A Novel Iterative Equalization Algorithm for Multicode CDMA System with V-BLAST Architecture

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Abstract—In this paper, a space-time iterative equalization algorithm for multi-code CDMA system with V-BLAST architecture (MC-VBLAST) is proposed. Also, a novel iterative equalization structure is obtained. The receiver performs two successive soft input soft output model, first from a MMSE equalizer and then from a Turbo decoder. Soft information got from previous iteration is used as a priori information for the next iteration. Simulation results demonstrate that the proposed algorithm offers significant performance gain over traditional, non-iterative receiver structures.

Keywords: CDMA, V-BLAST, MMSE, Turbo detection

I. INTRODUCTION

Space time codes have begun to be considered for wideband code division multiple access (CDMA) downlink systems to provide high data rate packet services. An attractive scheme for higer data rate is to combine multi-code CDMA with V-BLAST architecture (MC-VBLAST) [1] [2]. The scheme [3] [4] is used to achieve higher data rates than the conventional single antenna CDMA systems. In this scheme, multicode CDMA allows simultaneous data transmission over multiple spreading code channels destined for a single user. Meanwhile, on each spreading code channel, V-BLAST architecture offers multiple spatial data transmission.

In this paper we study the low complexity iterative equalization algorithm for MC-VBLAST system. Firstly, block-wise MMSE-based soft-in soft-output (SISO) detector based on [5] [6] is developed. We derive low complexity SISO equalization algorithm for MC-VBLAST block transmission system. This equalization algorithm is not only migrate the multiple access interference (MAI) but also co-antenna interference (CAI). Finally, simulation results show that the proposed algorithm significantly outperform the conventional coded MC-VBLAST system [3] [7].

The organization of the paper is as follows: The architecture of MC-VBLAST system is introduced in Section II. A MMSE-based SISO detector is studied in Section III Simulation results are presented in Section IV. Finally, the conclusion is given in Section 5.

II. SYSTEM MODEL

The notation adopted in this paper conforms to the following convention. Vectors are column vectors and are denoted in lower case bold: $\mathbf{x}$. Matrices are upper case bold: $\mathbf{A}$.

$D_m$ denotes the identity matrix of size $k \times k$. $e_i$ denotes the $i$th unit vector. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ represent conjugate, transpose, and Hermitian transpose, respectively. $\otimes$ denotes the Kronecker product. $E(\cdot)$ denotes the expected value operator. The covariance operator is given by $\text{Cov}(\mathbf{x}, \mathbf{y}) \triangleq E(\mathbf{x}\mathbf{y}^H) - E(\mathbf{x})E(\mathbf{y}^H)$. $\text{diag}\{\mathbf{a}\}$ stands for a diagonal matrix with $\mathbf{a}$ on its diagonal.

The MC-VBLAST system block diagram is shown in Fig. 1. The high data rate information sources, which have been turbo encoded, rate matched and interleaved, are demultiplexing into $NP$ substreams. $N$ is the number of transmit antennas. $P$ is the number of the spreading code channels. The $p$th $(p = 1, 2, \cdots, P)$ group of $N$ substreams is spread by the $p$th spreading code. The $m$th $(m = 1, 2, \cdots, M)$ substream of each group is summed, scrambled and transmitted over the $m$th antenna. Orthogonal pilot signals are also assigned to each antenna for channel estimation. The channel is modeled as frequency selective Rayleigh fading model with $L$ paths. After chip match filtering and sampling, the discrete signals received on the $m$th $(m = 1, 2, \cdots, M)$ receive antenna is given by

