Irregular LDPC Coded BICM in Image Transmission over Rayleigh Fading Channel

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Abstract—If the degree distribution is chosen carefully, the irregular LDPC codes can outperform the regular ones. In this paper, we proposed an LDPC coded BICM scheme in image transmission system to improve both efficiency and reliability. Simulation results show that LDPC codes are good coding schemes over fading channel in image communication. Simultaneously, irregular codes can obtain a code gain of about 0.7dB than regular ones when BER is 10^-4. So the irregular LDPC codes are more suitable for image transmission than the regular codes.

Keywords—irregular LDPC codes, degree distribution, density evolution, image transmission, BICM

I. INTRODUCTION

Low-density parity-check (LDPC) codes have near Shannon limit performance when decoded using an iteratively probabilistic algorithm [1]. Luby et al. constructed irregular LDPC codes and showed that irregular codes performed better than regular ones [2]. And then in [3-4] Richardson and Urbanke developed a density evolution algorithm to analyze, design the irregular LDPC codes. By this method, they constructed the LDPC codes that clearly beat the powerful Turbo codes [4]. On the other hand, wireless multimedia service is an important part of mobile communications. Channel coding is essential for improving communications reliability. We should consider efficiency together with reliability especially in bandwidth constraint region. We had examined the performance of regular LDPC codes in image transmission with BPSK mapping [5]. In this paper, we proposed an irregular LDPC coded Bit-interleaved Coded Modulation (BICM) scheme in order to obtain high spectrum efficiency and high reliability in image transmission system.

The paper is organized as follows. Section 2 gives a brief overview of irregular LDPC codes and density evolution algorithm. Section 3 presents the LDPC coded BICM schemes, especially the initialization of the decoding procedure. And then we focus on the simulation results and analysis in image transmission over Rayleigh fading channel. Finally, the conclusion is made in section 4.

II. IRREGULAR LDPC CODES

A. Degree Distribution

An LDPC code is a linear code specified by a very sparse parity-check matrix in which the number of element 1 is very small compared to the number of element 0. A LDPC code can be represented by a Tanner graph according to its parity-check matrix. The Tanner graph related to a \( M \times N \) parity-check matrix \( H \), i.e., the block length \( N \) and the information source block length \( K = N - M \), has \( N \) variable nodes (or bit nodes) and \( M \) check nodes (or function nodes). And if and only if there is a 1 at the \( i \)th row and the \( j \)th column in \( H \), there is an edge between the \( i \)th variable node and the \( j \)th check node. The degree of a node is the number of edges connecting with it. Then Regular LDPC codes are those for which all nodes of the same type have the same degree. Consequently for the irregular codes, the variable nodes and the check nodes can have different degree.

Let \( \lambda_i(\rho_i) \) represents the fraction of edges emanating from variable (check) nodes of degree \( i \) and \( d_v(d_c) \) denotes the maximum variable (check) node degree. Then an LDPC code ensemble is determined by its generating function \( \lambda(x) \) and \( \rho(x) \) of the degree distributions for the variable and check node, where

\[
\lambda(x) = \sum_{i=2}^{d_v} \lambda_i \cdot x^{i-1} \quad \rho(x) = \sum_{i=2}^{d_c} \rho_i \cdot x^{i-1}
\]

Fig. 1 shows a Tanner graph of an irregular LDPC code, whose generating function is \( \lambda(x) = 0.4x + 0.6x^2 \) and \( \rho(x) = 0.6x^2 + 0.4x^3 \), \( d_v = 3 \), \( d_c = 4 \), \( \lambda_2 = 0.4 \), \( \lambda_3 = 0.6 \) and \( \rho_2 = 0 \), \( \rho_3 = 0.6 \), \( \rho_4 = 0.4 \), respectively.

B. Density Evolution

Density Evolution algorithm can be used to analyze the capacity of LDPC codes under message passing decoding and find the degree distribution of capacity-approaching LDPC codes [3-4].
Consider a regular \((d_v, d_c)\)-LDPC code. The decoding process is best viewed as a message passing algorithm on the Tanner graph defined by \(H\). Fig. 2 is the message passing route of variable node \(b_1\) corresponding to Fig. 1. Messages are computed at the nodes and are exchanged between neighboring nodes along the edges iteratively.

![Figure 2. Message passing route of variable node b1 corresponding to Fig. 1](image)

Each variable node gets the initial messages from the channel output. Then each check node gathers all the incoming extrinsic messages from its \(d_v-1\) neighboring variable nodes and processes the messages. Similarly, each variable node receives messages from its \(d_c-1\) neighboring check nodes. Variable nodes and check nodes exchange messages iteratively. Finally, a variable node gathers all the information available and makes a decision. This two-step procedure is repeated.

