPD 1 vary in a similar pattern as $A_t$, i.e. lower during the day when the network is busy, and higher at night when losses tend to be lower; the contrary is true for beta SPD’s. In this sense, SPD 1 produces performance based prices, while SPD 2, 3 and 4 produce congestion sensitive prices.

An increase in network capacity is simulated by increasing the thresholds $T_L^U$ and $T_U^L$. In this situation, the provider is able to reduce losses of the customer’s data. Consequently (Fig. 3–b), the provider would expect to see price increase with a SPD that rewards zero losses, while price decrease with beta shaped SPD’s.

The Hurst parameter $H$ of the $A_t$ process indicates the level of burstiness of $A_t$. As shown in Figure 3–c, the effect of the burstiness of $A_t$ on prices is not obvious. However, the early peak with $H = 0.95$ implies that even when the network is only moderately loaded, the network performance may be deteriorated if $A_t$ is burstier.

Fig. 3–d shows the price variations with 3 file size distributions with different combinations of the shape and scale parameters. The average file size in scenario (1) is the same as in the baseline setting (Fig. 3–a), while in scenario (2) and (3) the average file sizes are both 5 times larger than in the baseline setting. Compared with the baseline scenario, the shape parameters of the file size distributions in scenario (1) and (2) are smaller; the file size distributions are flatter, and there tend to be more occurrences of large files in the tail of the distributions. The shape parameter in scenario (3) is the same as the baseline scenario, but the scale parameter is 5 times larger; therefore, the increased file size is more evenly distributed through the whole distribution. Comparing prices in these scenarios, and with the baseline scenario (Fig. 3–a), we can see that prices are more sensitive to a lot of moderately large files, as in scenario (3), than to a small number of extremely huge files (scenario (1) and (2)).

V. CONCLUSION

We have developed a two-component spot pricing framework for intra-domain expected bandwidth contracts with a loss based QoS guarantee. A nonlinear pricing scheme is used in pricing for cost recovery, and a utility based options pricing approach is developed to price the risky aspects of the loss based QoS guarantee. We implemented the options pricing framework using a demonstrative contract. Simulation analysis indicates that depending on the choices of SPD’s, the price of the risk in the service may be either performance based, or congestion sensitive. Changes in network conditions such as expanded capacity, changes in characteristics of network traffic, may affect prices through changing the probabilities of the customer’s data losses.

REFERENCES