$$\mathbf{r}_m = \mathbf{D}_m \mathbf{G}_m \mathbf{A} \mathbf{s} + \eta_m$$

Where $\mathbf{r}_m \in \mathbb{C}^J$, $J$ is the processing gain. $\mathbf{D}_1 = \cdots = \mathbf{D}_M = [\mathbf{d}_1, \cdots, \mathbf{d}_P] \in \mathbb{C}^{J \times LP}$ is the spreading matrix, where $\mathbf{d}_p = [\mathbf{d}_{1,p}, \cdots, \mathbf{d}_{LP,p}]$ where $\mathbf{d}_{l,p}$ is the energy normalized spreading sequence of $p$th spreading code channel over $l$th path, which is the product of spreading code and scrambling code. $\mathbf{G}_m = \mathbf{I}_P \otimes \mathbf{g}_m \in \mathbb{C}^{LP \times NP}$ is complex channel coefficient matrix. Where $\mathbf{g}_m = [\mathbf{g}_{1,m}^T, \mathbf{g}_{2,m}^T, \cdots, \mathbf{g}_{NP,m}^T]^T$, $\mathbf{g}_{l,m} = [g_{1,1,m}, \cdots, g_{1,L,m}]$, $g_{n,l,m}$ is the complex channel coefficient.
between \(n\)th transmit antenna and \(m\)th receive antenna over \(l\)th path. \(A = \text{diag} [A_{1,1}, \ldots, A_{N,1}, \ldots, A_{N,N}] \in \mathbb{R}^{N \times NP}\) is assumed equal for different data sub-streams, \(E_0\) is the average power per data stream. \(s = [s_{1,1}, \ldots, s_{N,1}, \ldots, s_{N,N}]^T \in \Phi^{NP}\) is the data vector with the modulation symbol alphabet \(\Phi\). \(n_m \in C^J\) is the complex channel noise vector with variance \(N_0/2\) per dimension. Stacking the received vectors to form a \(NM\)-dimensional vector, we can get the received signal model as follows

\[
r = Hs + n,
\]

respectively, where \(H = DGA, \quad r = [r_0^T, r_1^T, \ldots, r_{N-1}^T]^T, \quad D = [D_0^T, D_1^T, \ldots, D_{M-1}^T]^T, \quad G = [G_0^T, G_1^T, \ldots, G_{M-1}^T]^T, \quad n = [n_0^T, n_1^T, \ldots, n_{M-1}^T]^T\).

III. ITERATIVE EQUALIZATION ALGORITHM FOR MC-VBLAST SYSTEM

The basic idea of iterative equalization is to use a SISO detector, which is able to accept not only channel values but also apriori information about the symbols to be detected. This apriori information can be obtained in various ways, e.g. from the feedback of an outer decoder. At the same time, the outer decoder is also a SISO decoder, e.g. using BCJR algorithm [8], [9] and Viterbi algorithm [10]. The SISO detector can be classified as MAP/ML and MMSE detectors. For channels with large delay spreads and for large constellation sizes, MAP/ML-based detector suffers from impractically high computational complexity. So the MMSE-based SISO detector proposed by Wang and Poor [5] for turbo multiuser detection and its extended version by Reynolds [11] and Tuchler [12], [13] for iterative equalization are very attractive.

From the Gauss-Markov theorem [14], the MMSE solution of (2) is given by

\[
\hat{s} = \hat{s} + VH[HVH + \sigma^2I_{KM}]^{-1}(r - H\hat{s}),
\]

(3)

where \(\hat{s} \triangleq \hat{E}(s), \quad \text{Cov}(n, n) = \sigma^2I_{KN}, \quad V \triangleq \text{Cov}(s, s) = \text{diag}(v_0, \ldots, v_{KN-1})\). Without the existence of apriori information, we assume \(\hat{s} = 0, \quad V = I_{KN}\). Then equation (3) becomes the usual solution for linear MMSE equalization. When transmitted symbols are unknown, it is possible to get the mean and the variance from coded data LLRs provided by the SISO channel decoder of the previous iteration. The mean and the variance [5], [12]

\[
\hat{s}_k = \sum_{\alpha(d)} \alpha(d)P[s_k = \alpha(d)],
\]

\[
v_k = \sum_{\alpha(d)} |\alpha(d)|^2 P[s_k = \alpha(d)] - |\hat{s}_k|^2,
\]