For the Binary-input Symmetry-output channel and BPSK mapping, equally distributed information bit 0 and 1 are mapped to \(-1\) and \(+1\). We use log-likelihood ratios (LLRs) as messages. If

\[
q_i^l = \log \frac{p(x = +1 | t)}{p(x = -1 | t)}
\]

is defined as the output message of the \(i\)th variable node at the \(l\)th iteration, where \(x\) is the bit value and \(t\) denotes all the information available to the node up to this iteration. Likewise, we use

\[
r_j^l = \log \frac{p(x = +1 | r)}{p(x = -1 | r)}
\]

as the output message of the \(j\)th check node at the \(l\)th iteration, where \(x\) is the bit value and \(r\) denotes all the information available to the node up to this iteration. Under message passing decoding, at the \(l\)th iteration, every check node collects the messages from its \(d_v-1\) neighboring variable nodes by

\[
\tanh \frac{r_j^l}{2} = \prod_{i=1}^{d_v-1} \tanh \frac{q_i^{l-1}}{2}
\]

and then sends the resulting messages back to the variable node. The output message of a variable node is equal to the sum of all LLRs, i.e.,

\[
q_i^l = q_i^0 + \sum_{j=1}^{d_c-1} r_j^l
\]

where \(q_i^0\) is the initial message from the channel.

Assume the girth of the cycle in Tanner graph is large enough. Consequently the incoming messages to every node are independently identically distributed (i.i.d.) random variables. Let \(P^l\) represents the probability density function of each \(q_i^l\) and \(R^l\) represents the probability density function of every \(r_j^l\). From (2), the density of \(q_i^l\) is calculated by convolution of densities of \(r_j^l\)

\[
P^l = P^0 \ast \bigotimes_{j=1}^{d_v-1} R^l = P^0 \otimes (R^l)^{\otimes (d_v-1)}
\]

which can be efficient done in the Fourier domain by (4)

\[
P^l = 3^{-1} \left[ \mathfrak{F}[P^0] \left( \mathfrak{F}[R^l] \right)^{d_v-1} \right] \]

where \(*\) and \(\otimes\) are convolution operator and \(\mathfrak{F}[\cdot]\) is operator of Fourier Transform.

On the other hand, we take logarithms on each side to (1) to convert the product into a sum, i.e.,

\[
\left\{ \text{sgn} \left( \frac{r_j^l}{2} \right), \log \left| \tanh \frac{r_j^l}{2} \right| \right\} = \sum_{i=1}^{d_c-1} \left\{ \text{sgn} \left( \frac{q_i^{l-1}}{2} \right), \log \left| \tanh \frac{q_i^{l-1}}{2} \right| \right\}
\]

where \(\text{sgn} \ x\) is 0 if \(x \geq 0\) and 1 otherwise. As in the first case, density evolution of this step can be done numerically in the Fourier domain.

The density evolution can be generalized to irregular codes by

\[
R^l = \Gamma^{-1} \left( \rho \left( \Gamma \left( p^{l-1} \right) \right) \right) = \Gamma^{-1} \left( \sum_{i=2}^{d_c-1} \rho_i \left( \Gamma \left( p^{l-1} \right) \right)^{\otimes (i-1)} \right)
\]

\[
P^l = P^0 \otimes \lambda (R^l) = P^0 \otimes \sum_{i=2}^{d_c-1} \lambda_i \cdot (R^l)^{\otimes (i-1)}
\]

where the definition of function \(\Gamma\) and \(\Gamma^{-1}\) are described in [4].

Under the independent condition, the error probability is independent to the transmitted codeword. Then assume all one codeword is transmitted. Therefore, if the message of a variable node is larger than 0, the message is correct, otherwise it is incorrect. So in the proceeding of density evolution, the decoding error probability is computed by

\[
P_e^l = \int_{-\infty}^{0} P^l(z) dz
\]

which is the function of the channel parameter (noise power) and code degree distribution. When the degree distribution of a code and the desired error probability are fixed, we can calculate the threshold of the channel parameter, referred as the capacity of the LDPC code under message passing decoding [3]. At the same time, fix the desired error probability and change the degree distribution, the threshold of channel parameter is changed. The degree distribution is optimal if it corresponds to the largest channel parameter [4].
The irregular code in our simulation is one of optimal codes by density evolution algorithm [6].