of the transmitted symbols \(s_k, k = 0, 1, \ldots, KN - 1\) are functions of the apriori information \(L(\hat{x}_{k,i}), i = 0, 1, \ldots, M_c - 1\) (\(M_c\) is the number of bit per constellation symbol) since

\[
P[s_k = \alpha(d)] = \prod_{i=0}^{M_c-1} P(x_{k,i} = d_i) = \prod_{i=0}^{M_c-1} \frac{\exp\left[\tilde{d}_iL(\hat{x}_{k,i})\right]}{1 + \exp\left[\tilde{d}_iL(\hat{x}_{k,i})\right]},
\]

where

\[
\tilde{d}_i \triangleq \begin{cases} 1, & d_i = 1 \\ -1, & d_i = 0 \end{cases}.
\]

Here, \(d\) is an \(M_c \times 1\) vector of data bits, and \(\alpha(d)\) is a constellation symbol which is obtained using the mapping function (e.g., Gray mapping). From (3), the estimate symbol of \(\hat{s}_k\) depends on all of the apriori information. In order that \(\hat{s}_k\) is independent from the apriori, we redefine the vector \(\hat{s}\) and matrix \(V\) as follows

\[
\hat{s}_j = \begin{cases} 0, & j = k \\ \hat{s}_j, & j \neq k \end{cases} \quad [V]_{j,j} = \begin{cases} 1, & j = k \\ v_j, & j \neq k \end{cases}.
\]

Then the estimate of the \(k\)th transmitted symbol \(\hat{s}_k\) can be written as

\[
\hat{s}_k = e_k^H H^H \left[HVH + \sigma^2I_{KM} + (1 - v_k)H e_k e_k^H H^H \right]^{-1} \left[r - H(\hat{s} - e_k \hat{s}_k)\right] - e_k^H \hat{s}_k^H
\]

(8)

Defining the matrix \(\Sigma \triangleq \sigma^2I_{KM} + HVH\) in (8) and applying the matrix inversion lemma [15], we can write

\[
\hat{s}_k = \frac{e_k^H H^H \Sigma^{-1}}{1 + (1 - v_k) e_k^H H^H \Sigma^{-1} H e_k} \left[r - H(\hat{s} - e_k \hat{s}_k)\right] = \kappa_k w_k^H \left[r - H \hat{s} + H e_k \hat{s}_k\right] = \kappa_k \hat{s}_k'
\]

(9)

where

\[
w_k^H = e_k^H H^H \Sigma^{-1}, \quad \rho_k = e_k^H H^H \Sigma^{-1} H e_k,
\]

\[
\kappa_k = [1 + (1 - v_k) \rho_k]^{-1},
\]

\[
\hat{s}_k' = w_k^H \left[r - H \hat{s} + H e_k \hat{s}_k\right].
\]

Let \(\hat{s}' = [\hat{s}_0', \hat{s}_1', \ldots, \hat{s}_{KN-1}']^T\) and \(\rho = [\rho_0, \rho_1, \ldots, \rho_{KN-1}]^T\), and then we get

\[
\hat{s}' = H^H \left[\sigma^2I_{KM} + HVH\right]^{-1} \left[r - H \hat{s} + \text{diag}(\rho) \hat{s}\right]
\]

(10)

Using another matrix inversion lemma

\[
CD(A + BCD)^{-1} = (C^{-1} + DA^{-1}B)^{-1}DA^{-1},
\]

we can express (10) as

\[
\hat{s}' = (\sigma^2I_{KN} + H^H HV)^{-1} \left[H^H r - H^H H \hat{s} + \text{diag}(\rho) \hat{s}\right],
\]

(12)

where

\[
\rho_k = e_k^H (\sigma^2I_{KN} + H^H HV)^{-1} H^H H e_k.
\]