### III. SIMULATIONS AND RESULTS

#### A. Simulation System

The simulation system is described in Fig. 3. The 256×256 grayscale uncompressed image Lenna is tested as the image source.

![Block diagram of simulation system](image)

Figure 3. Block diagram of simulation system

BICM is one of coded modulation schemes. There is one encoder and one decoder in BICM and an ideal bit-interleaver as well. In general, the encoding sequence is first interleaved by an ideal interleaver and then is modulated. For LDPC codes, the parity-check matrix is constructed in random, which incorporated interleaving property into LDPC codes. LDPC codes can make the encoded bits statistically independent by itself. Therefore the interleaver can be omitted in the system which decreases the complexity of system.

Because of the image data, we choose the code length is \(N=8196\) and code rate is \(R=1/2\). And we constructed a regular code and an irregular code in order to compare the performance of the two type codes. Code 1 is a regular code of which each variable node has 3 edges and each check node has 6 edges. And then the generating function of Code 1 is \(\lambda(x) = x^7\) and \(\rho(x) = x^3\). Code 2 is irregular, of which the maximum variable degree is 10. The degree distribution of Code 2 is from [6] and the generating function of Code 2 is

\[
\lambda(x) = 0.292439x + 0.253636x^2 + 0.060454x^3 + 0.031610x^5 + 0.361861x^9
\]

\[
\rho(x) = 0.007254x^5 + 0.979220x^6 + 0.013526x^7
\]

#### B. Initialization for LDPC coded BICM

Before decoding, the modulator must transmit the channel output messages to the LDPC decoder. For MQAM modulation, \(K = \log_2 M\) bits are mapped to a symbol. Fig. 4 shows the constellation of Gray mapping of 16QAM. There are \(M\) symbols in the alphabet and the \(m\)th symbol in constellation is represented by \((s_x^m, s_y^m)\), \(1 \leq m \leq M\). If a symbol \((s_x, s_y)\) is transmitted over Rayleigh fading channel, the received symbol \((r_x, r_y)\) is

\[
r_x = a \cdot s_x + n_1
\]

\[
r_y = a \cdot s_y + n_2
\]

where \(a > 0\) is the Rayleigh fading amplitude known as channel State Information (SI) and \(n_1, n_2\) are i.i.d. AWGN component with zero mean and power spectrum density \(N_0/2 = \sigma^2\).

If the Channel State Information is perfect known, the likelihood probability of the \(m\)th symbol conditioned the received symbol \((r_x, r_y)\) is calculated used (9).

\[
P_m = \Pr\{s_x^m, s_y^m \mid r_x, r_y\}
\]

(9)

Here \(P_m\) represents the probability that the received symbol \((r_x, r_y)\) is transmitted from \((s_x^m, s_y^m)\). Assuming that the transmitted symbols are equally distributed and by the Bayes equation, (9) is rewritten as

\[
P_m = C \cdot \Pr\{r_x, r_y \mid s_x^m, s_y^m\}
\]

\[
= C \cdot \frac{\exp[-\frac{(r_x - a \cdot s_x^m)^2 + (r_y - a \cdot s_y^m)^2}{2\sigma^2}]}{1 \leq m \leq M}
\]

(10)

where \(C\) is a constant for all the \(P_m\).

As a set of probability distribution, the \(\{P_m\}\) must be normalized to \(\{P'_m\}\). Then according to the mapping scheme, the probability that every transmitted bit \(b_i (1 \leq i \leq K)\) among the symbol \((r_x, r_y)\) is 1 or 0 can be computed. The symbol ensemble is divided into two parts with respect to \(b_i\) is 1 or 0. For 16QAM with Gray mapping, the probability of the first bit \(b_0\) is 1 is computed by sum up

\[
P'_m, \quad m = 8, 9, 10, 11, 12, 13, 14, 15
\]

and the probability of the last \(b_3\) is 1 is computed by sum up

\[
P'_m, \quad m = 1, 3, 5, 7, 9, 11, 13, 15
\]

Thus, the likelihood ratio of a bit is obtained

![16QAM constellation with Gray mapping](image)

Figure 4. 16QAM constellation with Gray mapping
\[
L(b_j) = \frac{\Pr\{b_j = 1 | (r_x, r_y)\}}{\Pr\{b_j = 0 | (r_x, r_y)\}} = \sum_{j \in U_1} P_j \sum_{j \in U_0} P_j
\] (11)

Here \(U_1\) denotes the symbols ensemble that the bit \(b_j\) of the symbol is 1 and \(U_0\) denotes the symbols ensemble that the bit \(b_j\) is 0.