(13)
If we assume the output of the detector represents the output of an equivalent AWGN channel having \( s_k \) as its input symbol, we may write [5]

\[
\hat{s}_k = \mu_k s_k + \tilde{n}_k,
\]

where \( \mu_k \) is the equivalent amplitude of the output signal and \( \tilde{n}_k \sim N(0, \sigma^2_{\tilde{n}_k}) \) is a Gaussian noise sample. We may compute the parameters \( \mu_k \) and \( \sigma^2_{\tilde{n}_k} \) as follows [12]

\[
E[\hat{s}_k | s_k = \alpha(d)] = \kappa_k \rho_k \alpha(d) = \mu_k \alpha(d),
\]

\[
\sigma^2_{\tilde{n}_k} = \text{Cov}[\hat{s}_k, s_k = \alpha(d)] = \kappa_k \rho_k (1 - v_k \rho_k).
\]

Then the extrinsic information delivered by the SISO detector is given by [12]

\[
L_e(\hat{x}_{k,i}) = \log \sum_{\hat{d} \in S_{i,1}} \exp \left[ \frac{-|\hat{s}_k - \mu_k \alpha(d_k)|^2}{2\sigma^2_{\tilde{n}_k}} + \frac{1}{2} \tilde{d}_{[i]}^T L_{[i]} \right]
\]

\[
- \log \sum_{\hat{d} \in S_{i,0}} \exp \left[ \frac{-|\hat{s}_k - \mu_k \alpha(d_k)|^2}{2\rho_k (1 - \rho_k \rho_k)} + \frac{1}{2} \tilde{d}_{[i]}^T L_{[i]} \right]
\]

\[
= \log \frac{\sum_{\hat{d} \in S_{i,1}} \exp \left[ \frac{-|\hat{s}_k - \mu_k \alpha(d_k)|^2}{2\rho_k (1 - \rho_k \rho_k)} + \frac{1}{2} \tilde{d}_{[i]}^T L_{[i]} \right]}{\sum_{\hat{d} \in S_{i,0}} \exp \left[ \frac{-|\hat{s}_k - \mu_k \alpha(d_k)|^2}{2\rho_k (1 - \rho_k \rho_k)} + \frac{1}{2} \tilde{d}_{[i]}^T L_{[i]} \right]}
\]

(17)

where \( S_{i,1} = \{d_{i|1} = 1\}, S_{i,0} = \{d_{i|1} = 0\} \).

(18)

\( \tilde{d}_{[i]} \) denotes the subvector of \( \tilde{d} \) obtained by omitting its \( i \)th element \( \tilde{d}_{i|1} \), and \( L_{[i]} \) denotes the vector of all the apriori information values, also omitting \( L(\hat{x}_{k,i}) \). Notice that \( \hat{s}_k, \mu_k \) and \( \sigma^2_{\tilde{n}_k} \) all have the nonnegative number \( \kappa_k \), and then it can be cancelled in computing (17). That is the reason why we use (12) as the final detection results. The complexity of (17) can be reduced by using the Logarithm Jacobian [16], then the extrinsic information of (17) becomes

\[
L_e(\hat{x}_{k,i}) \approx \max_{d \in S_{i,1}} \left\{ \frac{-|s_k' - \rho_k \alpha(d)|^2}{2\rho_k (1 - \rho_k \rho_k)} + \frac{1}{2} \tilde{d}_{[i]}^T L_{[i]} \right\}
\]

\[
- \max_{d \in S_{i,0}} \left\{ \frac{-|s_k' - \rho_k \alpha(d)|^2}{2\rho_k (1 - \rho_k \rho_k)} + \frac{1}{2} \tilde{d}_{[i]}^T L_{[i]} \right\}
\]

(19)

The extrinsic information of the SISO detector will be de-interleaved to be used as the input to SISO decoder.

A standard binary turbo decoder generates the soft decisions only for the information bits, the extrinsic part of which are to be used in the next iteration as a priori information. In this design, the soft decisions for the coded bits as well as information bits are needed for the iterative detection. Let \( x_{k,i}, i = 0, 1, \ldots, u \) be the information bits and the coded bits associated with the transition from time \( k - 1 \) to \( k \). And let the trellis states at level \( k - 1 \) and at level \( k \) be denoted by the integer \( \zeta' \) and \( z \) respectively. Then for the bit \( i \) (information bit or coded bit), the soft decision can be calculated by

\[
L_D(\hat{x}_{k,i}) = \log \frac{P(x_{k,i} = 1 | y)}{P(x_{k,i} = 0 | y)} = \log \frac{\sum_{\{z', \zeta, z_{k,i} = 1\}} p(z', z, y)}{\sum_{\{z', \zeta, z_{k,i} = 0\}} p(z', z, y)}
\]

(20)

IV. SIMULATION RESULTS

We use simulation results to demonstrate the performance of the proposed iterative equalization algorithm for MC-VBLAST system. In all simulations, Walsh code is used as spreading code and complex PN sequence with length \( 2^{15} - 1 \) is used as scrambling code. The spreading factor is assumed 32 and a rate \( R = 1/3 \) turbo code of memory 3 with (recursive) feedback polynomial \( 1 + D^2 + D^3 \) and feedforward generator polynomial \( 1 + D + D^3 \) is used. The data frame consists of 3200 data bits and 3 tail bits. Both the inner interleaver of turbo encoder and the outer interleaver use the bit S-random interleavers [17] with \( S = 78 \) and \( S = 110 \), respectively. After the Gray mapped 16-QAM, the number of spreading code channel is 5, each antenna block which consists of 3840 data chips is formed. We assumed perfect channel estimation and timing acquisition. The channel between each transmit and receive antenna are assumed to be independent flat quasi-static Rayleigh fading channel. Two equal power quasi-static Rayleigh fading paths are considered and the time delay between two paths is one chip. The number of decode for turbo code is four times and the algorithm is Log-MAP.

The performance of different number of transmitter antenna and receive antenna are compared, such as \( (N, M) \) is \( (4, 4) \) and \( (4, 8) \) in Fig.2. The simulation results show that the proposed iterative equalization algorithm is better than non-iterative detection algorithms. Moreover, the system performance will be improved as the number of iterative equalization is increased. The performance of iterative equalization algorithm’s number between 4 or 5 are not distinctness. There are less 0.2dB gains at BER \( 10^{-4} \) when the number of turbo detection increased from 4 to 5 in Fig.2. For practical application, we had better choose the appropriate number of iterative equalization for decrease the computation complexity.

The proposed iterative equalization algorithm exists the tradeoff between the number of decode and the number of iterative detection. If no limitation on complexity, the more the number of turbo decode the better the system performance. However, after several turbo decode, the performance will be increased slowness as increase the number of iterative equalization. Meanwhile, if turbo code adopt one decode, it will be also increase the system performance when increase the number of detection, but converge of BER will be very slow. It is difficult to analysis the problem other than simulation. We compare four schemes pair (the number of decode, the number of iterative equalization): I(1, 8), II (2, 4), III (3, 4), IV (4, 4). From the computation complexity, scheme I needs 9 times decode and 8 times iterative equalization, scheme II needs 10 times decode and 4 times iterative equalization, scheme
III needs 15 times decode and 4 times iterative equalization, scheme IV needs 20 times decode and 4 times iterative equalization. The performance of scheme IV is the best, but the complexity is also the highest. The complexity of II and scheme III is higher than that of I, but the performance is far better than I. One time decode for turbo code is not enough to make full use of the advantage of Turbo code. So iterative equalization cannot get enough soft information, the system performance will be improved slowly. We choose the scheme III to get the tradeoff between the complexity and performance and only lost BER gain 0.2dB compared with the scheme IV.

V. CONCLUSIONS

In this paper we have proposed a low complexity iterative equalization algorithm for MC-VBLAST system. The performance of the proposed algorithm at different iterative time is studied. The system performance of different number of equalization and channel decoding is also studied. Simulation results show that the proposed receivers significantly outperform the conventional MC-VBLAST system with the rate-1/3 turbo code. This iterative equalization can be applied for future high data rate wireless mobile communication system.

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Fig. 2: BER Performance comparison of Ant. Pair (4,4)

Fig. 3: BER Performance comparison of different iterative equalize and decode.