C. Simulation Results and Analysis

We applied regular and irregular LDPC codes with 16QAM/64QAM constellation in image transmission over Rayleigh Channel, and examined the performances by PSNR (Peak-Signal-Noise-Ratio), BER (Bit-Error-Ratio) and the visual images of the recovered images, respectively.

For 16QAM scheme, Tab. 1 is the PSNR values with different SNR (\(E_b / N_0\)). Fig. 5 depicts the BER curves of the two types of LDPC schemes. And Fig. 6 gives the recovered visual images when SNR is 5.4 dB respectively.

**TABLE I. PSNR VALUE OF THE LDPC CODES WITH 16QAM**

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>5.2</th>
<th>5.4</th>
<th>5.6</th>
<th>5.8</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR (Code1)</td>
<td>9.98</td>
<td>11.51</td>
<td>15.30</td>
<td>20.68</td>
<td>32.88</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>4.6</th>
<th>4.8</th>
<th>5.0</th>
<th>5.2</th>
<th>5.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR (Code2)</td>
<td>12.10</td>
<td>17.54</td>
<td>28.33</td>
<td>31.81</td>
<td>37.25</td>
</tr>
</tbody>
</table>

For 64QAM modulation, Tab. 2, Fig. 7 and Fig. 8 are the results of 64QAM scheme.

**TABLE II. PSNR VALUE OF THE LDPC CODES WITH 64QAM**

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>9.4</th>
<th>9.8</th>
<th>10</th>
<th>10.2</th>
<th>10.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR (Code1)</td>
<td>13.41</td>
<td>14.86</td>
<td>16.68</td>
<td>20.04</td>
<td>37.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SNR(dB)</th>
<th>8.8</th>
<th>9.2</th>
<th>9.6</th>
<th>9.8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSNR (Code2)</td>
<td>13.71</td>
<td>19.57</td>
<td>25.43</td>
<td>32.09</td>
<td>38.08</td>
</tr>
</tbody>
</table>

From the simulation results, we see that the LDPC channel coding schemes with 16QAM/64QAM can achieve good performance in image transmission over Rayleigh fading channel. We can obtain high spectrum efficiency as well as high reliability at the same time. The distorted images can be nearly recovered by using regular Code 1 and irregular Code 2 with 16QAM at SNR about 6.0 dB, 5.2 dB respectively. When 64QAM modulation is utilized, at SNR about 10.6dB and 9.8dB the images are nearly recovered.

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At the same time, we can see the performance distinction among the regular LDPC codes and irregular ones.

When BER is lower than \(10^{-4}\), the irregular code absolutely surpasses the regular code. Code 2 has a coding gain of about 0.7 dB than Code 1. Code 2 works better than Code 1 in image error-correcting system. For example, for the 16QAM scheme, at SNR 5.4 dB the PSNR value of Code 1 is 11.54, while the PSNR of Code 2 is 37.25. At the same time, from Fig. 6, we can see that at SNR 5.4dB irregular Code 2 corrected the image perfectly but regular Code 1 did not.

However, when the BER is beyond \(10^{-4}\), the BER curve of regular Code 1 comes to zero rapidly. Yet the curve of Code 2 change slowly, even didn’t change. There always exist few errors that couldn’t be corrected by Code 2.

All these can be explained by their degree distributions. For
Code 1, $\lambda_3 = 1, \rho_6 = 1$, i.e. all the variable node degree is 3 and all the check node degree is 6. But Code 2 has a lot of variable nodes whose degree is 2. Error correcting for these lower degree nodes is more difficult than that of higher degree nodes. So, there is an error floor for the irregular LDPC codes [7].

IV. CONCLUSION

From the above simulation results and analysis, we can draw the following conclusions:

1) At relatively lower SNR and relatively high spectrum efficiency, the noised images can be renewed well through LDPC codes. Therefore LDPC codes can be applied in both power and bandwidth limited region to overcome the noise. And with the parallel decoding algorithm and the interleaver omitted, the system complexity is largely reduced.

2) When BER is lower than $10^{-4}$, the irregular code outperforms the regular code both with 16QAM and 64QAM modulation. Code 2 has an advantage about 0.7dB than Code 1. And at BER $10^{-4}$, the images are already recovered perfectly. So the irregular LDPC codes are more effective than the regular codes in image transmission.

3) Because there are variable nodes with relatively lower degree, the irregular codes have error flat floor at the BER about $10^{-3}$, while the BER of regular codes comes to zero rapidly.

4) Just because of there are variable nodes with high degree the irregular LDPC codes possess the Unequal Error Protection characteristic. The important data bits can be arranged with higher degree.

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